

A Height Accuracy Study Based on RTK and PPK Methods Outside the Standard Working Range

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Abstract—The aim of this paper is to study and analyze the Jack Up Vessel (JUV) foundation height accuracy, with the objective of the precise installation of an Offshore Wind Farm (OWF), based on Real Time Kinematic (RTK) and Post-Processing Kinematic (PPK) modes applied on short and long baselines length. The offshore wind farm project is located far from the coastline, not always in the standard working range of RTK. The standard allowed vertical installation tolerance for foundations is less than 10 cm. Taking into account all error sources, deformation of the vessel, motion, lever arms that impact the height measurement of the foundation, it is required that RTK and PPK perform within an accuracy less than 5 cm. In this work, all measures will be evaluated according to the tolerance specification of ± 2.5 cm. The survey GNSS tests executed during the project on board of a JUV should be able to provide answers to the following questions: Despite the critical environment, does RTK method allow reaching the theoretical specifications? Does PPK improve accuracy compared to the RTK solution? What is the influence of the baseline length? How much of the time the results fall within the range tolerance? What is the ideal logging period in which accurate and reliable results can be obtained? What is the influence of the hardware and software variants used in testing process on the results accuracy? Based on the test results and analysis a clear description of the influence of different parameters in the OWF precise height measurement in challenging environment will be exposed.

Keywords—Real time kinematic; post-processing kinematic; high-precision positioning

I. INTRODUCTION

In this paper, we deal with Differential Global Navigation Satellite System (DGNSS) solutions that have been used to reach accurate measurement at the centimeter level [1] on board a Jack Up Vessel (JUV) with the objective of the precise installation of an Offshore Wind Farm (OWF) located far from the coastline, not always in the standard working range. Indeed, many sources of errors relating to the satellite position, the orbits anomaly, the spatial environment, the satellite clock bias, the receiver clock offset, motion of JUV, will decrease the position accuracy [2-7]. To reduce the effect of these errors a differential positioning is needed, there are two emerging and developing methods for doing this [8], the first with real-time and the second with post-processing namely respectively Real Time Kinematic (RTK) and Post-Processing Kinematic (PPK) (Fig. 1).

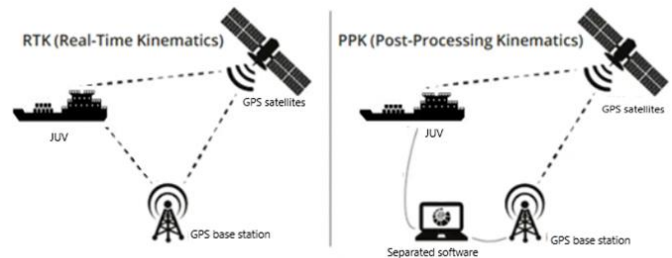


Fig. 1. RTK and PPK principle.

These two methods enhance the traditional method of deploying Ground Control Points (GCP) and become primary methods when we seek high-precision for any mapping, positioning or surveying task. It has been proven that RTK or PPK method extends the Precise Point Positioning (PPP), which needs a longer convergence time compared to differential methods, by incorporating a nearby local base station that perform and accelerate the integer ambiguity resolution, to reach a centimeter and sub-centimeter precision level [9, 10]. Therefore, the uses of GNSS have extended to many areas as Intelligent Transport System (ITS) including structural monitoring, geo-referencing of moving platforms and support for robotics and machine guidance, to name a few [11, 12]. Among the most relevant and recent works, we can cite the comparative study aimed to compare Unmanned Aerial Vehicle (UAV) based on RTK and PPK methods in mapping different surface types via five approaches: RTK Continuously Operating Reference Station (CORS) method, short-baseline PPK method obtaining corrections from a GNSS base station and three long-baseline PPK methods that obtained corrections from three Turkish RTK- (CORS) [13]. It is also promising to apply RTK-PPK to emerging industries like intelligent vehicle navigation and Low Earth Orbit (LEO) constellation augmented positioning [14].

RTK requires two receivers, a rover and a local base station, and a data radio connection between them. In this case, a local base station is placed at a known position. It can calculate the absolute distance of the satellites, based on its own position and the satellite navigation signals. It transmits the differences in distances to the rover as a differential correction. Then, the rover corrects its own distance, based on the correction data received from the base station. It is important to mention that it is not necessary to have a second receiver as a local base all the time. Alternatively, local services sharing base corrections over the Networked

Transport of RTCM via Internet Protocol (NTRIP) technology can be a good option. In case of PPK, there is no radio connection between the two receiver stations. The function of the base station, in the PPK case, is to continuously calculate the differential correction as a function of time and to store the data. The rover station stores the coordinates and data of the satellites used for its calculations as it moves. After the measurement, the data from the two stations can be compared according to the measurement times. After that, the correction can be calculated by separate software.

The present study evaluates the accuracy of the RTK and PPK methods to measure the foundation height of an Offshore Wind Farm (OWF). In standard GNSS positioning applications, it is a well-known fact that it is not possible to get the same precision for the vertical component as for the horizontal components. Indeed, since satellite sky distribution can never be homogenous on the vertical component, as there are no visible satellites under the horizon, errors on the pseudorange and phase observations propagate more adversely on the vertical component than on the horizontal components [15], which explain the strong correlation between the vertical component and some of the main systematic biases (receiver clock error and tropospheric bias). In addition, during the last decades there is a growing interest in being able to continuously monitor movements and deformations in man-made structures that have to withstand strong external forces like those in terrains that are subject to movements such as landslides, ground subsidence. Therefore, analyzing the Jack Up Vessel (JUV) foundation height accuracy in an OWF located far from the coastline, constitutes a real challenge for this work. During the project on board of a Jack Up Vessel (JUV), for the installation of OWF foundations vertical installation, tolerances of ± 10 cm are imposed. Taking into account all error sources, deformation of the vessel, motion, lever arms ... that impact the height measurement of the foundation, the vertical accuracies for GNSS must be lower than 5cm. Based on the theoretical specifications from the hardware used in this project which are Septentrio AsteRx-U & AsteRx-SB and Trimble SPS852 & SPS 855, the expected accuracy at 50 km would be 6 cm for Septentrio and 6.5 cm for Trimble. We will see later that the obtained measurements will be more precise than the indicated theoretical specifications.

