Quantum Cryptography Experiment using Optical Devices

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***Abstract*—The study of quantum cryptography is one of the great interest. A straightforward and reliable quantum experiment is provided in this paper. A half wave plate in linearly polarized light makes up a simplified polarization rotator. The polarization rotates twice as much as the half wave plate's fast axis' angle with the polarization plane when the half wave plate is rotated. Here, an experiment of message sharing is conducted to demonstrate quantum communication between parties. The unitary transformation is performed step by step using half-wave plates represented by the Mueller matrix. A simulation created using Python programming has been used to test the proposed protocol's implementation. Python was chosen because it can mathematically imitate the quantum state of superposition.**

Keywords—Half-wave plate; polarizer; photon beam splitter; Stoke Vector

I. Introduction

Numerous organizations, including those in the industrial and academic areas, have shown a great deal of interest in quantum cryptography. Numerous productive research projects over the past few years have demonstrated a great advancement in optical equipment development and quantum cryptography [1]. The term "flying quantum bits" is frequently used to describe photons, emphasizing both their quantum nature and their capacity to transport quantum information over long distances [2]. Light can be thought of as a transverse electromagnetic wave, as has already been established. So far, it only discussed light that is linearly polarized or plane-polarized, meaning that the electric field's orientation is constant despite changes in its amplitude and sign over time.

An optical filter known as a polarizer or polarizer is used to block light waves of certain polarizations while allowing light waves of others to flow through. A polarizer is also an optical device that uses natural light as its input and produces some kind of polarized light as its output [3]. This means that it may convert an undefined or mixed polarization light beam into polarized light by filtering it through a well-defined polarization beam. For instance, consider the superposition of two equal-amplitude, incoherent, orthogonal ̧p-states as a possible representation of unpolarized light. A linear polarizer is a device that separates these two elements, discarding one and keeping the other [4]. In fact, the BB84 protocol, the very first quantum key distribution approach, was put forth using the polarization of photons.

Normally, light is free to oscillate at an angle to its path in any direction. When light passes through a polarizing filter, it becomes polarized. But how can be telling experimentally whether a device is a linear polarizer or not? Only light in a ̧p-states will be transmitted if light is incident on a perfect linear polarizer. The orientation of that ̧p-state will be parallel to the transmission axis of the polarizer, which is a specific direction. Only the part of the optical field that is parallel to the transmitting axis will effectively pass through the device unchanged. Due to the full symmetry of unpolarized light, the reading from the detector will not change if the polarizer is rotated about the z-axis. Based on the output beam's intensity, the polarizer determines the polarization states.

Now take a look at a type of optical components called retarders or also known as wave plate, which are used to modify the polarization of incident waves. Half-wave plates ($\lambda/2$ plates) are the most common types of waveplates. Half wave plate is described as having $\frac{\theta}{2}$ phase shift [5]. Linearly polarized light can be rotated by half wave retarders to an angle that is double the angle between the fast axis of the retarder and the plane of polarization. The polarization rotates 90º when the fast axis of a half wave plate is positioned at 45º from the polarization plane.

Continuously adjusting the energy with a half-wave plate and polarizer is another effective strategy. The polarizer can select the polarization direction of light, whereas the half-wave plate can rotate the polarization direction of light, and the two together may provide continuous light energy adjustment. The half-wave plate may continually change the polarization orientation. Assume that the slow axis is at an angle, θ to the polarisation direction. The phase of the fast axis is incremented by one after passing through the polarizer, and the polarisation direction is rotated by an angle, θ. Light passes into the polarizer through the half-wave plate. The transmitted energy for the polarizer is determined by Malus' law (Eq. 1).

The optical beam splitter is a crucial component in determining the statistical properties of light. The beam splitter has been used in many different areas of optics. For instance, in the field of quantum information, the beam splitter is fundamental for teleportation, bell measurements, entanglement, and basic research on photons. A beam splitter is a tool that splits an input beam into two beams travelling in different directions [6].

The rest of this paper is organized as follows: Section II reviews the related works of quantum cryptography that use optical devices. Section III presents the experiment environment. Section IV describes the principle of operation with schematic. Section V discusses hardware implementation. The experiment setup is discussed in Section VI. Section VII describes the experiment simulation. Pseudo code explained in Section VIII. Finally, Section IX concludes this paper.

In this study, presented a straightforward method for polarization alignment in which a half-wave plate is used to rotate the polarization of incoming light with respect to the fiber axis.

