Enhanced IVIFN-ExpTODIM-MABAC Technique for Multi-Attribute Group Decision-Making Under Interval-Valued Intuitionistic Fuzzy Sets

Applications to College English Teaching Quality Evaluation

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***Abstract*—The evaluation of English teaching quality is crucial for enhancing teaching effectiveness. It helps teachers understand their teaching methods and students' learning outcomes through systematic assessment, thereby guiding teachers to adjust their teaching strategies. Additionally, the results of the evaluation provide decision-making support for educational management at schools, optimizing curriculum design and resource allocation. Regular evaluations of teaching quality motivate teachers for continuous professional development, improve teaching standards, and ensure that students achieve maximum growth and progress in their English learning journey. The assessment of college English teaching quality employs multi-attribute group decision-making (MAGDM). Techniques like Exponential TODIM (ExpTODIM) and MABAC are utilized to facilitate MAGDM. During the evaluation process, interval-valued intuitionistic fuzzy sets (IVIFSs) are utilized to handle fuzzy data. This research introduces a novel method, the interval-valued intuitionistic fuzzy number ExpTODIM-MABAC (IVIFN-ExpTODIM-MABAC) tailored for MAGDM under the framework of IVIFSs. To demonstrate its efficacy, a numerical example evaluating college English teaching quality is presented. Key contributions of this study include: (1) Extending the ExpTODIM-MABAC method to include IVIFSs with an Entropy model; (2) Utilizing Entropy to ascertain weights within IVIFSs; (3) Proposing the IVIFN-ExpTODIM-MABAC approach for MAGDM under IVIFSs; (4) Validating the approach with a numerical example and various comparative analyses of college English teaching quality.**

Keywords—Multi-attribute group decision-making (MAGDM); interval-valued intuitionistic fuzzy sets (IVIFSs); ExpTODIM approach; MABAC approach; college English teaching quality evaluation

I. Introduction

The evaluation of English teaching quality is a critical component in educational management and pedagogical improvement. It not only focuses on the teaching skills and methods of teachers but also encompasses students' learning outcomes, the teaching environment, and curriculum content among other aspects [1]. The purpose and significance of evaluating English teaching quality are extensive and can be analyzed in the following dimensions: (1) Enhancing teaching quality. Teaching quality evaluation, through regular checks and feedback mechanisms, helps teachers recognize deficiencies in their teaching methods and techniques, thereby encouraging them to take measures for improvement. This evaluation process promotes continuous learning and self-improvement among teachers, ensuring that teaching methods stay current [2]. Additionally, teaching quality evaluations help teachers better manage course pacing and adjust teaching strategies to ensure the appropriateness and effectiveness of the content delivered. (2) Promoting student learning. Another important goal of teaching quality evaluation is to monitor and enhance students' learning outcomes [3-5]. During this process, teachers can understand students' interests, motivations, and challenges in learning English and then adopt personalized teaching approaches to meet the needs of different students [6]. For instance, for students with weaker foundations, teachers might increase tutoring time and use more interactive and visual aids to enhance their interest and efficiency in learning. (3) Optimizing curriculum design. Through the assessment of teaching quality, educational institutions can obtain direct feedback on which teaching contents and methods are most effective, which is crucial for curriculum design and textbook compilation [7, 8]. Evaluation results help education decision-makers understand changes in market and student needs, thereby optimizing the course structure, introducing new teaching contents, or eliminating outdated educational elements [9]. (4) Ensuring fairness and transparency in teaching. Regular teaching quality evaluations provide clear and fair assessment standards for all teachers and students. This standardized evaluation process helps eliminate biases and subjectivity, ensuring that every student is educated and assessed under the same standards, thus promoting fairness and transparency in educational activities [10, 11]. (5) Driving educational innovation. The results of teaching quality evaluations are a crucial basis for promoting educational innovation. Through evaluations, educational institutions can discover the effectiveness of new teaching methods and technologies, encouraging teachers to try and implement innovative teaching strategies. For example, with the development of information technology, many schools have begun experimenting with new teaching models such as blended learning and flipped classrooms. Teaching quality evaluations can effectively monitor the actual effects and adaptability of these new methods [12-14]. In conclusion, the evaluation of English teaching quality is a key tool for enhancing teaching effectiveness, ensuring educational fairness, and promoting comprehensive student development. It not only helps teachers and educational institutions understand the actual effects of

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teaching but also compels educational policies and practices to continuously adjust and improve. Therefore, establishing and maintaining a scientific and comprehensive teaching quality evaluation system is fundamental to improving educational quality and achieving educational modernization. Through such an evaluation system, we can better meet students' learning needs, enhance teachers' teaching abilities, and optimize the allocation of educational resources [15-17].