According to previous works [16], the position information should be determined, in RTK mode, with accuracy in the range of centimeters in real time as the vessel sails. If the system also provides raw data in Receiver Independent Exchange (RINEX) format, it is also possible to determine the position using PPK method, with centimeter or sub-centimeter precision. In PPK mode, all calculations required for position correction are made post sailing by separate software.

During the project on board the JUV, we should be able to provide answers to the following questions: Despite the critical environment, does RTK method allow reaching the theoretical specifications? Does PPK improve accuracy compared to the RTK solution? What is the influence of the baseline length? How much of the time the results fall within the range tolerance? What is the ideal logging period in which accurate and reliable results can be obtained? What is the

influence of the hardware and software variants used in testing process on the results accuracy?

Based on the test results and analysis a clear description of the influence of different parameters in the OWF precise height measurement outside RTK working range and in challenging environment will be exposed.

II. TEST AND EXPERIMENTATION

This section describes the test and experimentation process, as well as hardware and software variants that will be used in testing process. The development of offshore energies requires ever larger cranes. For this, magic of evolution, boats have grown legs. One of these giants is the jack-up platform that has been developed for oil drilling and offshore wind. For the latter case, the location of the wind turbines must be precise and the height measurement must be accurate. The quality of the bottom having been previously studied, the foundations must rest in the intended place. For this, two factors are essential: controlling the position of the vessel and ensuring that it is stable so that the lifting is clean. To meet the first constraint a geolocation system and motion sensors are provided. Then come the feet that lift the ship and thus ensure that the crane will operate without being disturbed by the movement of the waves.

A. Test Location

Wind turbines require wind. They are therefore often installed in areas prone to strong storms. Although favorable weather windows are preferred, it is necessary to be able to withstand severe conditions. The test location is situated in the Baltic Sea in the most eastern part of Danish territorial waters and Danish Exclusive Economic Zone, directly next to the maritime border to Germany and Sweden. The closest distance to the Danish coastline (Island of Møn) ranges between 13 km and 39 km. The OWF project is located far from the coastline, not always in the standard working range of RTK.

B. Hardware GNSS Setup

The standard allowed vertical installation tolerance for foundations is less than 10 cm. Taken into account accumulative errors of other hardware in the setup required to determine the height of the foundation, it is required that the RTK performs within an accuracy of 5 cm. Based on the test results and analysis a clear definition and installation procedure (logging time, processing service, processing software and error budget) must be defined to determine the height measurement of an offshore base station outside the RTK working range. Two GNSS GA830 antennas were installed on board of the helideck of the JUV (Fig. 2), guaranteeing a clear view of the sky. The two GNSS antennas are installed close to each other and thus experience the same influence of any minor vessel movement.



Fig. 2. GA830 antenna.

For the purposes of this study, we use two Global Navigation Satellite System (GNSS) units namely Trimble and Septentrio units, five receiver brands referenced as DGPS4 (SPS852), SPS855, DGPS5 (AsteRx-U), Sept2 (AsteRx-U), Sept SB (AsteRx-SB).

We should note that the used GNSS constellation is composed from Global Positioning System (GPS), GLObal NAVigation Satellite System (GLO), Galileo (GAL) satellite system, and BeiDou (BEI) satellite system. We should note, again, that SPS852 was receiving GPS+GLO+BEI+GAL, DGPS 5 (AsteRx-U) was receiving GPS+GLO+BEI+GAL, Sept 2 (AsteRx-U) was receiving GPS+GLO+GAL and SPS855 was receiving RTX+(GPS+GLO+GAL). The AsteRx-SB remained working on RTK+(GPS+GLO+GAL)+(OSS KF B – baseline of 8 – 10 km), so it could be used as a reference.

In RTK or PPK modes, the receiver needs to know the type of antenna used at the base station in order to properly compensate for the phase center variation at the base. This information is typically included in the correction stream received from the base station. In this project, heights will be referred to the Antenna Reference Point (ARP) of DGPS5. One antenna is installed 0.080 m lower than the other antenna. Further, AsteRx units refer to ARP heights while Trimble units refer to Antenna Phase Center (APC) heights. Therefore, it is necessary to take into account the difference of 0.0885 m between ARP and APC of a GA830 antenna when comparing a Trimble receiver to an AsteRx receiver. An overview of the internal loggings and the time window is given in Table I.

TABLE I. OVERVIEW DATASETS AND TIMESLOTS USED FOR ANALYSIS

	TEST PURPOSE	TIME WINDOW OF DATASET (IN UTC TIME)
DATASET 1	Comparison RTK on different hardware units with a short baseline (± 8 km)	11/02/2021 19:00 – 12/02/2021 19:00
		13/02/2021 01:00 – 13/02/2021 21:00
		14/02/2021 01:30 – 14/02/2021 19:00
		15/02/2021 02:20 – 15/02/2021 12:00
DATASET 2	Comparison RTK (± 8 km) and RTK <i>GPSnet.dk</i> (29km from closest station)	10/02/2021 00:00 – 10/02/2021 23:59
DATASET 3	Comparison RTK (± 8 km) and PPP-RTK RTX	15/02/2021 16:00 – 16/02/2021 12:00
DATASET 4	Comparison RTK <i>GPSnet.dk</i> and PPP-RTK RTX	17/02/2021 00:00 – 17/02/2021 23:59
DATASET 5	Comparison RTK (± 8 km), PPP-RTK RTX and PPP	18/02/2021 18:00 – 19/02/2021 18:00
		20/02/2021 01:00 – 21/02/2021 01:00

C. Real Time RTK Performances

1) *Analysis of vessel settlement*: All tests are done on board of JUV in jacked up position. However, depending of the soil conditions it is possible that the legs can settle which can

have an influence on the height of the GNSS antennas since they are considered as static during the test processing. Before performing analysis on the RTK loggings, it was checked whether significant height changes could be determined during one jacked up position. Per location, the vessel was jacked up, the data was plotted for each GNSS receiver. A linear trend line is obtained and plotted on the graph (Fig. 3).

