Analysis of Doppler Effects in Underwater Acoustic Channels using Parabolic Expansion Modeling

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***Abstract*—Underwater communication systems play an important role in understanding various phenomena that take place within our vast oceans. They can be used as an integral tool in countless applications ranging from environmental monitoring to gathering of oceanographic data, marine archaeology, and search and rescue missions. Acoustic Communication is the viable solution for communication in highly attenuating underwater environment. However, these systems pose a number of challenges for reliable data transmission. Nonnegligible Doppler Effect emerges as a major factor. In order to support reliable high data rate communication, understanding the channel behavior is required. As sea trials are expensive, simulators are required to study the channel behavior. Modeling this channel involves solution to wave equations and validation with experimental data for that portion of the sea. Parabolic expansion model is a wave theory based acoustic channel model. This model applies Pade coefficients and Fourier coefficients as expansion functions to solve the wave equations. This work attempts to characterize the impact of Doppler Effect in the underwater acoustic channel using parabolic expansion models.**

Keywords—Doppler effect; underwater communication; acoustic channel models; parabolic expansion

I. Introduction

Wave equation is the theoretical basis of the mathematical models of acoustic propagation. It shows the relationship between sound pressure and vibration velocity of particle. Theory of acoustic propagation and solution to the frequency-domain wave equation are available in literatures [1-2]. There are different underwater acoustic propagation channel models available namely ray tracing, parabolic expansion models, normal mode propagation model multipath expansion model and fast field model. These models can be classified based on their range dependence into two types namely range independent models and range dependent models. Range independence indicates that the model considers a horizontally stratified ocean whose properties vary only as a function of depth. On the contrary, range dependence means that the model could consider some properties of the ocean medium that are allowed to vary as a function of range or the angle of the receiving antenna. According these definitions, normal-mode model, multipath expansion model and fast-field model are range independent while, ray-theoretical model and parabolic expansion model are range dependent.

Of the range dependent models, Ray-theoretical models characterize the propagation on the basis of geometry. Ray tracing techniques are available in literatures [3-5]. A multipath channel was modeled by Qarabaqi and Stojanovic, with ray tracing method [6]. Doppler Effect is modeled in these literatures by direct variation of the frequency component in the pressure field function. Parabolic expansion model is based on wave theory. The field functions obtained at low frequencies is accurate and provides an insight into antenna design. In this model, Doppler Effect is modeled as variation in the wave number of the field function. Parabolic expansion based modeling techniques are available in literatures [7-13].

This work attempts to characterize the impact of Doppler Effect due to source receiver motion in the underwater acoustic channel with parabolic expansion models using two different expansion functions namely Pade coefficients and Fourier coefficients to solve wave equations and analyze them with the environment data obtained from one of the Indian sea trials. This paper is organized as follows. Section 2 provides review of the parabolic expansion model. Section 3 explains the modeling of Doppler Effect using the parabolic expansion model and Section 4 discusses the results obtained and the work concludes with the key findings.

II. Parabolic Expansion Model

For sinusoidal frequency f_0, the wave equation for the acoustic pressure in an environment with azimuth symmetry is,

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial \psi^2}+\frac{\partial^2}{\partial z^2}+k^2\right)P=0 \tag{1}$$

Here, P is the acoustic pressure propagating in the (r,ψ, z) plane in dB re μPa, k is the wave number, r is the range in meters and z is the depth in meters , ψ is the azimuth in radians. k is the wave number in radians/meter; c is the velocity of the sound in the medium in meters/second. The second term in (1) provides the azimuth coupling between different radial element sets and is very small in general and thus is neglected

Parabolic Expansion Model assumes the medium is stratified, propagation is in vertical direction alone and the ocean bottom is flat and horizontal. This assumption is valid for all short range shallow water communication and useful in reducing computational complexity.

At far field, spreading and horizontal reflections are weak in case of short range and shallow depth, At range independent regions (smaller range over which horizontal variation is considered to be small), only forward wave propagation exists. To handle range dependence, the range is divided into several smaller range independent regions and thus the assumption is that the energy contained in the backward wave propagation is negligible. Thus the two dimension equation is reduced to one dimension.

For mathematical simplicity assume

$$P(r,z)=\left(\frac{1}{\sqrt{r}}\right)\phi(r,z)e^{ik_0 r} \tag{2}$$

$$Q=\left(\frac{\partial^2}{\partial z^2}+k^2\right) \tag{3}$$

Then (1) can be written as

$$\frac{\partial\phi}{\partial r}=i\left(\sqrt{Q}-k_0\right)\phi \tag{4}$$

Obtaining solution to ϕ requires finding $\sqrt{Q}$

A. Split step Fourier Algorithm

In split step algorithm the square root operator is split into two operators, one involving wave number and the other involving velocity.