II. Related Work

In the last few years, several quantum experiment protocols using optical devices have been proposed. The main objective of those experiments was to ensure that the messages are delivered well among the parties involved. This section reviews several quantum experiments that use optical devices such as polarizer and half-wave plate for stimulation how the sharing message operation happens in quantum network, discussing their basic concepts, and describing their advantages and drawbacks. Fig. 1 shows the example implementation of polarizer, which when light passes through a polarizing filter, and it becomes polarized.

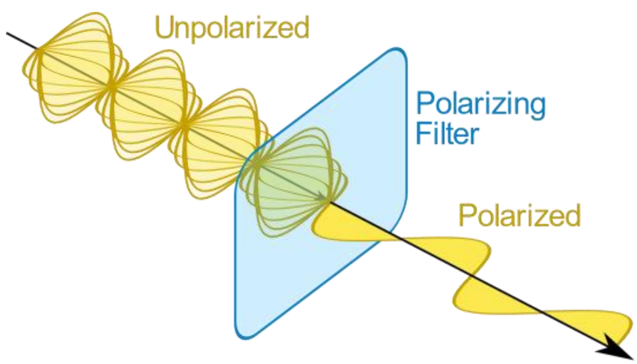

Fig. 1. Polarisation of light [7]

According to prior research, the quantum gate offers a number of advantages, including being simple to operate because parties do not always share information about the operators, requiring less precision than arbitrary rotational devices like half-wave plates (HWP) which enables precise measuring [8]. In 2019, Harun *et al.* proposed Hybrid M-ary in a braided single stage (HMBSS) [9]. The unitary transformation is performed step by step utilizing half-wave plates represented by the Mueller matrix. Five HWP was implemented in this protocol. For the purpose of authentication, the half wave plate's update angle or rotation is updated every 8 bits. The level of security has been raised at each stage due to the quick polarization changes, although it takes longer to send the information. The time required for message transmission and half-wave plate rotation is represented by the total transmission time. One can compute the transmission time as follows [9]:

$$Transmission\ Time = T_{msg} + T_{HWP} \tag{1}$$

where T_{msg} is the message transfer time and T_{HWP} is the half wave plate's time to update its angle during the encryption phase. The more the number of half wave plate implemented, the more time needed to encode each bit [10]. For example, the IV three-stage technique takes a long time because a single bit must exchange 3 times over the channel when using the three-stage protocol. The IV three-stage protocol requires Alice and Bob to use seven HWP for the polarization of bits in order to ensure the message's confidentiality, which lengthens the time it takes to encode each bit [11].

In 2021, with the use of single qubit unitary operations, Kang *et al.* have created a quantum message authentication mechanism that ensures the authenticity and integrity of the original message [12]. This protocol is made up of two parts: quantum encryption and a consistency check. Linear combinations of wave plates are used to implement the quantum encryption component. Assume that the secret key sequences have already been disclosed to Alice and Bob. This protocol implemented quantum encryption using half wave plates on Alice's side for the message authentication step.

Riggs *et al.* proposed Multi-Wavelength Quantum Key Distribution Emulation [13]. Multi-wavelength QKD can enable ternary and quaternary data transfer while removing the requirement to share a portion of the final key for eavesdropper detection. The half-wave plates were rotated before the laser pulse is sent in order to polarize the laser and measure it appropriately. In order for a half-wave plate to function, linearly polarized light must be rotated for the resulting polarization is twice as large as the angle between the optical axis and the incident polarized light [13]. In data transmission phase, through Alice's half-wave plate and Bob's half-wave plate, the laser beam is polarized. Bob's half-wave plate serves as the beam's measurement. At this point, the laser beam should only be vertically or horizontally polarized if Alice and Bob employed matching bases. If the resulting polarization is diagonal, the horizontally polarized light is transmitted while the vertically polarized light is reflected at the polarizing photon beam splitter cube. Just one of the detectors should experience the full intensity of the beam and light up if Alice and Bob employ matching bases.

III. Experiment Environment

In order to fully examine the performance of the proposed method, numerous parameter values must be determined. The quantum network simulation photon generation, encoding, decoding stage and schematic diagram utilized in this paper are described in this section.

A. Quantum Network Stimulation

Quantum networks including hardware components such as optical devices are costly to deploy since expert configuration is required and real-time experiments take a long time to complete. As a result, quantum communication protocols are tested and evaluated using simulation.

Because it can describe quantum states in mathematical form, the Python software was used to simulate the function of hardware components in a quantum communication environment. The Python software is also used to simulate the impact of light polarization in the implementation.

B. Photon Generation

A sender turns the original message into a binary string, which is then converted into a qubit via photon polarization. Multiple photons were created for each encoded bit [14], referring to one bit of information for the production of multiphoton, let's say three photons for each bit. The operation begins with the laser generating a laser beam, which is then passed over the quantum channel and transmitted through a beam splitter, which splits the beam into two channels, one with a 0° polarizer and the other with a 90° polarizer. The mirror then assembles the laser beam before combining it with a beam combiner.

