A Directional Audible Sound System using Ultrasonic Transducers

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***Abstract*—In general the audible sound has the characteristics of spreading, however the ultrasound is directional. This study used amplitude-modulating technique for an array of 8 ultrasonic transducers to produce directional audible sound beam. In this study sound field distribution for the directional audible sound beam has been investigated. The effect of different weightings varied with different frequency for the transducers on the directivity of the sound beam has also been evaluated. An H_∞ optimization method was used to calculate the optimal weightings of the transducers for better directivity of the sound beam. Different optimal weightings also added to the carrier and sideband frequencies to control the difference frequency's beam width and side lobe amplitude. The results showed that the beam width can be controlled and good directivity of the sound beam can be obtained by using the H_∞ optimization method.**

Keywords—ultrasound; amplitude-modulating; directional audible sound beam; weightings; H_∞ optimization method

I. Introduction

Generally, the audible sound frequency is the range from 20 Hz to 20 kHz; however, ultrasound is sound pressure with a frequency greater than the upper limit of human hearing i.e. 20 kHz as shown in Fig. 1. Recently the highly directional audible sound has been investigated for a few years. The audible sound has the characteristics of spreading, however the ultrasound is directional.

In the pass decades, the performance in the ultrasound systems and their beamforming has been studied. Beamforming is the concept of forming directional beams. Previous study [1] has discussed that two plane waves of different ultrasonic frequencies could generate the directional audible sound due to acoustical nonlinearity. There are mainly two new waves, one of which has a frequency equal to the decrease of the original two frequencies and the other equal to the difference frequency.

New ultrasonic waves whose frequencies correspond to the decrease and difference of the two ultrasonic signals will be produced. The nonlinear interaction of ultrasonic sound waves in air will produce this phenomenon. For instance, if there are two ultrasonic signals in the air at closely frequencies f_0 and f_1 as shown in Fig. 2, they will be transformed into 2 f_0, 2 f_1, $f_1 \pm f_0$ and other higher order harmonics in the signal [2]. The new components of 2 f_0, 2 f_1 and $f_1 + f_0$ in the air will be strongly attenuated rapidly with increasing distance from the transducer. However, the remaining frequency $f_1 - f_0$ is decay slowly with increasing distance from the transducer because of the relatively low absorption.

For simplicity, the ultrasonic transducer is fed with two ultrasonic signals f_0 at 40 kHz and f_1 at 41k Hz, then the new modulation frequency is 1 kHz ($f_1 - f_0$). The new frequency component of 1 kHz has lower absorption than other high-frequency terms. For this reason, this study will not discuss the new high-frequency terms. Instead, it will focus on and discuss the frequency subtraction component $f_1 - f_0$ of new frequency components.

Traditional in-car communication gadgets, such as cell phones or radios, generally adopt traditional loudspeakers to broadcast. This has several weaknesses. For instance, people in the car can all hear the sound. This makes it impossible to maintain privacy. Besides, people feel bothered and thus the ride quality would not be so pleasant. If cars are equipped with directional loudspeakers, each passenger can hear the different music or information. Thus, people would not feel bothered. Before directional loudspeakers are applied to vehicles, it is necessary to do through research on audible sound beams. Most of efforts were devoted to the preprocessing methods [3] and speaker design [4] for improving the system performance of the directional sound beam. On the other hand, several model equations have been presented to describe the propagation of finite-amplitude sound beams from the parametric array, which has been reviewed in Ref. [5]. More works were refer to the directional audible sound by using the parametric array [6, 7]. Yoneyama used an array of transducers to demodulate the broadband sound signals with reasonable loudness, and called the audio spotlight [5]. However, the approaches of beamforming require complex mathematics equations. As an alternative approach, the ultrasonic field is presented to a newly numerical computation [8, 9]. Also J. Yang. Et. [2] presented an algorithm to discuss the weighted parametric array.