Let

$$q=\left(\frac{Q}{k_0^2}-1\right) \tag{5}$$

The solution to (4) can also be written in terms of operators as:

$$\psi(r+\Delta r,z)=e^{ik_0\left(Q_{op}-1\right)\Delta r}\psi(r,z) \tag{6}$$

$$\left(1-Q_{op}\right)=T_{op}+U_{op}$$

$$T_{op}=1-\left[1-\left(\frac{k_z}{k_0}\right)^2\right]^{1/2}$$

$$U_{op}=1-\frac{c_0}{c(r,z)}$$

$$k_z=nk_0 \tag{7}$$

k_z is the depth dependant wave number, k_0 is the reference wave number and n is the refractive index; the operator Q_{op} is split into two terms viz T_{op} (velocity operator) and U_{op} (wave number operator).

Thus Split step Fourier Transform algorithm is implemented by (8).

$$\psi(r+\Delta r,z)=e^{-ik_0\frac{\Delta r}{2}U_{op}(r+\Delta r,z)}\times$$
$$IDFT\left\{e^{-ik_0\Delta r T_{op}(k_z)}\times DFT\left[e^{-ik_0\frac{\Delta r}{2}U_{op}(r,z)}\times\psi(r,z)\right]\right\} \tag{8}$$

B. Split Step Pade Algorithm

In split step Pade algorithm the Square root operator is approximated with Pade solution

$$\sqrt{1+q}\approx\frac{A+Bq}{C+Dq}=1+\sum_{i=1}^{n_p}\frac{a_i q}{1+b_i q};$$
$$a_i=\frac{2}{2n_p+1}\sin^2\left(\frac{i\pi}{2n_p+1}\right);b_i=\cos^2\left(\frac{i\pi}{2n_p+1}\right) \tag{9}$$

With this approximation, the solution to (4) reduces to the form which can be split into three terms each forming a matrix H(1),H(2),H(3). The matrices so obtained will in general have three or less term at every depth grid and hence they are called Tri diagonal matrices.

$$\left(\mathbf{H}(\mathbf{1})+\mathbf{H}(\mathbf{2})+\mathbf{H}(\mathbf{3})\right)\phi(r+\Delta r,z)=\phi(r,z) \tag{10}$$

$$H_{i,j}(1)=\int_z h_i(z)f_1(z)h_j(z)dz \quad i=j-1,\,j,\,j+1;\ f_1=\frac{1}{\rho} \tag{11}$$

$$H_{i,j}(2)=\int_z h_i(z)f_2(z)h_j(z)dz \quad i=j-1,j,j+1;\ f_2=\frac{1}{\rho c^2} \tag{12}$$

$$H_{i,j}(3)=\int_z h_i(z)\frac{\partial}{\partial z}\left(f_1(z)\frac{\partial}{\partial z}\left(h_j(z)\right)\right)dz \quad i=j-1,j,j+1 \tag{13}$$

$$h_i(z)=\begin{cases}0 & z\le z_{i-1}\\ z-z_{i-1}\big/z_i-z_{i-1} & z_{i-1}\le z\le z_i\\ 1-z-z_i\big/z_{i+1}-z_i & z_i\le z\le z_{i+1}\\ 0 & z\ge z_{i+1}\end{cases} \tag{14}$$

where ϕ(r,z) is the propagating field; ρ is the density; z is the depth, and h_i (z) is the linear finite element basis function.

Gaussian elimination method is applied to solve the system of equations represented in Matrix form to obtain the numerical solution of the parabolic wave equation.

III. Modeling Doppler Effect using the Parabolic Expansion Model

Doppler Effect is caused by transceiver motion. When the source is moving and the receiver is stationary, the shift in transmit frequency along the various vertical angle of transmission is given by

$$f_n^{'}\approx f_T\left(1+\frac{v_s}{c}\cos\left(\theta_n-\phi_s\right)\right);n=1,2,\ldots,N_z \tag{15}$$

f_T is the Transmit frequency in Hz; is the source velocity in m/s and ϕ_s is the angle in radians θ_n the source makes with the horizontal while moving and are the discretely sampled vertical angle in radians along which the transmission loss is calculated.