MAGDM is a collaborative decision-making process that brings together decision-makers from various backgrounds to evaluate multiple options across several attributes [18-21]. This method is prevalent in diverse sectors such as economics, management, engineering, and public policy, where complex decisions need balanced and comprehensive assessments. The essence of MAGDM lies in its ability to utilize the collective intelligence and resources of a group to pinpoint the most effective decision, considering a broad spectrum of factors. The purpose of MAGDM is multifaceted. Primarily, it aims to amalgamate diverse perspectives into a single decision-making process [22-25]. This integration is crucial in complex scenarios where no single decision-maker possesses enough information or expertise to cover all aspects of a decision comprehensively [26-28]. By involving multiple stakeholders, the decision-making process becomes more holistic and inclusive, thus enhancing the decision's quality and breadth. Moreover, MAGDM enhances the rationality and acceptance of decisions. Group discussions and evaluations increase transparency and fairness, fostering a decision-making environment where outcomes are more likely to be rational and broadly accepted. This is particularly important in settings where decisions have significant social, economic, or environmental impacts. Another critical aim of MAGDM is to optimize resource allocation. In environments where resources are scarce, this method facilitates a rational distribution based on thorough analysis and group consensus, ensuring that every decision maximizes the potential benefits from the available resources [29-32]. MAGDM also serves as a robust framework for dealing with complexity and uncertainty. By incorporating various data points and expert opinions, it provides a methodological approach to handle uncertainties inherent in dynamic environments, thereby stabilizing the decision-making process against external fluctuations [33-37]. The significance of MAGDM extends beyond just making effective decisions. It also enhances collaboration and communication among team members, who must share information and discuss options thoroughly before arriving at a consensus [38-42]. This interaction not only improves the decision outcomes but also strengthens team dynamics and cooperation. Furthermore, MAGDM promotes adaptability. As situations evolve and new information emerges, this decision-making framework can adapt, allowing decision-makers to revise their strategies promptly and efficiently [43-48]. This flexibility is vital in responding to changing conditions and maintaining the relevance and effectiveness of decisions. Lastly, the process of MAGDM increases both transparency and accountability. By documenting each decision-making step and its basis, the process ensures that all actions are traceable and decision-makers are accountable. This transparency is crucial for maintaining public trust, especially in decisions that affect broader communities or have significant public implications [38, 39, 49, 50]. In practical terms, MAGDM is seen in actions such as government policy formulation, corporate strategic planning, and new product development. For instance, in urban planning, government officials utilize MAGDM to integrate considerations like cost, public opinion, and environmental impact to devise comprehensive, sustainable urban transportation systems. In conclusion, MAGDM is more than just a decision-making tool; it is a critical process that taps into collective expertise to tackle complex problems efficiently and effectively. As the challenges in various fields grow more intricate, the role of MAGDM becomes increasingly vital, making it an indispensable element in the decision-making landscape of modern society. The evaluation of college English teaching quality involves MAGDM. IVIFSs [51] are used to represent fuzzy information during this evaluation. Various decision techniques have been proposed for MAGDM, managing the traditional ExpTODIM [52, 53] and MABAC [54, 55] techniques separately. However, there have been few or no techniques that combine the entropy model [56] with ExpTODIM [52, 53] and MABAC [54, 55] techniques under IVIFSs. Consequently, the IVIFN-ExpTODIM-MABAC technique is constructed to handle MAGDM under IVIFSs. A numerical example for evaluating college English teaching quality alongside different comparative analyses is provided to demonstrate the IVIFN-ExpTODIM-MABAC technique. The main objectives and motivations of this research include: (1) Enhancing the ExpTODIM-MABAC approach by integrating IVIFSs with the entropy framework. (2) Utilizing entropy to ascertain weights within IVIFSs. (3) Introducing the IVIFN-ExpTODIM-MABAC method for MAGDM in the context of IVIFSs. (4) Offering a numerical example and performing diverse comparative analyses to verify the IVIFN-ExpTODIM-MABAC model's effectiveness in assessing college English teaching quality.

The structure of the research is systematically organized. Section II introduces the IVIFSs. In Section III, the IVIFN-ExpTODIM-MABAC method is comprehensively proposed for MAGDM using IVIFSs. Section IV offers a numerical example that evaluates the quality of college English teaching, accompanied by comparative analyses. Section V concludes the paper with final remarks.

II. Preliminaries

Atanassov [51] performed the IVIFSs.