Fig. 1. The range of ultrasound

There has been little work done on the beam width control in directional loudspeakers. Without the beam width control, the directivity of the sound beams is fixed, and it cannot meet the various needs while applying the directional loudspeakers to vehicles. In order to find a way to better control the directivity of the sound beams, papers concentrated on this issue have been closely reviewed so far, and the Chebyshev method has proven to be the only effective method on this issue [2, 8, 9].

However, this paper discusses the theoretical simulation controlling the directivity of the sound beams using the uniform linear array (ULA) composed of eight transducers. An H_∞ optimization method is used to investigate the weighting distribution of uniform linear array, and the influence on the spreading angle of the sound beam. The method proposed in the study can improve the performance of the directional audible sound for all frequencies. The optimal weighting values of the transducers corresponding to specific sound beam spreading angle can be used to control the beam width and the amplitude of the side lobe.

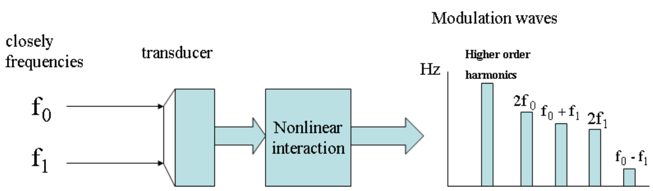

Fig. 2. Nonlinear phenomenon in the air

II. Design of Directional Audible Sound Systmes

In this section the formulation for designing directional audible sound systems using an H_∞ control method is presented. The directional audible sound can be expressed by the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation which accurately describes the diffraction, absorption, and nonlinearity of ultrasound in a parametric array as follows [10].

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_0}{2}\nabla_\perp^2 p + \frac{\delta}{2c_0^3}\frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^2 p^2}{\partial z^2}, \tag{1}$$

where

- p = acoustic pressure
- z = coordinate along the axis of the beam propagation direction
- τ = retarded time
- c_0 = small signal sound speed
- ρ_0 = ambient density
- δ = sound diffusivity
- β = coefficient of nonlinearity
- $\nabla_\perp^2$ = transverse Laplacian operator

Because of this successive approximations, a quasi-linear solution of the form $p = p_1 + p_2$ is assumed. Therefore p_1 is the linear solution of (1) for the primary pressure at frequency ω and p_2 is a small correction to p_1 at the second-harmonic frequency 2ω.

The definition becomes:

$$\begin{aligned}\mathbf{p}_1(\mathbf{r},\mathbf{z},\tau) = \frac{1}{2\mathbf{j}}[\mathbf{q}_{1\mathrm{a}}(\mathbf{r},\mathbf{z})\mathbf{e}^{\mathrm{j}\omega_a\tau} \\ + \mathbf{q}_{1\mathrm{b}}(\mathbf{r},\mathbf{z})\mathbf{e}^{\mathrm{j}\omega_b\tau}] + \mathbf{c.c.},\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{p}_2(\mathrm{r},\mathrm{z},\tau) = \frac{1}{2\mathrm{j}}[\mathrm{q}_{2\mathrm{a}}(\mathrm{r},\mathrm{z})\mathrm{e}^{\mathrm{j}2\omega_a\tau} + \mathrm{q}_{2\mathrm{b}}(\mathrm{r},\mathrm{z})\mathrm{e}^{\mathrm{j}2\omega_b\tau} \\ + \mathrm{q}_+(\mathrm{r},\mathrm{z})\mathrm{e}^{\mathrm{j}\omega_+\tau} + \mathrm{q}_-(\mathrm{r},\mathrm{z})\mathrm{e}^{\mathrm{j}\omega_-\tau}] + \mathrm{c.c.},\end{aligned} \tag{3}$$

Then q_1 and q_2 are the complex pressure amplitudes, *c.c.* denotes the complex conjugate of preceding terms.

q_1 can be obtained from a homogeneous equation as:

$$\frac{\partial q_1}{\partial z} + \frac{i}{2k}\nabla_\perp^2 q_1 + \alpha_1 q_1 = 0 \tag{4}$$