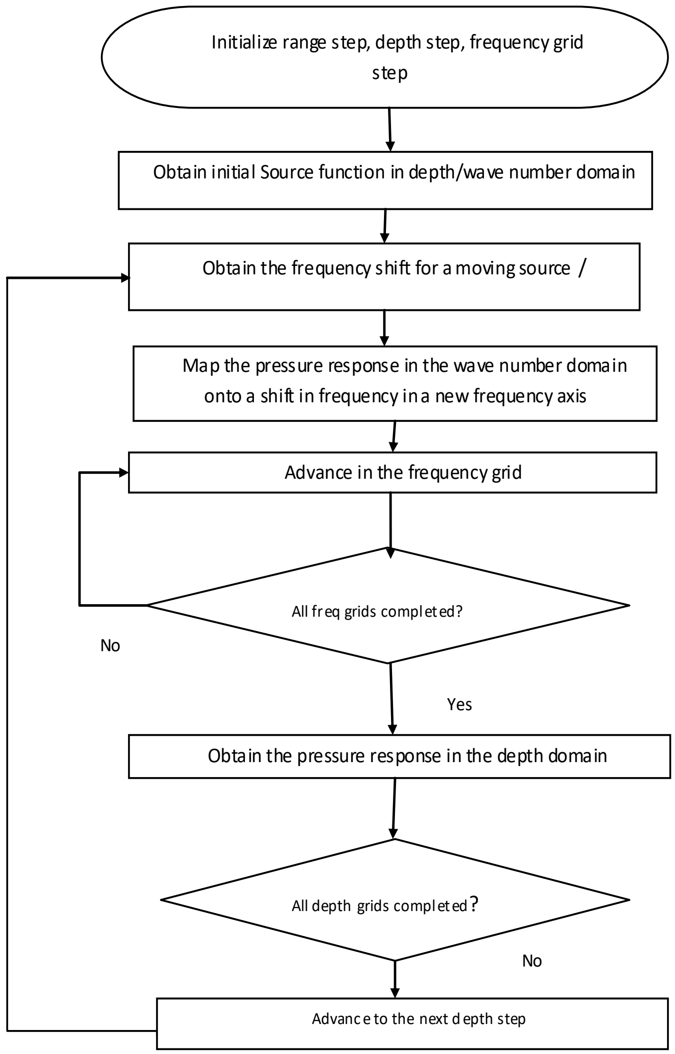

Fig. 1. Parabolic Expansion Method.

Consider the receiver is placed at the same depth and the source is moving along the horizontal axis. The spread in frequency by source motion is incorporated to the output field by distributing the source function into discrete frequency grids defined by (15) and applying smoothing between frequency cells through interpolation.

$$f_i = f_T - \frac{N_f}{2}\Delta f + (i-1)\Delta f;\; f_i = 1,2,\ldots,N_f, \tag{16}$$

Where N_f is the number of frequency grids and

$$\Delta f = \frac{f_{\max} - f_{\min}}{N_f};$$
$$f_{\max} = \max\left(f_n^{'}\right);\; f_{\min} = \min\left(f_n^{'}\right); \tag{17}$$

The effect of receiver movement is incorporated by a shift in frequency in a new frequency axis that accounts for both source and receiver motion. The Doppler shift for each wave number and their maxima and minima are defined as:

$$\Delta F_n = f_T \frac{|v_r|}{c_0}\cos(\theta_n - \phi_r)$$
$$\Delta F_{\max} = \max_n(\Delta F_n);$$
$$\Delta F_{\min} = \min_n(\Delta F_n);\; \forall n = 1,2,\ldots N_z \tag{18}$$

where the receiver speed and direction of motion are represented by v_r, ϕ_r respectively.

These Doppler shifts modify the extremes of the frequency axis depending on the direction of receiver motion. Thus the new frequency axis is defined as:

$$f''_{max} = \begin{cases} f_{max} + \Delta F_{min} & v_r < 0 \\ f_{max} & otherwise \end{cases}$$
$$f''_{min} = \begin{cases} f_{min} + \Delta F_{max} & v_r > 0 \\ f_{min} & otherwise \end{cases} \tag{19}$$

The new PE field is generated by interpolation across frequency grid. This is the initial field and it is propagated through each range cell by transforming it to wave number domain and the field at the receiver is calculated using Split step Pade and Split step Fourier algorithm. The steps involved in Doppler modeling are depicted in Fig. 1.

IV. RESULTS

Nominal values of the density of sea water for different depth and salinity are tabulated in Table I [14] and are depicted in Fig. 2.