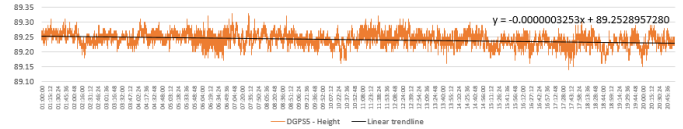


Fig. 3. Linear trendline of DGPS 5 height in function of time of Jack up 2.

The slope coefficient of this trend line was then recalculated and the average settlement per 24 h measured (Table II). To give an indication of the influence on our specific loggings during these tests, the average settlement for each logging is measured. The loggings were respectively 24 h, 20 h, 17.5 h and 9.5 h long.

TABLE II. JUV SETTLEMENT CALCULATION PER GNSS UNIT PER LOCATION

	JACK UP1 12/02-24H	JACK UP2 13/02-20H	JACK UP3 14/02-17.5H	JACK UP4 15/02-9.5H
AVERAGE SETTLEMENT PER 24 HOUR (m)				
DGP S4	-0.013	-0.039	-0.001	-0.063
DGP S5	0.006	-0.028	-0.011	-0.019
SEP T 2	0.002	-0.028	-0.011	-0.019
SEP T SB	0.006	-0.030	-0.011	-0.026
SPS8 55	0.004	-0.028	-0.006	-0.019
AVERAGE SETTLEMENT FOR EACH LOGGING (m)				
DGP S4	-0.013	-0.032	-0.001	-0.025
DGP S5	0.006	-0.023	-0.008	-0.008
SEP T 2	0.002	-0.023	-0.008	-0.008
SEP T SB	0.006	-0.025	-0.008	-0.010
SPS8 55	0.004	-0.024	-0.004	-0.008

First of all, one can observe a significantly higher value of height change for DGPS4 in two out of four locations. For the second location (Jack up 2), the settlement is high for all the devices. So, the problem arises mainly for Jack up 4. One might think this could be caused by the vessel movement and the load change due to unloading wind turbine parts. However, as mentioned before the two GNSS antennas are installed close to each other, and thus experience the same vessel movement. However, the other units do not show this same trend. If, we compare the average settlement per 24 h and the average settlement for 9.5 h logging for DGPS4-height, we note a significant difference between them, so we can think that the shorter time logging period may be the cause of this significant deviation.

For the first, third and fourth logging, it can be concluded that there is no significant settlement which has to be taken into account. All values (except for DGPS4) are 0.010 m or lower. Only on the second location, the subsidence coefficient is higher than 0.022 m for all GNSS receivers. This can be considered as a settlement. The surface might be softer at this location. However, it will not be applied to the data, since we cannot say with certainty that this is the cause of the height difference.

2) *Analysis of RTK height and standard deviation:* In the following tests, a more in-depth analysis was done to evaluate whether real time RTK performs as within ± 2.5 cm accuracy on a short baseline (<10 km), as specified in the hardware manufacturer specifications. Note that this analysis is based on in total 71 hours of data on four different locations.

As per manufacturer specifications Septentrio baseline length should be less than 40 km and Trimble baseline length should be less than 30 km (Table III). Based on these specifications, we can expect a Root Mean Square (RMS) error of respectively 18 mm (Septentrio) and 23 mm (Trimble) with a baseline of 8 km. In the below paragraphs, the results are compared to the results of the Septentrio AsteRx-SB, which is chosen as vertical reference, since it is connected to the base station (OSS KF B – baseline of 8 – 10 km) + RTK during all the performed tests.

TABLE III. MANUFACTURERS RTK SPECIFICATIONS

ASTERX-U & ASTERX-SB	Vertical accuracy 1cm + 1 ppm RMS	Open sky conditions RMS levels Baseline < 40km RTK fixed ambiguities
SPS852 & SPS 855	Vertical accuracy 15mm + 1ppm RMS	Baseline < 30km

3) *Statistics full loggings:* The average height of each logging at 8 km baseline and the difference between the units and the AsteRx-SB are listed in Table IV.

TABLE IV. RTK AVERAGE HEIGHT AND DIFFERENCE

	DGPS4 SPS852	DGPS 5 ASTERX- U	ASTERX- U SEPT 2	ASTE RX-SB	SPS855
HEIGHT (m)					
JACK UP 1	89.177	89.166	89.166	89.166	89.182
JACK UP 2	89.253	89.241	89.241	89.240	89.256
JACK UP 3	89.206	89.200	89.200	89.199	89.216
JACK UP 4	89.199	89.189	89.189	89.189	89.206
DIFFERENCE (m)					
JACK UP 1	0.011	0.000	0.000	-	0.016
JACK UP 2	0.012	0.001	0.000	-	0.015
JACK UP 3	0.007	0.001	0.001	-	0.017
JACK UP 4	0.010	0.000	0.000	-	0.018
TOTAL AVG. DIFF.	0.010	0.000	0.000	-	0.016

Note that the vessel aims to always jack up at the same keel height. For this reason, the height for each jack up is

similar. Table IV shows that the average results of all receivers lay close to each other. However, the results of Septentrio units are almost similar; we note a maximum of 0.001 m difference. The Trimble units show slightly higher differences, but the differences stay within 0.020 m, which is still within specifications. This does not mean that the Trimble results are less good than Septentrio units but indicates that the results may also differ depending on the receiver brand.

The difference of results between different GNSS brands could be caused by several factors; processing of results, usage of different antenna models. Depending on the brand and unit, Trimble relative, National Geodetic Survey (NGS) absolute or NGS relative antenna model are used. This can induce differences between ARP and APC. For the GA830 antenna, the difference between NGS relative and absolute is 0.017 mm. For Trimble SPS855 unit, Trimble relative is used, which explain the difference observed in Table IV for this unit.

The average Standard Deviation (SD) of a long logging is already at the limit of the specifications (Table V).

TABLE V. STANDARD DEVIATION OF THE UNITS ON RTK FOR EACH JACK UP

	DGPS4 SPS852 (m)	DGPS 5 ASTERX-U (m)	ASTERX-U SEPT 2 (m)	ASTER X-SB (m)	SPS8 55 (m)
JACK UP 1	0.017	0.021	0.021	0.020	0.020
JACK UP 2	0.020	0.022	0.022	0.020	0.020
JACK UP 3	0.016	0.021	0.021	0.019	0.019
JACK UP 4	0.019	0.021	0.021	0.019	0.020
Total	0.018	0.021	0.021	0.019	0.020

Table V shows that the average standard deviation of the data of the full logging is similar for all units and for a long logging it is already at the limit of manufactures specifications. The values are between 0.160 m and 0.220 m. This shows that on long term loggings, the stability of different brands is similar and within specifications.