Definition 1 [51]. The IVIFSs is introduced:

$$FI = \left\{ \left\langle FM(\zeta), FN(\zeta) \right\rangle \middle| \zeta \in X \right\} \tag{1}$$

where $FM(\zeta) \subset [0,1]$ is membership and $FN(\zeta) \subset [0,1]$ is non-membership, and $FM(\zeta), FN(\zeta)$ meet condition: $0 \le \sup FM(\zeta) + \sup FN(\zeta) \le 1$, $\forall \zeta \in X$. For convenience, $FI = ([FA, FB],[FC, FD])$ is IVIFN.

Definition 2 [57]. Let $FI_1 = \left(\left[FA_1, FB_1\right], \left[FC_1, FD_1\right]\right)$ and $FI_2 = \left(\left[FA_2, FB_2\right], \left[FC_2, FD_2\right]\right)$ be IVIFNs, the operation laws are performed:

$$FI_1 \oplus FI_2 = \begin{pmatrix} \left[FA_1 + FA_2 - FA_1 FA_2, FB_1 + FB_2 - FB_1 FB_2\right], \\ \left[FC_1 FC_2, FD_1 FD_2\right] \end{pmatrix} \quad (2)$$

$$FI_1 \otimes FI_2 = \begin{pmatrix} \left[FA_1 FA_2, FB_1 FB_2\right], \\ \left[FC_1 + FC_2 - FC_1 FC_2, FD_1 + FD_2 - FD_1 FD_2\right] \end{pmatrix} \quad (3)$$

$$\lambda FI_1 = \left(\left[1-\left(1-FA_1\right)^{\lambda}, 1-\left(1-FB_1\right)^{\lambda}\right], \left[\left(FC_1\right)^{\lambda}, \left(FD_1\right)^{\lambda}\right]\right), \lambda > 0 \quad (4)$$

$$FI_1^{\lambda} = \left(\left[\left(FA_1\right)^{\lambda}, \left(FB_1\right)^{\lambda}\right], \left[1-\left(1-FC_1\right)^{\lambda}, 1-\left(1-FD_1\right)^{\lambda}\right]\right), \lambda > 0 \quad (5)$$

From Definition 2, several properties are established.

$$\text{(1) } FI_1 \oplus FI_2 = FI_2 \oplus FI_1, FI_1 \otimes FI_2 = FI_2 \otimes FI_1, \left(\left(FI_1\right)^{\lambda_1}\right)^{\lambda_2} = \left(FI_1\right)^{\lambda_1 \lambda_2};$$

$$\text{(2) } \lambda\left(FI_1 \oplus FI_2\right) = \lambda FI_1 \oplus \lambda FI_2, \left(FI_1 \otimes FI_2\right)^{\lambda} = \left(FI_1\right)^{\lambda} \otimes \left(FI_2\right)^{\lambda};$$

$$\text{(3) } \lambda_1 FI_1 \oplus \lambda_2 FI_1 = \left(\lambda_1 + \lambda_2\right) FI_1, \left(FI_1\right)^{\lambda_1} \otimes \left(FI_1\right)^{\lambda_2} = \left(FI_1\right)^{\left(\lambda_1 + \lambda_2\right)}.$$

Definition 3 [58]. Let $FI_1 = \left(\left[FA_1, FB_1\right], \left[FC_1, FD_1\right]\right)$ and $FI_2 = \left(\left[FA_2, FB_2\right], \left[FC_2, FD_2\right]\right)$ be IVIFNs, both the score value (SV) and accuracy value (AV) of FI_1 and FI_2 are performed:

$$SV\left(FI_1\right) = \frac{FA_1 + FA_1\left(1 - FA_1 - FC_1\right) + FB_1 + FB_1\left(1 - FB_1 - FD_1\right)}{2} \quad (6)$$

$$SV\left(FI_2\right) = \frac{FA_2 + FA_2\left(1 - FA_2 - FC_2\right) + FB_2 + FB_2\left(1 - FB_2 - FD_2\right)}{2} \quad (7)$$

$$AV\left(FI_1\right) = \frac{FA_1 + FC_1 + FB_1 + FD_1}{2}, \quad (8)$$

$$AV\left(FI_2\right) = \frac{FA_2 + FC_2 + FB_2 + FD_2}{2} \quad (9)$$

For TI_1 and TI_2, from Definition 3, then

$$\text{(1) } if\ SV\left(FI_1\right) < SV\left(FI_2\right),\ FI_1 < FI_2;$$

$$\text{(2) } if\ SV\left(FI_1\right) > SV\left(FI_2\right), FI_1 > FI_2;$$

$$\text{(3) } if\ SV\left(FI_1\right) = SV\left(FI_2\right), AV\left(FI_1\right) < AV\left(FI_2\right),\ FI_1 < FI_2;$$

$$\text{(4) } if\ SV\left(FI_1\right) = SV\left(FI_2\right), AV\left(FI_1\right) = AV\left(FI_2\right),\ FI_1 = FI_2.$$