Then an inhomogeneous equation for q_2 is presented as:

$$\frac{\partial q_2}{\partial z} + \frac{i}{4k}\nabla_\perp^2 q_2 + \alpha_2 q_2 = \left(\frac{\beta k}{2\rho_0 c_0^2}\right) q_1^2 \tag{5}$$

Where $k=\omega/c$ becomes the wave number, whereas $\alpha_n=\delta n^2\omega/2c^3$ is the thermos viscous attenuation coefficient at frequency $n\omega$. For a parametric speaker, we are only interested in the audible sound beam at difference frequency. Assume that the sound beam is caused to be at difference frequency $\omega_- = \omega_a - \omega_b$ ($\omega_a > \omega_b$), then ω_a and ω_b become the two primary frequencies. The result of the complex pressure amplitude q_- is:

$$q_-(r',z') = -\frac{\pi\beta k_-}{\rho_0 c_0^2}\int_0^z\int_0^\infty q_{1a}(r',z')\, q_{1b}^*(r',z')\, G_-(r,z|r',z')\, r'dr'dz' \tag{6}$$

Thus $k_- = \omega_-/c$, q_{1a} q_{1b} are the complex pressure amplitudes for frequencies ω_a and ω_b, relatively, G-(r, z|r ______, z ______)is the Green's function presented as:

$$G_-(r,z|r',z') = \frac{ik_-}{2\pi(z-z')}J_0\left(\frac{k_- rr'}{z-z'}\right)\exp\left[-\alpha_-(z-z') - \frac{ik_-(r^2+r'^2)}{2(z-z')}\right] \tag{7}$$

Then $\alpha_- = \delta\omega_-^2/2c^3$, and J_0 become the zeroth-order Bessel function. Being simple, we consider the sound beams produced by a primary source with Gaussian amplitude shading. While the complex pressure amplitude $q_1(r, 0)$ at primary frequency can be determined as:

$$q_1(r,0) = p_0 \exp\left[-\left(\frac{r}{a}\right)^2\right] \tag{8}$$

On the other hand p_0 is the peak source pressure and a is the effective source radius.

Then, the linear solution can be deduced as:

$$q_1(r,z) = \frac{p_0 e^{-\alpha_1 z}}{1 - iz/z_0}\exp\left[-\frac{(r/a)^2}{1 - iz/z_0}\right] \tag{9}$$

Where $Z_0 = \frac{1}{2}ka^2$.The far-field solution of the directivity is given by

$$D_1(k,\theta) = \exp\left[-\frac{1}{4}(ka)^2 \tan^2\theta\right] \tag{10}$$

Assume that we only considered one far-field, according to a bi-frequency Gaussian source, the far-field directivity of different frequency is described by the product of the directivity function, i.e.:

$$D_{-}(\theta) = D_{1a}(\theta) D_{1b}(\theta) \tag{11}$$

Thus, for a bi-frequency Gaussian source the far-field directivity of the difference frequency is described by the product of the directivity functions of the primary waves.

Assume that a group of M ultrasonic transducers is arranged in a uniform linear array (ULA) with an inter element spacing of d as shown in Fig. 3. An observation point is set in the far-field of the array at an angle, θ with respect to the normal of the transducer array aperture. If each transducer is weighted with a weighting, ω_m for m=0, 1, 2, . . ., M−1, the array response function [11] can be derived as:

$$H(\omega\tau) = \frac{1}{M}\sum_{m=0}^{M-1} w_m e^{im\omega\tau} \tag{12}$$

Where,

$\tau = d/c \sin\theta_0$ is the time delay.

ω_m , m = 0,1,2,3…..M-1 is the weighting for each transducer

θ is the angel with respect to the axis of the beam.