Table VI shows how many data per second per logging are not within the ± 2.5 cm tolerance. We note that 12% up to 24% of the logged data is not within specifications, thus 76% up to 88% of the measurements during these 71 loggings at 8 km baseline are within tolerance.

TABLE VI. PERCENTAGE OF POSITIONS PER SECOND NOT WITHIN ± 2.5 CM OF THE 24 HOUR AVERAGE OF THE LOGGING, PER JACK UP

	DGPS4 SPS852	DGPS 5 ASTERX-U	ASTERX-U SEPT 2	ASTER X-SB	SPS 855
JACK UP 1	16%	22%	22%	19%	19%
JACK UP 2	18%	23%	24%	20%	23%
JACK UP 3	12%	21%	21%	17%	18%
JACK UP 4	16%	24%	24%	18%	20%

One of the scopes of the next tests was to try to define the ideal logging period in which accurate and reliable results can be obtained with a minimum of critical operational time. Below, the averages will be evaluated for 1h and 15 min periods.

4) *Statistics logging per hour*: For the statistical analysis of the data per hour, average heights and standard deviations per hour are evaluated. In total, 71 hourly average values are calculated. The minimum, maximum and average values are calculated and we estimated how much of these values are not within the predetermined tolerance of ± 2.5 cm. This is done in order to evaluate the performance of RTK when logging a specific period. The results per hour are summarized in Table VII and Table VIII. Table VII exposes the height difference between one hour average and total logging average and Table VIII exposes the standard deviation of RTK per hour.

TABLE VII. HEIGHT DIFFERENCE BETWEEN 1 HOUR AVERAGE AND TOTAL LOGGING AVERAGE

	DGPS4 SPS852 (m)	DGPS 5 ASTERX- U (m)	ASTERX -U SEPT 2 (m)	ASTE RX-SB (m)	SPS8 55 (m)
MIN	-0.024	-0.016	-0.016	-0.016	- 0.018
MAX	0.016	0.018	0.024	0.017	0.020
AVERAGE	0.000	0.000	0.000	0.000	0.000
% OUTSIDE TOLERANCE (± 2.5 CM)	0.0%	0.0%	0.0%	0.0%	0.0%

TABLE VIII. STANDARD DEVIATION OF RTK RESULTS PER HOUR

	DGPS4 SPS852 (m)	DGPS 5 ASTER X-U (m)	ASTER X-U SEPT 2 (m)	ASTE RX-SB (m)	SPS855 (m)
MIN	0.000	0.000	0.000	0.000	0.000
MAX	0.025	0.032	0.032	0.029	0.025
AVERAGE	0.015	0.019	0.019	0.017	0.017
% OUTSIDE TOLERANCE (± 2.5 CM)	0.0%	1.3%	2.7%	1.3%	2.7%

Logging with RTK during 1 hour gives 100% of the time values within 0.025 m difference from the average over 24 h, with a maximum difference observed of 0.024 m over the 71 hours of data, which is in accordance but so close to the tolerance limit. The SD minimum over 1 hour is no longer that 0.000, but has maximum values of 0.032 m. We note that 97.3 to 100% of the time, the SD is less than 0.025 m, which is considered a very good percentage. There are no significant differences between the units and brands, although the Trimble SPS852 shows the best results. We can conclude that 1 hour of logging is a good period to achieve fairly accurate results.

5) *Statistics logging per 15 min*: The same exercise was done with the same data, per 15 min. For in total 285 loggings

of 15 min, the average height difference between 15 min average and 24 h average as well as the standard deviation have been calculated and analyzed. The results are summarized in Table IX.

TABLE IX. HEIGHT DIFFERENCE BETWEEN 15' AVERAGE AND 24 HOUR AVERAGE

	DGPS4 SPS852 (m)	DGPS 5 ASTERX- U (m)	AsteRx- U SEPT 2 (m)	AsteR x-SB (m)	SPS8 55 (m)
MIN	-0.053	-0.036	-0.036	-0.035	- 0.032
MAX	0.029	0.029	0.031	0.023	0.027
AVERAGE	0.000	0.000	0.000	0.000	0.000
% OUTSIDE TOLERANCE (± 2.5 CM)	2%	4%	5%	2%	3%

The 15 min loggings in RTK are between 95 – 98% of the time within ± 0.025 m tolerance to total logging average. There are no significant differences between the brands and devices. The maximum average difference observed over all 15 min loggings is -0.053 m. All minimum and maximum values now exceed the ± 2.5 cm limit. This was not yet the case when analyzing the data per hour.

The minimum SD according to 15 min is no longer 0.000, but has maximum values of 0.033 m. The average SD is less than the tolerance 95.1% to 99.3% of the time (Table X). There are no significant differences between the units and brands. Overall, the height is also stable over periods of 15 min, but the period of 1 h shows higher performances.

TABLE X. STANDARD DEVIATION OF RTK RESULTS PER 15'

	DGPS4 SPS852 (m)	DGPS 5 ASTERX- U (m)	AsteRx- U SEPT 2 (m)	AsteR x-SB (m)	SPS8 55 (m)
MIN	0.000	0.000	0.000	0.000	0.000
MAX	0.031	0.030	0.033	0.030	0.033
AVERAGE	0.014	0.017	0.017	0.016	0.016
% OUTSIDE TOLERANCE (± 2.5 CM)	0.7%	3.9%	4.9%	1.8%	3.9%

6) *Trend analysis of height plot*: In the analysis below, the overall performances were analyzed by looking into detail the time series GNSS height graphs. For each logging, a graph is made per hour. The trend of the data was analyzed visually, the vertical axis shows a gridline per 5 cm and each device is visualized in its specific colour.

In Fig. 4, two graphs from two days are shown. When visually looking to the graphs, it was observed that the data sometimes show a different behavior – trend between the different brands.