Definition 4[59]. Let $FI_1 = \left(\left[FA_1, FB_1\right], \left[FC_1, FD_1\right]\right)$ and $FI_2 = \left(\left[FA_2, FB_2\right], \left[FC_2, FD_2\right]\right)$, the IVIFN Hamming distance (IVIFNHD) and IVIFN Euclidean distance (IVIFNED) are performed:

$$IVIFNHD\left(FI_1, FI_2\right) = \frac{1}{4}\begin{bmatrix} \left|FA_1 - FA_2\right| + \left|FB_1 - FB_2\right| \\ + \left|FC_1 - FC_2\right| + \left|FD_1 - FD_2\right| \end{bmatrix} \quad \text{(10-a)}$$

$$IVIFNED\left(FI_1, FI_2\right) = \sqrt{\frac{1}{4}\begin{bmatrix} \left(FA_1 - FA_2\right)^2 + \left(FB_1 - FB_2\right)^2 \\ + \left(FC_1 - FC_2\right)^2 + \left(FD_1 - FD_2\right)^2 \end{bmatrix}} \quad \text{(10-b)}$$

The IVIFWA and IVIFWG approach is performed [60].

Definition 5 [60]. Let $FI_j = \left(\left[FA_j, FB_j\right], \left[FC_j, FD_j\right]\right)\left(j = 1, 2, \cdots, n\right)$ be IVIFNs, the IVIFWA approach is performed:

$$IVIFWA_{f\omega}(FI_1, FI_2, \ldots, FI_n) = \overset{n}{\underset{j=1}{\oplus}}\left(f\omega_j FI_j \right)$$
$$= \left(\begin{array}{l} \left[1-\prod_{j=1}^{n}\left(1-FA_j\right)^{f\omega_j}, 1-\prod_{j=1}^{n}\left(1-FB_j\right)^{f\omega_j} \right], \\ \left[\prod_{j=1}^{n}\left(FC_j\right)^{f\omega_j}, \prod_{j=1}^{n}\left(FD_j\right)^{f\omega_j} \right] \end{array} \right) \quad (11)$$

where $f\omega = \left(f\omega_1, f\omega_2, ..., f\omega_n \right)^T$ be weight numbers of $FI_j = \left(\left[FA_j, FB_j \right], \left[FC_j, FD_j \right] \right)$, $f\omega_j > 0, \sum_{j=1}^{n} f\omega_j = 1.$

Definition 6[61]. Let $FI_j = \left(\left[FA_j, FB_j \right], \left[FC_j, FD_j \right] \right) \left(j = 1, 2, \cdots, n \right)$ be IVIFNs, the IVIFWG approach is performed:

$$IVIFWG_{f\omega}\left(FI_1, FI_2, \ldots, FI_n\right) = \overset{n}{\underset{j=1}{\otimes}}\left(FI_j \right)^{f\omega_j}$$
$$= \left(\begin{array}{l} \left[\prod_{j=1}^{n}\left(FA_j\right)^{f\omega_j}, \prod_{j=1}^{n}\left(FB_j\right)^{f\omega_j} \right], \\ \left[1-\prod_{j=1}^{n}\left(1-FC_j\right)^{f\omega_j}, 1-\prod_{j=1}^{n}\left(1-FD_j\right)^{f\omega_j} \right] \end{array} \right) \quad (12)$$

where $f\omega = \left(f\omega_1, f\omega_2, ..., f\omega_n \right)^T$ be weight numbers of $FI_j = \left(\left[FA_j, FB_j \right], \left[FC_j, FD_j \right] \right)$, $f\omega_j > 0, \sum_{j=1}^{n} f\omega_j = 1.$

III. IVIFN-ExpTODIM-MABAC Approach for MAGDM with Entropy

A. IVIFN-MAGDM Issues

Then IVIFN-ExpTODIM-MABAC approach is performed for MAGDM. Let $FA = \left\{ FA_1, FA_2, \cdots, FA_m \right\}$ be alternatives and $FG = \left\{ FG_1, FG_2, \cdots, FG_n \right\}$ be attributes with weight values $f\omega = \left(f\omega_1, f\omega_2, ..., f\omega_n \right)^T$, where $f\omega_j \in [0,1]$, $\sum_{j=1}^{n} f\omega_j = 1$ and experts $FE = \left\{ FE_1, FE_2, \cdots, FE_q \right\}$ with weight values be $fw = \left\{ fw_1, fw_2, \cdots, fw_q \right\}$. Then, IVIFN-ExpTODIM-MABAC technique is put forward for MAGDM.