C. Encoding and Decoding

For encoding stage, the beam is directed towards the HWP at this point in order to modify the polarization of the beam using a secret polarization angle. While for decoding stage, the detector will detect whether the bit is 0 or 1 after go through beam.

D. Schematic Diagram

The basic polarization of light as one of the strategies to encode the photons is required by the laser that emitted numerous photons through a multi-stage process. The Stokes parameter, which is a value of a linear polarization angle, is used to determine the polarization state of light [15]. The Stokes vector is used to determine light polarization as it passes through an optical system. The following vector form can be used to represent the four Stokes parameters [16]:

$$s = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{2}$$

where s_0 indicates total optical beam intensity, s_1 indicates linear horizontal or vertical polarised light, s_2 indicates +45° or -45° polarised light, and s_3 indicates right circular or left circular polarised light. s_1 and s_2 will be affected by the simple rotation operator, but s_3 will stay unaffected.

The polarizer determines the polarization states based on the intensity of the output beam. The intensity output can be determined using Malus's law, given as [17]–[19]:

$$I_O = I_I \, cos^2(\theta) \tag{3}$$

where θ is the secret polarization angle for the bits, I_O is the output intensity, I_I is the input intensity.

$$\frac{1}{2} \times [1 \; cos(2\theta) \, sin(\theta) \; 0] \times = \frac{1}{2} \times [1 + cos(2\theta)] \tag{4}$$

where S is the input bit, Equation 2's condensed form is obtained as below:

$$\frac{1 + cos(2\theta)}{2} = cos^2 \theta \tag{5}$$

In this study, 90° of linearly polarized light represents bit 1 and 0° of linearly polarized light represents bit 0 in matrices, as shown in Table I.

TABLE I. STOKES VECTOR AND STATE OF POLARIZATION

Stokes Vector	State of polarization	Bit representation
$s: \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	Light that isn't polarized	-
$s: \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$	Linearly polarized light at 0°	0
$s: \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$	Linearly polarized light at 90°	1

IV. PRINCIPLE OF OPERATION WITH SCHEMATIC

Following is a detailed explanation of the full experimental process for the hardware over free space optics (FSO) illustrated in Fig. 3. The protocol's implementation was divided into three different phases which are encoding, rotation transformation and decoding phase.

A. Encoding

Alice generates a state with a 0 linear polarization at the start of the protocol using a 0^0 polarizer. The polarization of light is accomplished by Alice and Bob using two sets of polarizers to change a bit into a quantum state referred as a qubit, described by a Mueller matrix [10].

$$M_{pol} = \frac{1}{2} \begin{bmatrix} 1 & \cos(2\theta) & \sin(2\theta) & 0 \\ \cos(2\theta) & \cos^2(2\theta) & \cos(2\theta)\sin(2\theta) & 0 \\ \sin(2\theta) & \cos(2\theta)\sin(2\theta) & \sin^2(2\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

where θ is the degree of polarisation angle that is set from 0º to 180º and M_{pol} is the polarizer's rotation.

The beam is directed towards the HWP at this point in order to modify the polarization of the beam using a secret polarization angle. The HWP is angled at a certain angle and is updated for each batch of bits.

B. Rotation Transformations Stage

This section discusses the configuration of the unitary transformations used with half wave plates as well as the selection of the half wave plates' rotation angle (θ) with regard to the horizontal axis. This decision relies on the Mueller matrix formalism to guarantee that the half wave plate setup's input and output polarisation angles are equivalent.

1) Half wave plate operation: Fig. 2 shows the implementation of both polarizer and half wave plate. A half wave plate generates a 180° polarization shift between the fast and slow axes of a wave plate. A HWP that is installed on a rotator and will be rotating at a random angle decided upon by Alice and Bob is also included. Given that the Stokes vector has four dimensions, Mueller matrices, as shown by [20], can be used to characterize the optical components of the HWP device as four by four matrices. The HWP operation's rotation is shown as [21]:

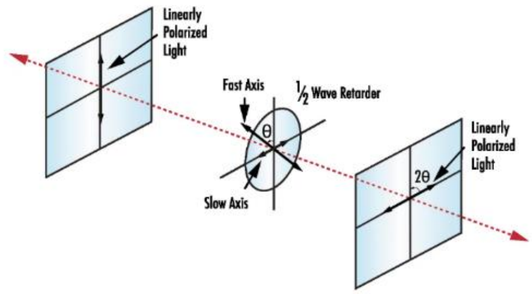

Fig. 2. Polarizer and half wave plate are employed [7]

$$M_{HWP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\theta) & \sin(4\theta) & 0 \\ 0 & \sin(4\theta) & -\cos(4\theta) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (7)$$

Where

$$0^0 < \theta < 180^0 \quad (8)$$

C. Decoding Stage

A beam combiner is used to combine the signals that come out of the channel. When the beam goes through the 0° and 90° polarizers, the detector will detect whether the bit is 0 or 1.