In selecting proper weightings, the side lobes level can be made suitable at the expense of the beam width of the main lobe. From (12), it showed that the maximum of the main lobe exists on the broadside of the ULA ($\theta = 0$). However, the maximum of the main lobe can be changed by adding a phase shift or delay to each transducer. If the ULA is to be steered in the direction θ_0, time delay ($m\tau_0$) has to be added to *m*th transducer before transmitting the signal into the air. The time delay τ_0 can be calculated as $\tau = (d/c)\times\sin\theta_0$, and the array response of the delay-and-sum beamforming becomes:

$$H(\omega\tau) = \frac{1}{M}\sum_{m=0}^{M-1} w_m e^{-im\omega(\sin\theta_0 - \sin\theta)d/c_0} \tag{13}$$

Then, the far-field directivity of the weighted primary sources array for frequency ω_a, $D_{1a}(\theta)$ can be appeared:

$$D_{1a}(\theta) = D_1(k_a,\theta) H(k_a,\theta) \tag{14}$$

When $D_1(k_a, \theta)$ is the aperture directivity shown in (10) for frequency ω_a, and the far-field array response $H(k_a, \theta)$ is indicated in (12) with w_{am} and ω_a instead of w_m and ω. Similarly, the far-field directivity for primary frequency ω_b, $D_{1b}(\theta)$ can be shown as:

$$D_{1b}(\theta) = D_1(k_b,\theta) H(k_b,\theta) \tag{15}$$

$D_1(k_b, \theta)$ and $H(k_b, \theta)$ are written as (10) and (12) for frequency ω_b with w_{bm} instead of w_{am}. Therefore, the beam pattern of the audible frequency is produced by substituting (12) and (13) into (14):

$$D_{-}(\theta) = D_1(k_a,\theta) H(k_a,\theta) D_1(k_b,\theta) H(k_b,\theta) \tag{16}$$

Thus, by substituting (10),(13) into (16) ,the directivity of far-field difference frequency equation can be written as:

$$\begin{aligned} D_{-}(\theta) = {} & \exp[-\frac{1}{4}(k_a * a)^2 \tan^2\theta] * \frac{1}{M}\sum_{m=0}^{M-1}\omega_m e^{-jm\omega(\sin\theta_0 - \sin\theta)d/c} * \\ & \exp[-\frac{1}{4}(k_b * a)^2 \tan^2\theta] * \frac{1}{M}\sum_{m=0}^{M-1}\omega_m e^{-jm\omega(\sin\theta_0 - \sin\theta)d/c} \end{aligned} \tag{17}$$

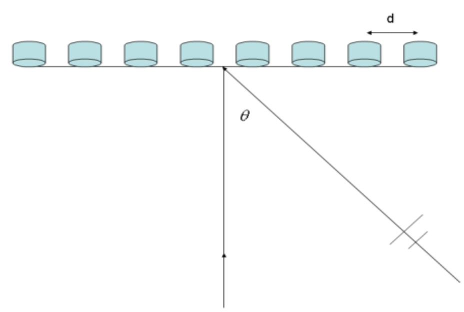

Fig. 3. Single array of M ultrasonic transducers

In this work the H_∞ optimization method is used to control the directivity of ultrasonic transducers. The design formulation can be written as:

Minimize σ
Subject to

$\|D_{-}(\theta_{sidelobe1})\|_\infty < \sigma$ $\quad -40<\theta<-1/2\Delta$

$\|D_{-}(\theta_{sidelobe2})\|_\infty < \sigma$ $\quad 1/2\Delta<\theta<40$

$\|D_{-}(\theta_{mainlobe})\|_\infty - \|D_{-}(\theta_{sidelobe1})\|_\infty > \varepsilon$ $\quad -1/2\Delta<\theta<1/2\Delta$

$\|D_{-}(\theta_{mainlobe})\|_\infty - \|D_{-}(\theta_{sidelobe2})\|_\infty > \varepsilon$ $\quad -1/2\Delta<\theta<1/2\Delta$ (18)

where

σ is the real number.

Δ is the beam width, and ε is the amplitude difference between the main lobe and side lobe.