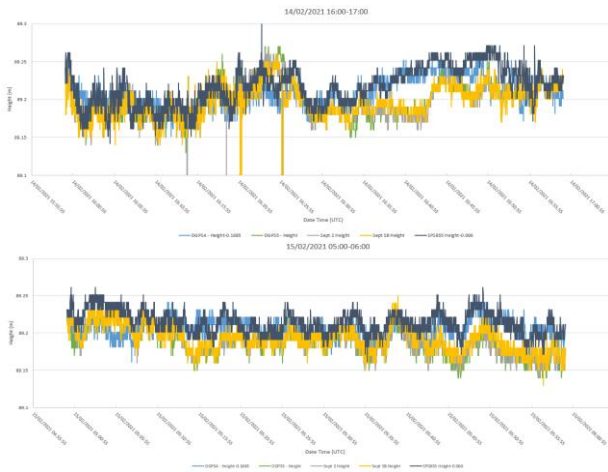


Fig. 4. Height results of five units in RTK short baseline.

Indeed, the three Septentrio units mostly match very well, despite the fact that these are two different models, which can be noted on the Sept AsteRx-SB (yellow graph) compared to the DGPS5 AsteRx-U (green graph) and Sept2 AsteRx-U (grey graph). In general the results of both Trimble resemble more to each other than to the height results of the Septentrio units, but the two Trimble units are different models as well SPS852 (light blue graph) and SPS855 (dark blue graph), which explains the small differences between the two Trimble graphs.

It can also be seen that the Trimble units in Fig. 4 are always slightly higher than the Septentrio units. Therefore, it can be said that the receivers from the same brand tend to follow the same behavior. This can be confirmed on these periods 14/02 16:41 – 16:45 and 16:48 – 16:50 as well as 15/02 05:45 – 05:50, where the difference between the Trimble and the Septentrio units achieved 0.040 m for several minutes. Knowing that the Trimble SPS855 is connected to the same GNSS antenna as the Septentrio units, this clearly proves the differences in results caused by the RTK engine and justify the use of different brands.

D. Post-Processing PPK Performances

In this section, we evaluate the PPK performance on different baseline lengths. Indeed, data is post-processing using different base stations on different distances (base line lengths). As a check, post-processing was also performed with base station OSS KF B. Base stations on land are obtained from two external correction services: GPSNET.DK and SWEPOS (Table XI).

TABLE XI. OVERVIEW BASE STATIONS FOR SINGLE BASE POST-PROCESSING

Denmark - GPSNET.DK	STEG	±30km
	FAXE	±50km
	ORHO	±60km
	GDS1	±70km
Sweden - SWEPOS	SKAN	±40km
	SMYG	±50km

Furthermore, data of 3 out of 5 receivers was post-processed: DGPS5 (AsteRx-U), Sept2 (AsteRx-U) and DGPS4 (SPS852). Data is post-processed for one jack up cycle of 24 h from 19 h on 11/02 until 19 h on 12/02. This was done in a static and kinematic way. Static post-processed data is compared to results from online post-processing services (AUSPOS and CSRS-PPP) and kinematic post-processed is compared to real time data. For post-processing, two software programs are used: Trimble Business Center (TBC) V5.10 and Qintertia V2.2.5847. Static post-processing will be done in TBC, kinematic post-processing will be done in TBC and Qintertia.

7) *Static mode*: As mentioned below, static post-processing is done in TBC with seven base stations (OSS KF B, STEG, SKAN, SMYG, FAXE, ORHO and GDS1) and one rover (DGPS 5). The static results were compared to AUSPOS and CSRS-PPP online services and were summarized on Table XII. The height of DGPS5 is sent to online services.

DGPS5 and Sept 2 are expected to have the same results since it concerns the same receiver type and the same antenna. Differences between the online services are ranging between 1 mm for the AsteRx-U to 7 mm for the SPS852. OSS KF B and all other base stations seem to match slightly better with the AUSPOS data than with CSRS-PPP data as shown on Table XIII.

Despite the superiority of AUSPOS online service, we always remain within specifications for the two online services for all baseline lengths. This shows that post-processing results match well with online services even with longer baselines (Table XIII).

TABLE XII. HEIGHT DGPS5 SENT TO ONLINE SERVICES AND POST-PROCESSED WITH DIFFERENT BASE STATIONS

	SPS852 (m)	DGPS 5 ASTERX-U (m)	SEPT 2 ASTERX-U (m)
AUSPOS	89.207	89.213	89.213
CSRS-PPP	89.200	89.211	89.212
OSS KF B (8.4KM)	89.206	89.220	89.221
STEG (29.2KM)	89.220	89.221	89.221
SKAN (42.3KM)	89.218	89.223	89.223
SMYG (49.4KM)	89.224	89.230	89.230
FAXE (51.9KM)	89.208	89.212	89.212
ORHO (62.5KM)	89.213	89.219	89.220
GDS1 (74.1KM)	89.210	89.215	89.217

TABLE XIII. DIFFERENCE BETWEEN ONLINE SERVICES AND POST-PROCESSED DATA FROM DIFFERENT BASE STATIONS

	SPS852 (m)	DGPS 5 ASTERX-U (m)	SEPT 2 ASTERX- U (m)
OSS KF B (8.4KM)			
AUSPOS	-0.001	0.007	0.008
CSRS-PPP	0.006	0.009	0.009
STEG (29.2KM)			
AUSPOS	0.013	0.008	0.008
CSRS-PPP	0.020	0.010	0.009
SKAN (42.3KM)			
AUSPOS	0.011	0.010	0.010
CSRS-PPP	0.018	0.012	0.011
SMYG (49.4KM)			
AUSPOS	0.017	0.017	0.017
CSRS-PPP	0.024	0.019	0.018
FAXE (51.9KM)			
AUSPOS	0.001	-0.001	-0.001
CSRS-PPP	0.008	0.001	0.000
ORHO (62.5KM)			
AUSPOS	0.006	0.006	0.007
CSRS-PPP	0.013	0.008	0.008
GDS1 (74.1KM)			
AUSPOS	0.003	0.002	0.004
CSRS-PPP	0.010	0.004	0.005

8) Kinematic mode

a) Trimble Business Center: Kinematic post processing was done in TBC with five base stations (OSS KF B, STEG, FAXE, ORHO and GDS1) and one rover (DGPS 5). Data will be compared to the real time DGPS 5 data.