V. Hardware Implementation

Table II lists the hardware components utilized in the simulation experiments, along with their descriptions. The schematic diagram is explained in the next section.

The hardware components listed in Table II are deployed into the simulation environment at this point. Depending on the requirements of the experiment, the hardware can be put in fiber optic or free space optic.

TABLE II. Hardware Component [15]

Components	Description
Laser	Light intensity detector
Beam Splitter	A 50/50 beam splitter is used to split a laser beam into equal-intensity beams.
Polarizer	Filter the laser beam to produce a 0° or 90° polarized light beam that indicates bit 0 or 1.
Half wave plate	Encrypt the laser's data and generate superposition states based on angles.
Light intensity detector	Check whether the photons are 0 or 1.

VI. Experimental Setup

Fig. 3 shows the implementation of HWP in message sharing procedure. Alice will have two half-wave plates (HWPs), whereas Bob will have one HWP. In our implementation, a linearly polarized laser serves as the photon source. Depending on whether the input bit is 0 or 1, polarizers are used to filter a beam of light to pass a 0^0 or 90^0 degree polarized beam [8]. Alice generates a state with a 0 linear polarization at the start of the protocol using a 0^0 polarizer. The polarization technique generates a quantum bit (qubit) by encrypting the classical bits with photons. After encrypting the classical bits to the photons, Alice will use HWP-1 to execute a transformation at the angle $\boldsymbol{\theta_A}$, this produces the superposition state, $|\psi$. The superposition states hold the secret message that Alice and Bob will communicate in quantum ways.

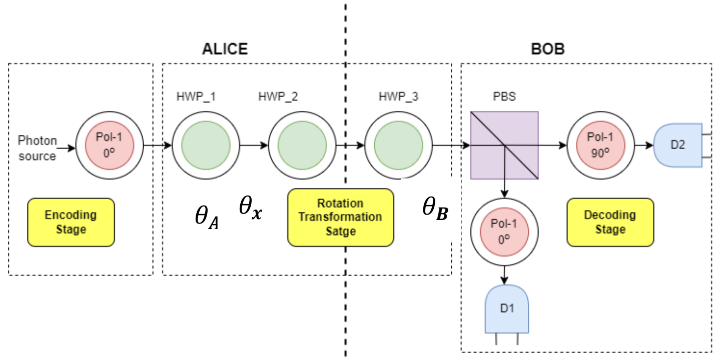

Fig. 3. Schematic of the experimental set-up over free space optics

In order to increase the security of the protocol, it is noted that HWP-1 at the angle $\boldsymbol{\theta_A}$ is the authentication key specified from 0^0 to 45^0. The transformation coupled with the encoded bit is then applied by Alice using HWP-3, which will be set at an angle $\theta_x = 0$ for bit 0 or $\theta_x = 45$ for bit 1.

$$Message\ X \Rightarrow bits \in \{0,1\}$$

$$\{0,1\}^n \Rightarrow \begin{cases} |0\rangle, S_{out} = M_{HWP}\,(0^0)\ x\ S_{in} \\ |1\rangle, S_{out} = M_{HWP}(45^0)\ x\ S_{in} \end{cases} \quad (9)$$

where S_{in} is the input light's input state comes from the polarizer Stokes parameter sequence, which is represented as

$$S_{in} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} (10)$$

and S_{out} is the output light's input state from the polarizer Stokes parameter sequence:

$$S_{out} = \begin{bmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} \quad (11)$$

The input for this study is a 0° state. Consequently, the Stokes parameter's output is described by

$$S_{out} = M_{HWP}\,(\theta_x) \times S_{in} (12)$$

After passing through the HWP, the light's polarization angle may be calculated [28,29] by:

$$\theta = \cos^{-1} \frac{S_{in}\,[1{:}3].S_{out}[1{:}3]}{||\,S_{in}\,[1{:}3]\,||\ x\ ||\,S_{out}\,[1{:}3]\,||} \quad (13)$$

where "." represents the multiplication operator and $||.||$ represents the norm operator.