Substituting (17) into (18) gets:

$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{sidelobe1}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe1})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2\theta_{sidelobe1}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe1})d/c}<\sigma$$

-40<θ<-1/2Δ

$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{sidelobe2}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe2})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2{}_{sidelobe2}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe2})d/c}<\sigma \qquad 1/2\Delta<\theta<40$$

$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{mainlobe}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{mainlobe})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2\theta_{mainlobe}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{mainlobe})d/c}-$$
$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{sidelobe1}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe1})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2\theta_{sidelobe1}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe1})d/c}>\varepsilon$$

-1/2Δ<θ<1/2Δ

$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{mainlobe}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{mainlobe})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2\theta_{mainlobe}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{mainlobe})d/c}-$$
$$\exp[-\frac{1}{4}(k_a*a)^2\tan^2\theta_{sidelobe2}]*\frac{1}{M}\sum_{m=0}^{M-1}w_a(f_a)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe2})d/c}*$$
$$\exp[-\frac{1}{4}(k_b*a)^2\tan^2\theta_{sidelobe2}]*\frac{1}{M}\sum_{m=0}^{M-1}w_b(f_b)e^{-jm\omega(\sin\theta_0-\sin\theta_{sidelobe2})d/c}>\varepsilon$$

-1/2Δ<θ<1/2Δ (19)

The performance of optimization is according to the different concerns of the constraints. During the simulation, different constrains would lead to different levels of performances. We used optimization method to find the optimal weightings which varied with frequency. The constraints in (19) are not the only possible types of constrains. Commonly, the different types of constrains can results in different performance, i.e. the beam width and amplitude of the side lobe in this work, for example.

III. Simulation Results

In this section the simulation results for the directivity of the audible sound beam created by using the H_∞ optimization method are presented, and then compared to those obtained by using Chebyshev weighting method [2]. In this study, the number of weightings is 16 for frequencies f_a and f_b as shown in Fig. 4. Therefore there are 16 design variables, i.e. $\{h_{a0}, h_{a1}, \dots, h_{a7}\}\{h_{b0}, h_{b1}, \dots, h_{b7}\}$,

The carrier frequency of the ultrasonic transducer array is set as 40 kHz, i.e. f_a = 40 kHz. The demodulated signal is at the frequency from 500 to 20,000 Hz with 500 Hz interval. A total of M = 8 ultrasonic transducer array is used with inter-element spacing, d = 9.7 mm. The effective source radius is set at a = 3.85 mm and the speed of sound c is 344 ms^{-1}. The weightings, h_{an} and h_{bn}, are calculated for difference frequency's beam width, $\theta\_ = 20^0$, 40^0 and 60^0 using the proposed method.

Figs. 5, 6 and 7 show the difference frequency's directivity for $\theta_{-} = 20^0$, 40^0 and 60^0 respectively. Figs. 5(a), 6(a) and 7(a) are the difference frequency's directivity using Chebyshev weighting method [8], and Figs. 5(b), 6(b) and 7(b) are the difference frequency's directivity using the optimization method for weightings varied with frequency proposed in the paper. From the figures it can be seen that the amplitude in the side lobe using the proposed method is lower than that using Chebyshev weighting method. This is because the optimization method tried to find the optimal weightings which minimize the amplitude of the side lobe and subject to the amplitude difference between the main lobe and the side lobe. Therefore the optimization method proposed in this paper performs better than Chebyshev weighting method. As can be seen from the figures the beam width can also be controlled using the optimization method. This is because the difference frequency's directivity is the product of two primary frequency's directivities, its beam width always takes on the narrowest beam width of the two primary waves. It can obviously be seen that a constant beam width is achieved for all frequencies using the proposed method.

The highest side lobe amplitude with the proposed method gets attenuated more compared to the method with Chebyshev weighting. This is since the amplitude of the side lobe region is minimized through all the frequencies. Therefore the lower side lobe amplitude is obtained using the propose method.

IV. Conclusions

In this paper the H_∞ optimization method has been proposed for designing directional audible sound beam. The directional audible sound beam was generated by using a uniform linear array composed of eight ultrasonic transducers with different weightings varied with frequency. The performance of the beam width control using the proposed method has been evaluated. It can be seen that the proposed method could effectively control the beam width and the level of the side lobes for the audible sound beam. It is verified by the simulation results that the lower side lobes level could be obtained by using the proposed method. Therefore the proposed method could improve the system performance compared to that with the Chebyshev weighting method.