From Table XIV, we note that all data are almost continuously in RTK Fix and does not decrease with distance. Let us also note that Post-Processed (PP) data are a bit more in RTK Fix mode than the real time solution.

TABLE XIV. PERCENTAGE OF EPOCHS IN RTK FIXED, PER BASE STATION

Real Time OSS KF B	PP OSS KF- B (m) (8.4KM)	PP STEG (m) (29.2KM)	PP FAXE (m) (51.9KM)	PP ORHO (m) (62.5KM)	PP GDS1 (m) (74.1KM)
99.92%	99.98%	99.98%	99.98%	99.98%	99.98%

Overall, data seem to follow the same trend as the real time DGPS5 data (Fig. 5). However, in the graph of Fig. 5 is visible that the datasets tend to spread up to ± 0.100 m mainly for the OSS KF B case.



Fig. 5. Kinematic post processed height data from TBC vs real time DGPS 5 data (20-21h).

Table XV gives an overview of the statistics of the kinematic post-processing of the full logging of 24 h.

TABLE XV. AVERAGES AND SD OVER 24 HOUR KINEMATIC PROCESSING

REAL TIME OSS KF B	PP OSS KF B (m) (8.4KM)	PP STEG (m) (29.2 KM)	PP FAXE (m) (51.9KM)	PP ORHO (m) (62.5KM)	PP GDS1 (m) (74.1KM)
AVERAGE HEIGHT FOR 24H					
89.166	89.153	89.191	89.189	89.193	89.191
STANDARD DEVIATION FOR 24H					
0.021	0.019	0.026	0.033	0.041	0.036
HEIGHT DIFF. WITH REAL TIME DGPS5 FOR 24H					
-	-0.013	0.025	0.023	0.027	0.025

The average of 24 h data processed at +25 km baseline show results that are 2.5 cm or more higher compared to the real time height (Table XV). Average over 24 h of the check base OSS KF B, is 1.3 cm lower. We can still deduce that the standard deviation and the differences between the real time and post-processing height increase with the baseline length. Note that the differences are higher than the static logging for the same period.

In the Table XVI, we present the difference between 1 h average and 24 h average.

When taking averages per hour, 4.2% of the time the values for STEG were outside this 2.5 cm upper/lower tolerance from the 24 h average, with a maximum absolute difference of 4.5 cm. While, this is $\pm 8.3\%$, for baseline distance of 50-75 km (FAXE, ORHO and GDS1). The maximum absolute difference for all base stations is 9.4 cm.

TABLE XVI. REAL TIME AND POST PROCESSING (TBC) STATISTICS FOR 1 HOUR LOGGINGS OVER A 24 HOUR PERIOD

	DIFFERENCE BETWEEN 1H AVERAGE AND 24H AVERAGE					
	Real time OSS KF B (m)	PP OSS KF B (m) 8.4km	PP STEG (m) 29.2km	PP FAXE (m) 51.9km	PP ORH O (m) 62.5k m	PP GDS1 (m) 74.1km
MIN	-0.012	-0.011	-0.020	-0.028	-0.018	-0.027
MAX	0.013	0.015	0.045	0.094	0.089	0.086
AVERAGE	0.000	0.000	0.000	0.000	0.000	0.000
% OUTSIDE TOLERAN CE (± 2.5 CM)	0.0%	0.0%	4.2%	8.3%	8.3%	8.3%
SD PER HOUR						
MIN	0.014	0.011	0.009	0.010	0.010	0.010
MAX	0.032	0.026	0.052	0.075	0.128	0.080
AVERAGE	0.020	0.017	0.019	0.020	0.025	0.024
% OUTSIDE TOLERAN CE (± 2.5 CM)	4.2%	4.2%	4.2%	12.5%	20.8%	20.8%

These results can mean that RTK logging during one hour with a base station at a distance of 30 km in 95.8% can be considered long enough to define the coordinate within tolerance. And for 50-75 km (FAXE, ORHO and GDS1) this percentage decreases to 91.7% and in 8.3% of the time there is a risk of reaching values that exceed the tolerance threshold.

The differences between 1 h and 24 h confirm that there is no significant difference in real time and post-processed results from shorter baseline, since real time and PP OSS KF B show similar results and are both 100% of the time within tolerance.

When looking to the SD values, an increase of percentage outside of tolerance can be noted with the baseline length. For 62 km and 74 km baseline, only $\pm 79.2\%$ of the loggings per one hour are within tolerance, which is a fairly low percentage. Note that these last distances exceed the manufacturer requirement of 40 km.

However, STEG base station is less than 40 km from the rover, which is according to the manufacturer requirements. Despite that, 4.2% of the time the value per hour is outside of the ± 2.5 cm tolerance. For this reason, it was looked more in detail what happens there. It was noted that 13-15% of all the values outside of tolerance are falling in a timepan of less than 1 hour (02:59-03:40, 04:27-04:40, 18:05-18:44). This first timepan with a lot of high values is seen in Fig. 6. This clearly shows that data is not matching despite the RTK fixed solution.

No specific reason can be given to these decreased results. It could be related with ionosphere and troposphere and thus baseline. These effects tend to increase the vertical error with baseline length. STEG with the shortest baseline (29.2 km) shows slightly less disturbance than GDS1, ORHO and FAXE.

The same exercise as done per hour, was done per 15 min. The real time and post processing statistics are summarized in Table XVII.

With a baseline of more than 25 km, averages over a period of 15 minutes, values are outside the 2.5 cm upper/lower tolerance for 13% up to 19% of the time. Heights post-processed with the base station OSS KF B are more within tolerance than the real time result (1% versus 6%). The minimum and maximum values increase significantly with the baseline length. The range of the average height values per 15' is already ± 0.200 m for base stations at 50 km or further.



Fig. 6. Kinematic post processed data from TBC vs. real time DGPS 5 data (3-4h).