Bob receives the optical beam from Alice that contains the message, and uses a HWP-3 set at an angle θ_A to remove his transformation. After passing it via a polarization beam splitter, Bob will receive in a beam polarizer at either 0^0 or 90^0 degrees. Accordingly, if θ_x = 45, the output light will be vertically polarized and if θ_x = 0, the output light will be horizontally polarized. Detectors will determine if the bit is 0 or 1.

VII. Experimental Simulation

In this section, the proposed protocol's basic description is covered. First of all, the message that has been converted to binary code is encrypted into a quantum bit or also known as qubit. The proposed protocol is then used to communicate the qubit from Alice to Bob as follows.

1) Let's say Alice gets a photon ready: The first qubit, $|0\rangle$ is what Alice wishes to transmit, and it is encrypted to the photons using the first defined angle from the first method, $\theta_A = 30^0$. Alice prepares photons according to *Equation (10)* and encrypts them by rotating HWP according to *Equation (7)*:

$$\varphi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4(30)) & \sin(4(30)) & 0 \\ 0 & \sin(4(30)) & -\cos(4(30)) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 0.8660254 \\ 0 \end{bmatrix} \tag{14}$$

2) Alice then performs the transformation corresponding with the encoded bit. The angle is set to θ_x = 0° if bit 0 is being transmitted or θ_x = 45º if bit 1 is being sent.

$$\varphi_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4(0)) & \sin(4(0)) & 0 \\ 0 & \sin(4(0)) & -\cos(4(0)) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ -0.8660254 \\ 0 \end{bmatrix} \tag{15}$$

3) When Bob gets φ_2, he rotates it using the angle of the authentication key, $\theta_A = -30^0$, to decode it. The original message is delivered to Bob. The result of the φ_2 represents the state $|0\rangle$ that Alice sent.

$$\varphi_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\big(4(-30)\big) & \sin\big(4(-30)\big) & 0 \\ 0 & \sin\big(4(-30)\big) & -\cos\big(4(-30)\big) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -0.5 \\ -0.8660254 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

VIII. Pseudo Code

The algorithms in this work serve as an exact list of instructions that carry out the steps in the process of transmitting messages via optical devices. The steps include encoding, rotation of HWP and decoding stage. The pseudo-code of the protocol is detailed in Algorithm 1.

Algorithm 1

```
Notation:
R(θ) ← is the rotation of HWP using Equation (6)
θ_A ← Angle of HWP at Alice
θ_x ← Angle of HWP at Alice either 0º or 45º
θ_B ← Angle of HWP at Bob
Alice prepares a photon using Equation (9)
Encoding Stage: Generate a photon to represent a qubit after the
    laser beam passed through a linear polarizer:
Alice generates a state with a 0º linear polarization using a 0º
    polarizer.
Rotation of HWP Stage
R(θ_A) |φ_0⟩ = |φ_1⟩
R(θ_x) |φ_1⟩ = |φ_2⟩
if bit 0 is being transmitted then
  θ_x = 0º
else if bit 1 is being sent then
  θ_x = 45º
end if
R(θ_B) |φ_2⟩ = |φ_3⟩
Decoding Stage: The polarizer detects the photon's polarization
    states
Bob receives beam polarizer at either 0º or 90º degrees
if θ_x = 0º then
   the output light will be horizontally polarised
else if θ_x = 45º then
   the output light will be vertically polarised
end if
Detectors will determine if the bit is 0 or 1
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IX. Conclusion

Photonic polarization qubits are frequently employed in quantum computation and quantum communication due to their robustness in transmission and ease of manipulation. But the usage of optical devices such as half wave plate and polarizer might high costs. The more the number of half-wave plates implemented in an experiment, the more times needed to transmit the message. This is because of changing the polarization angle of an optical device for encoding purposes, more time is required. These circumstances have led to an increase in source redundancy, which in turn causes a rise in transmission time. After consideration, this experiment implements three half-wave plates only. Bear in mind that this experiment is to help to stimulate how was the quantum cryptography works on message sharing among the parties since it can describe quantum states in mathematical form. This study was written to explain that quantum experiments can be carried out using optical devices. And the stimulation shown can be used as a guideline for other researchers. This paper also contributes an alternative way through Python software to simulate the function of each hardware component and uses the polarization of the quantum state to encrypt messages in a quantum communication environment.

Acknowledgment

This research was supported by Ministry of Higher Education (MOHE) through Fundamental Research Grant Scheme (FRGS/1/2021/ICT11/UTHM/03/1). We also want to thank to the Government of Malaysia which provide MyBrain15 programme for sponsoring this work under the self-funded research grant and L00022 from Ministry of Science, Technology and Innovation (MOSTI).

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