Acknowledgment

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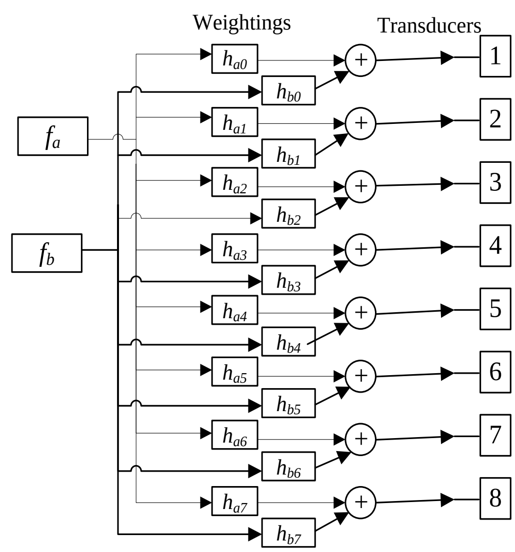
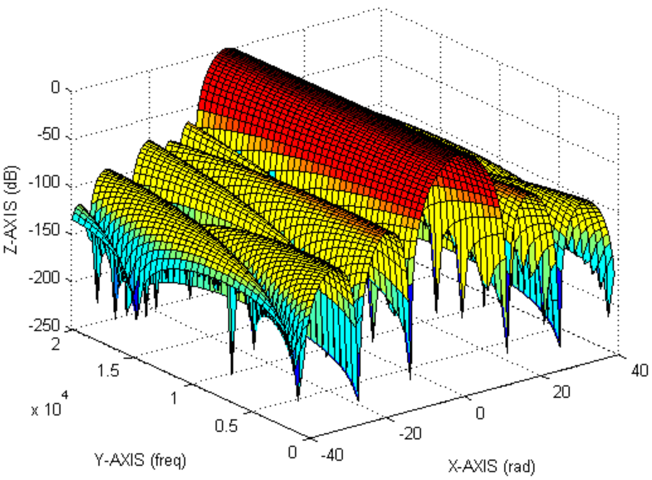
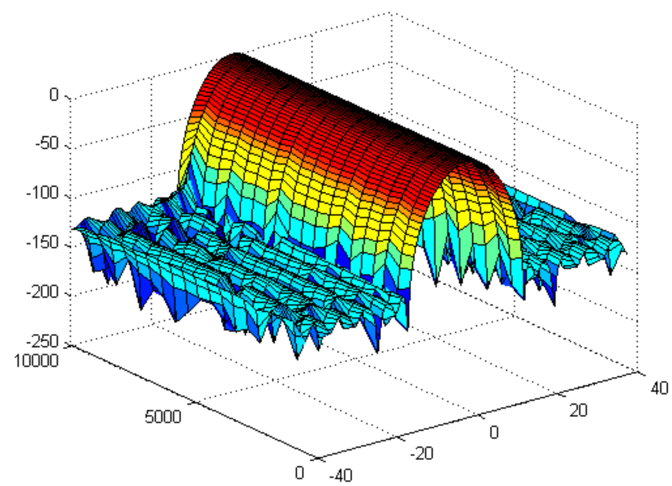


Fig. 4. Block diagram for beam width control systems with different weightings

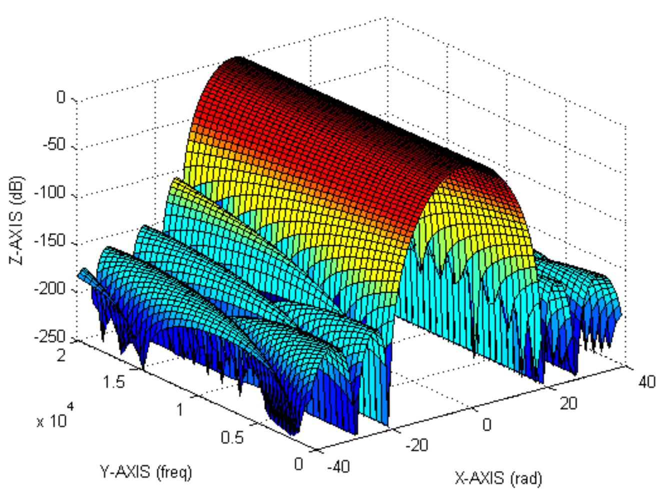


(a)