TABLE I. DENSITY VALUES OF SEAWATER

Temperature (°C)	Salinity				
	5	*10*	*20*	*30*	*35*
0	3.97	8.01	16.07	24.10	28.13
5	4.01	7.97	15.86	23.74	27.70
10	3.67	7.56	15.32	23.08	26.97
15	3.01	6.85	14.50	22.15	25.99
20	2.07	5.86	13.42	20.98	24.78
25	0.87	4.62	12.10	19.60	23.36
30	0.57	3.15	10.57	18.01	21.75

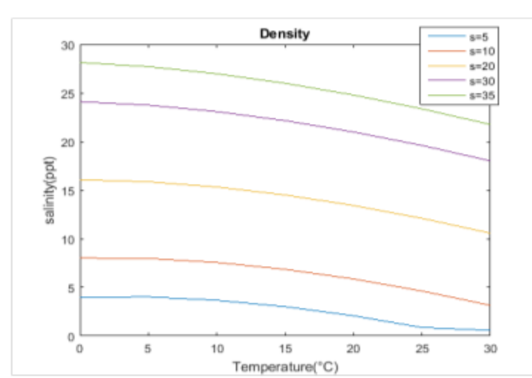

Fig. 2. Density Vs Temperature and Salinity.

ETOPO5 is the bathymetry derived for the Indian Ocean region (20°E to 112° E and 38°S to 32°N). The datasets are provided in ASCII, NETCDF and BINARY formats. [15].The data set is plotted in Fig. 3 and the plot infers that the bathymetry is uniform.

Under INDMOD project, the ship ORV Sagar Kanya was deployed at (43° E) for the measurements of depth, Temperature, salinity, speed profile[16]. The observed datasets for duration of one hour is considered and the results are plotted in Fig. 4.

The sound speed profile obtained from the Sagar Kanya experiment data is used as a reference to define the environment profile for both the algorithms.

The input parameters considered is shown in Table II. The pulse shown in Fig. 5 is transmitted. Amplitude of the pulse is inversely proportional to the bandwidth considered.

The split step Fourier algorithm has an explicit wave number operator; the transmission loss across frequency component is shown in Fig. 6.

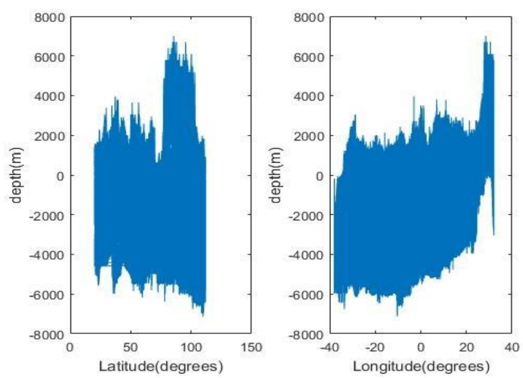

Fig. 3. (a) Latitude Vs Depth (b)Longitudeude Vs Depth Obtained from ETOPO5 Data.

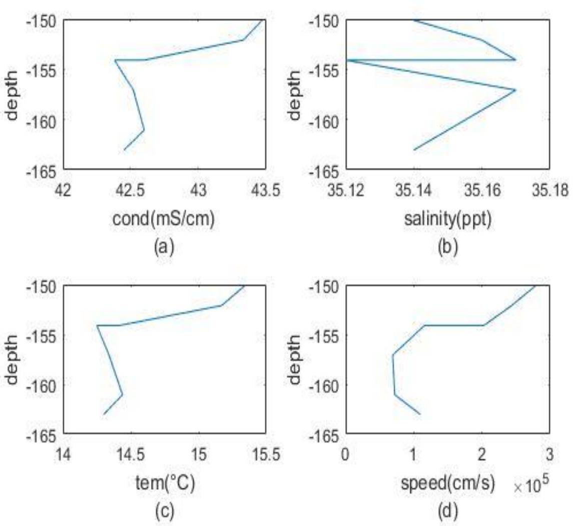

Fig. 4. Variation of (a) Temperature (b) Salinity (c) Conductivity (d) Speed with Respect to Depth Obtained from Sagar Kanya Experiment Data.

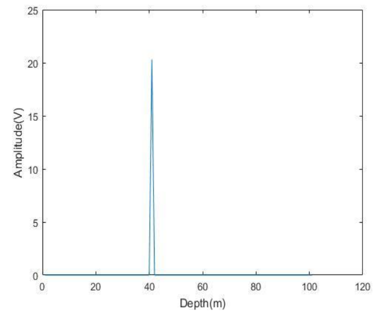

Fig. 5. Transmitted Pulse in Depth domain

The receiver is placed at different depths as shown in Fig. 7 to study the frequency range received by the receiver.