TABLE XVII. REAL TIME AND POST PROCESSING (TBC) STATISTICS FOR 15' LOGGINGS OVER A 24 HOUR PERIOD

	DIFFERENCE BETWEEN 15' AVERAGE AND 24H AVERAGE					
	Real time OSS KF B (m)	PP OSS KF B (m) 8.4km	PP STEG (m) 29.2km	PP FAXE (m) 51.9km	PP OR HO (m) 62.5 km	PP GDS1 (m) 74.1km
MIN	-0.036	-0.027	-0.045	-0.039	-0.038	-0.043
MAX	0.029	0.024	0.095	0.172	0.145	0.154
AVERAGE	0.000	0.000	0.000	0.000	0.000	0.000
% OUTSIDE TOLERANCE (± 2.5 CM)	6%	1%	18%	13%	14%	19%
SD PER 15'						
MIN	0.000	0.005	0.001	0.002	0.002	0.002
MAX	0.031	0.034	0.060	0.084	0.219	0.127
AVERAGE	0.017	0.014	0.014	0.015	0.018	0.018
% OUTSIDE TOLERANCE (± 2.5 CM)	4%	3%	4%	8%	11%	13%

The SD shows the same trend, the values increase by baseline length. However, it is quite particular that the percentages of values outside of tolerance are smaller than those per hour. This is due to the fact that over short periods the SD varies little.

b) *Qinertia*: The second software that can be used for kinematic post-processing is Qinertia. Processing has been done as well with this software, to get a better view on the consistency between Post-Processing (PP) software. This also gives a better view on the reliability of the PP results. The same data were processed: DGPS 5 was processed with four base stations (STEG, FAXE, ORHO and GDS1). Data are compared to the real time DGPS 5 data.

First of all, Table XVIII shows the percentage of the time that RTK fixed solution is achieved.

TABLE XVIII. PERCENTAGE SOLUTION MODE IN RTK FIXED

Real Time OSS KF B	PP STEG (m) (29.2KM)	PP FAXE (m) (51.9KM)	PP ORHO (m) (62.5KM)	PP GDS1 (m) (74.1KM)
99.92%	85.16%	76.09%	72.68%	57.28%

RTK fixed solution decreases significantly with distance. This is a big contrast with TBC, where all base stations could achieve RTK fixed for 99.98% of the epochs. We can deduce that Qinertia has more difficulties to recover RTK fixed solution even for shorter baseline.

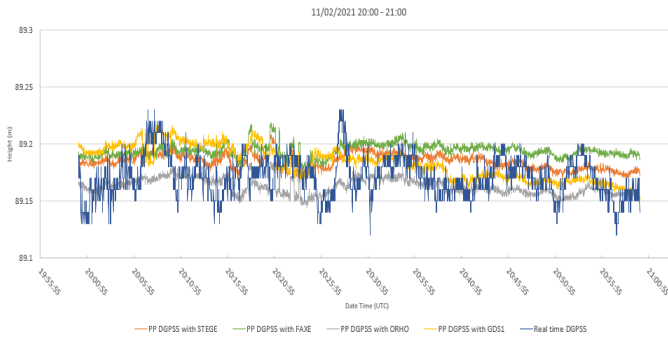


Fig. 7. Kinematic post processed data from qinertia vs real time DGPS 5 data (20-21h).

Overall, data seem to match the real time DGPS 5 data (Fig. 7). It can be observed that the real time data seems choppy, this is due to a difference in rounding between QINSy loggings and Qinertia: QINSy data has two decimals while Qinertia has three decimals.

On an average of 24 h STEG, FAXE, ORHO and GDS1 data are very close to the real time height within 1 cm. These differences are much smaller than those of TBC processed heights. The SD has much higher values, but in the same line as the TBC results. Longer is the baseline, higher is the standard deviation (Table XIX).

TABLE XIX. AVERAGES AND SD OVER 24 HOUR

REAL TIME OSS KF B	PP STEG (m) (29.2KM)	PP FAXE (m) (51.9KM)	PP ORHO (m) (62.5KM)	PP GDS1 (m) (74.1KM)
AVERAGE HEIGHT FOR 24H				
89.166	89.161	89.159	89.167	89.176
STANDARD DEVIATION FOR 24H				
0.021	0.017	0.036	0.031	0.059
HEIGHT DIFF. WITH REAL TIME DGPS5 FOR 24H				
-	-0.005	-0.007	0.001	0.010

It looks like Qinertia loses RTK fixed location even at a shorter distance. But if Qinertia achieves RTK fixed, the results are very precise. This is not the case with data processed by TBC. This can be seen by comparing Fig. 6 with Fig. 7.

From Table XX, the difference per hour with the 24 h average is still quite low for a 30 km baseline, but with longer baselines this increases. Also the difference between minimum and maximum values follows this same trend, showing a 0.137 m difference at GDS1.

ORHO shows better results than FAXE, but it must be mentioned that the data in this graph are partially distorted since the data was often not in RTK fixed. Only RTK fixed results are included in the summary in Table XX, which implies that the averages are not always based on 24 h values and 1 h of data does not always actually includes 3600 values, in case RTK Fix is lost in the meantime.

TABLE XX. REAL TIME AND POST PROCESSING (QINERTIA) STATISTICS FOR 1 HOUR LOGGINGS OVER A 24 HOUR PERIOD

	REAL TIME OSS KF B (m)	PP STEG (m) (29.2KM)	PP FAXE (m) (51.9KM)	PP ORH O (m) (62.5K M)	PP GDS1 (m) (74.1K M)
% RTK FIXED	99.92%	85.16%	76.09%	72.68%	57.28%
DIFFERENCE BETWEEN 1H AVERAGE AND 24H AVERAGE					
MIN	-0.012	-0.016	-0.039	-0.029	-0.059
MAX	0.013	0.025	0.044	0.057	0.078
AVERAGE	0.000	0.000	0.001	-0.003	-0.005
% OUTSIDE TOLERANCE (±2.5CM)	0.0%	0.0%	22.2%	12.5%	58.8%
SD PER HOUR					
MIN	0.014	0.004	0.005	0.007	0.003
MAX	0.032	0.037	0.102	0.020	0.041
AVERAGE	0.020	0.012	0.018	0.013	0.016
% OUTSIDE TOLERANCE (±2.5CM)	4.2%	4.8%	16.7%	0.0%	5.9%
HEIGHT DIFFERENCE COMPARED TO REAL TIME AVERAGE					
MIN	-	-0.030	-0.049	-0.028	-0.046
MAX	-	0.013	0.024	0.056	0.086
AVERAGE	-	-0.005	-0.006	-0.002	0.005
% OUTSIDE TOLERANCE (±2.5CM)	-	4.8%	11.1%	12.5%	35.3%

As it was suggested earlier that the RTK fixed solution from Qinertia is generally speaking better at longer distances than TBC, this is no longer valid after analyzing the data more in detail. This is also confirmed when looking at the time series graphs of Fig. 8. Indeed, a reduced availability of data can be observed, only the closest station (STEG) seems to give useful data. Further, it can be observed that data after losing RTK fixed solution can be less accurate. It should also be noted that the heights jumps can only be caused by the Qinertia engine, which is not handling data well at larger distances.