(b)

Fig. 5. Difference frequency's directivity for $\theta_{\_} = 20^{0}$. (a) Chebyshev weighting method. (b) Optimization method for weightings varied with frequency



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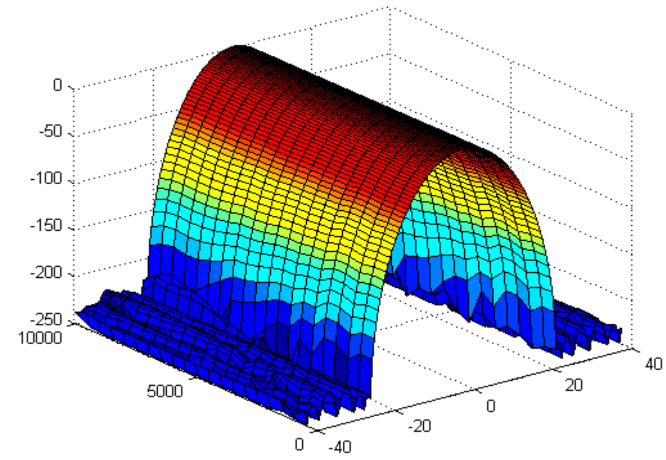
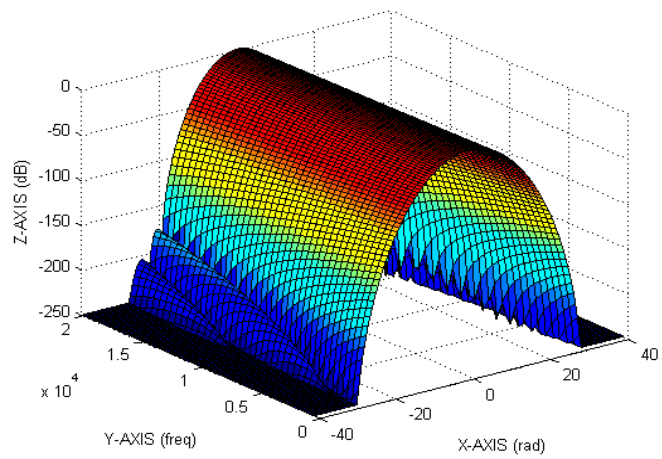
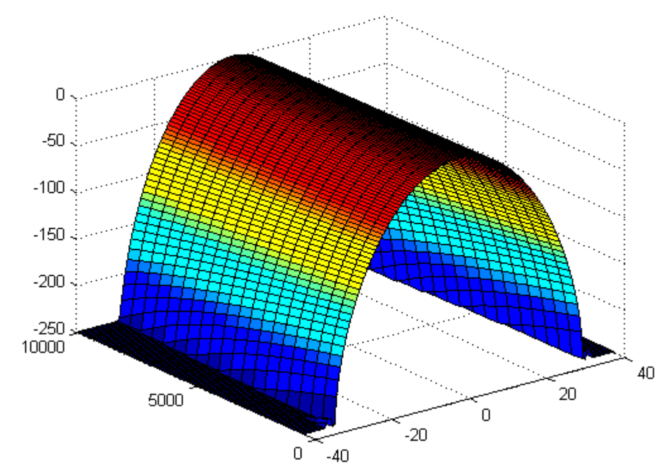


(b)

Fig. 6. Difference frequency's directivity for $\theta_{\_} = 40^{0}$. (a) Chebyshev weighting method. (b) Optimization method for weightings varied with frequency



(a)



(b)

Fig. 7. Difference frequency's directivity for $\theta_{\_} = 60^{0}$. (a) Chebyshev weighting method. (b) Optimization method for weightings varied with frequency