TABLE II. PARAMETERS OF SPLIT STEP FOURIER ALGORITHM

Parameters	Values
Range (r)	5Km
Max Depth (Zmax)	100m
Source Frequency (f_T)	3800Hz
Sound Speed C(Z)	1460-1540m/s
Source Depth (Zsrc)	40 m
Source Angle (Φs)	pi/4
Source Velocity (Vs)	20m/s
Receiver Angle(Φr)	pi/4
Receiver Velocity (Vr)	5m/s

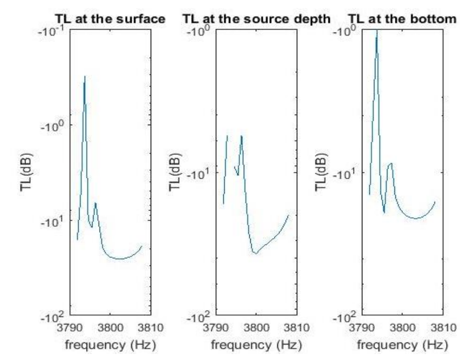

Fig. 6. Transmission Loss at 5Km with source frequency 3.8KHz; vr=5m/s; phir=pi/4 rad

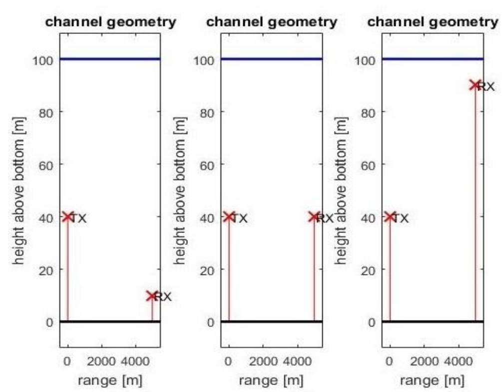

Fig. 7. Channel Geometry (a)Zg=10m; (a)Zg=40m; (a)Zg=90m;

Similar to the previous case, the pulse shown in Fig. 5 is transmitted from the source depth 40m. The input parameters considered are shown in Table III.

Fig. 8 shows the corresponding received pulse. Results show that when the receiver position (receiver depth) is near to the source depth, stronger signals are received.

Consider the source frequency in the lower, mid, high band say 5.3 Hz, 9.8 Hz, 14.2 Hz Fig. 9 depicts the attenuation in the amplitude of the received pulse as a function of depth.

Consider the receiver motion; Let the receiver be moving with the speed of 215 m/s, 2150 m/s 21500 m/s at an angle 45 degrees to the source. The corresponding Doppler shift from equation 11 is 1Hz, 10Hz, 100Hz, respectively. The frequency shift with respect to receiver motion is shown in Fig. 10.

TABLE III. PARAMETERS FOR SPLIT STEP PADE ALGORITHM

Parameters	**Values**
Range (r)	5Km
Max Depth (Zmax)	100m
Source Frequency (ft)	9.8Hz
Bandwidth	8.9Hz
Sound Speed C(Z)	1460-1540m/s
Coherence Bandwidth	0.01-0.89Hz

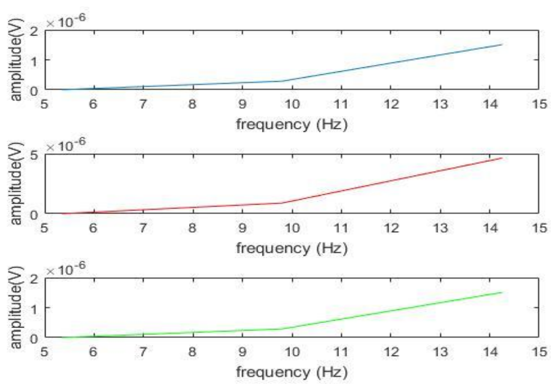

Fig. 8. Received Pulse at Depths Zg=10m (top))Zg=40m (mid) ; Zg=90m, (bottom).