The same exercise is done at a 15 min interval (Table XXI). Here, the same remark must be made about the distortion on the data. Since the results are for quite some time not in RTK fixed, the averages are not based on 24 x 4 quarters.

The difference in height compared to the 24 h average shows the same trend as the data per hour. While real time data was 100% within tolerance, this is now only 93.7% of the time. The percentage of values outside tolerance increases. At a distance of 74 km, only 43.4% of the values are within tolerance, this rate is low even for 1 h average logging. This proves the superiority of 1 h average logging versus 15 min average logging.



Fig. 8. Kinematic post processed data from TBC vs real time DGPS 5 data (8-9h).

TABLE XXI. REAL TIME AND POST PROCESSING (QINERTIA) STATISTICS FOR 15' LOGGINGS OVER A 24H PERIOD

	REAL TIME OSS KF B (m)	PP STEG (m) (29.2K M)	PP FAXE (m) (51.9K M)	PP OR HO (m) (62.5 KM)	PP GDS1 (m) (74.1K M)
% RTK FIXED	99.92%	85.16%	76.09 %	72.6 8%	57.28%
DIFFERENCE BETWEEN 15' AVERAGE AND 24H AVERAGE					
MIN	-0.036	-0.026	-0.100	0.03 9	-0.068
MAX	0.029	0.057	0.067	0.09 4	0.092
AVERAGE	0.000	0.000	0.000	0.00 0	0.001
% OUTSIDE TOLERANCE (±2.5CM)	6.3%	3.6%	23.3%	23.2 %	56.6%
SD PER 15'					
MIN	0.000	0.000	0.000	0.00 2	0.003
MAX	0.031	0.039	0.168	0.12 9	0.192
AVERAGE	0.017	0.008	0.013	0.01 1	0.017
% OUTSIDE TOLERANCE (±2.5CM)	4.2%	3.6%	8.0%	4.2%	13.2%
HEIGHT DIFFERENCE 15' COMPARED TO REAL TIME AVERAGE 15'					
MIN	-	-0.043	-0.115	- 0.04 3	-0.054
MAX	-	0.047	0.051	0.09 5	0.110
AVERAGE	-	-0.005	-0.007	0.00 2	0.011
% OUTSIDE TOLERANCE (±2.5CM)	-	15.7%	31.5%	33.3 %	45.3%

The SD values are increasing with distance. For the longest baseline, 86.8% of the SD are smaller than 0.025 m, which is quite good. This shows that Qinertia results are in general stable over time and there is not much difference between 1 h average logging and 15 min average logging.

Height difference 15 min logging compared to real time average 15 min logging out of tolerance is 10-20% higher per 15 min than per 1h. However, the averages match well with the real time data. The extreme values show min-max ranges of respectively 0.090 m, 0.166 m, 0.138 m and 0.164m. While at 1 h averages, only GDS1 min-max range was above 0.100 m. However, we always observe an increase of the outside tolerance rate of the height difference between post-processing and real time for both 1 h and 15 min logging.

III. CONCLUSION

The tests in this work were performed to find all the elements that go into the precise measurement of the height of OWF outside the standard RTK working range. For the installation of OWF foundations vertical installation tolerances of ±10 cm are imposed. Taking into account all error sources, deformation of the vessel, motion, lever arms that impact the height measurement of the foundation, the vertical accuracies for GNSS must be lower than 5 cm. In these tests, we considered the specifications of ±2.5 cm tolerance.

Based on test results, following conclusions are made:

- GNSS units:

The statistical analysis does not show a real difference between the different units, but a finer analysis of the time series graphs shows that overall all the receivers follow the same trend. But in some periods, units from the same brand follow the same trend and show significant variations from the other brands. Sometimes these variations are bigger than the tolerance, even if the same antenna is used. This confirms that using different receivers from different brands is necessary for reliable measure.

- Baseline length:

Data of DGPS5 were post-processed with base stations on different lengths. For shorter baseline the results are similar for both RTK mode and PPK mode. This is no longer the case for longer baseline. If, we are outside the manufacturer specification range +30 km for DGPS5, the results show a clear decrease in quality, but remains mostly within the recommended standards.

- RTK vs PPK:

During the tests, it was proved that for shorter baseline; according to manufacturer specification length PPK performances exceed RTK performances, despite the fact that for longer baseline PPK results show a decrease in quality. For static post-processing, the results of the different base stations match well with online services. For kinematic post-processing, the choice of the software program is important. The percentage of epochs in RTK fixed, per base with TBC software is very interesting and it looks like Qinertia has more difficulties to recover RTK fixed solution even for shorter

baseline. But, if Qinertia achieves RTK fixed, the results are very interesting, even for a short period logging. Although, if we want to reach at least a 95% certainty of having a logging within tolerance, the logging must be longer one hour or more and the base station should be within the manufacturer specifications range.

In conclusion, this paper shows that for precise installation of an Offshore Wind Farm (OWF) located far from the coastline, and to reach better measurement quality and centimeter accuracy outside the standard working range, it is important to take into account several parameters, like statistical parameters, settlements of the device brand, baseline length, time logging interval, and the choice of the software program. This work is of utmost importance in the GNSS application process and should open new possibilities in high accuracy positioning. In future work, we will test a high precision correction services from a Virtual Reference Station (VRS) method. This VRS service can provide RTK and PPK correction based on a network of base stations which can give stable results even in larger areas (longer baselines). This should improve the performances and maximize the centimeter level availability in challenging conditions.

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