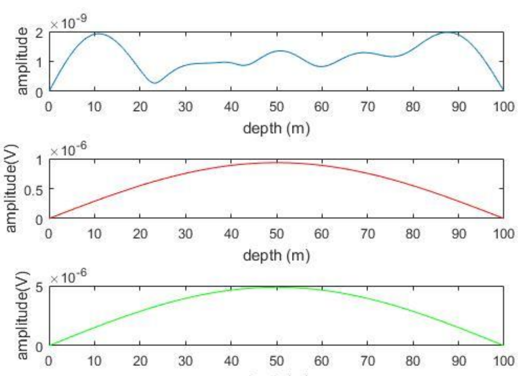

Fig. 9. Attenuation vs Depth Curve for the Source Frequencies 5.3Hz(Top), 9.8Hz(Mid), 14.2Hz(Bottom).

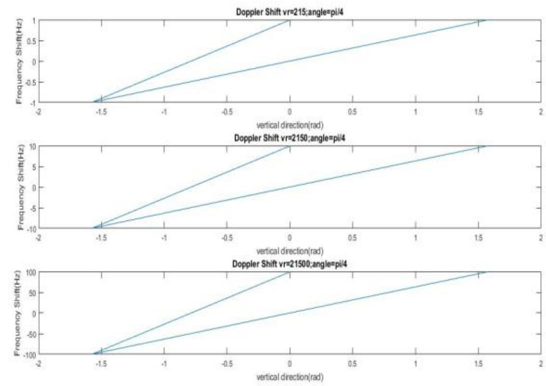

Fig. 10. Doppler Shift with Receiver Height =40m; Range=5Km (Top) vr=215 m/s;(Mid) vr=2150 m/s;(Bottom) vr=21500 m/s.

Fig. 11 depicts the fast fading scenario. The output power for various Doppler shifts is plotted against delay.

As the Doppler shift increases, coherence time decreases and hence more and more delayed components (resolvable multipath components) are received over the same symbol period.

Fig. 12 depicts the frequency selective fading scenario where different frequency components undergo different transmission loss. The transmission loss of frequency samples is plotted for various coherence bandwidths. It is observed that more samples undergo a flat fading when the coherence bandwidth is more as in Fig. 12 (bottom).

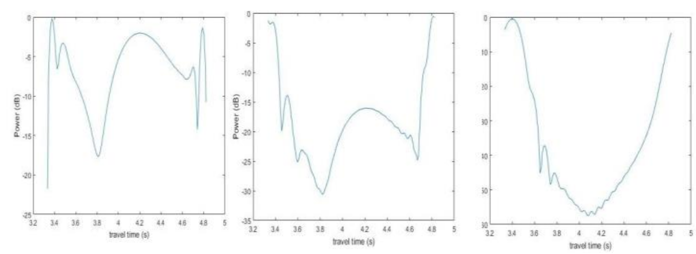

Fig. 11. Received Pulse with Receiver Height =40m; Range=5Km with Doppler Shift =100Hz; (Top); Doppler Shift =10Hz;(Middle); Doppler Shift =1Hz(Bottom).

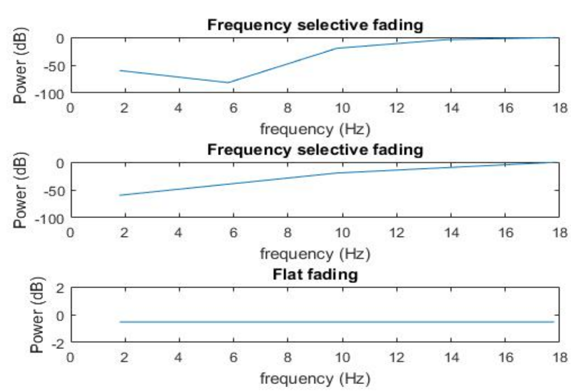

Fig. 12. Received Pulse with Receiver Height =40m; Range=5Km with Coherence Bandwidth =4.45 Hz; (top); Coherence Bandwidth =8.9 Hz (Middle); Coherence Bandwidth =20 Hz (Bottom).

V. Conclusions

This Paper analyses the parabolic expansion based modeling approach to model the Doppler Effect under the environment parameters that depicts the ocean scenario in India. The present data set depicts a flat environment as only a portion of the data set is considered. The future study is directed towards the analysis for environment data observed for a longer duration. The receiver picks up stronger signals when they are around the source depth. Split step Pade method characterize the platform motion through wave number variation at each depth grid. Unlike the previous work in [13], the present work considers a wider bandwidth. At increased frequency of operation, the inversion of tridiagonal matrix results in nearly singular values reducing the accuracy. The accuracy of the results can be enhanced by considering Split step Fourier coefficients as they have an explicit wave number domain operator. Future work is directed towards developing a beam forming algorithm for Doppler compensation.

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