Step 1. Perform the IVIFN-matrix $FR^{(t)} = \left[FR_{ij}^{(t)} \right]_{m\times n} = \left(\left[FA_{ij}^{(t)}, FB_{ij}^{(t)} \right], \left[FC_{ij}^{(t)}, FD_{ij}^{(t)} \right] \right)_{m\times n}$ and average matrix $FR = \left[FR_{ij} \right]_{m\times n}$:

$$FR^{(t)} = \left[FR_{ij}^{(t)} \right]_{m\times n} = \begin{array}{c} \\ FA_1 \\ FA_2 \\ \vdots \\ FA_m \end{array} \begin{array}{c} \begin{array}{cccc} FG_1 & FG_2 & \ldots & FG_n \end{array} \\ \left[\begin{array}{cccc} FR_{11}^{(t)} & FR_{12}^{(t)} & \ldots & FR_{1n}^{(t)} \\ FR_{21}^{(t)} & FR_{22}^{(t)} & \ldots & FR_{2n}^{(t)} \\ \vdots & \vdots & \vdots & \vdots \\ FR_{m1}^{(t)} & FR_{m2}^{(t)} & \ldots & FR_{mn}^{(t)} \end{array} \right] \end{array} \quad (13)$$

$$FR = \left[FR_{ij} \right]_{m\times n} = \begin{array}{c} \\ FA_1 \\ FA_2 \\ \vdots \\ FA_m \end{array} \begin{array}{c} \begin{array}{cccc} FG_1 & FG_2 & \ldots & FG_n \end{array} \\ \left[\begin{array}{cccc} FR_{11} & FR_{12} & \ldots & FR_{1n} \\ FR_{21} & FR_{22} & \ldots & FR_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ FR_{m1} & FR_{m2} & \ldots & FR_{mn} \end{array} \right] \end{array} \quad (14)$$

Based on IVIFWG technique, the $FR = \left[FR_{ij} \right]_{m\times n} = \left(\left[FA_{ij}, FB_{ij} \right], \left[FC_{ij}, FD_{ij} \right] \right)_{m\times n}$ is performed in Eq. (15):

$$FR_{ij} = \left(FR_{ij}^{(t)} \right)^{fw_1} \otimes \left(FR_{ij}^{(t)} \right)^{fw_2} \otimes \cdots \otimes \left(FR_{ij}^{(t)} \right)^{fw_q}$$
$$= \left(\begin{array}{l} \left[\prod_{t=1}^{q}\left(FA_{ij}^{(t)}\right)^{fw_t}, \prod_{t=1}^{q}\left(FB_{ij}^{(t)}\right)^{fw_t} \right], \\ \left[1-\prod_{t=1}^{q}\left(1-FC_{ij}^{(t)}\right)^{fw_t}, 1-\prod_{t=1}^{q}\left(1-FD_{ij}^{(t)}\right)^{fw_t} \right] \end{array} \right) \quad (15)$$

Step 2. The $FR = \left[FR_{ij} \right]_{m\times n} = \left(\left[FA_{ij}, FB_{ij} \right], \left[FC_{ij}, FD_{ij} \right] \right)_{m\times n}$ is normalized to $NFR = \left[NFR_{ij} \right]_{m\times n} = \left(\left[FA_{ij}^N, FB_{ij}^N \right], \left[FC_{ij}^N, FD_{ij}^N \right] \right)_{m\times n}$ in Eq. (16, 17).

For benefit attributes:

$$NFR_{ij} = \left(\left[FA_{ij}^N, FB_{ij}^N \right], \left[FC_{ij}^N, FD_{ij}^N \right] \right) = \left(\left[FA_{ij}, FB_{ij} \right], \left[FC_{ij}, FD_{ij} \right] \right) \quad (16)$$

For cost attributes:

$$NFR_{ij} = \left(\left[FA_{ij}^N, FB_{ij}^N \right], \left[FC_{ij}^N, FD_{ij}^N \right] \right) = \left(\left[FC_{ij}, FD_{ij} \right], \left[FA_{ij}, FB_{ij} \right] \right) \quad (17)$$

B. Perform the Attributes Weight through Entropy

Step 3. The weight numbers are important for MAGDM [62-64]. Entropy technique [56] is put forward weight numbers under IVIFSs. The normalized decision matrix is performed in Eq. (18):

$$NDM_{ij} = \frac{\begin{pmatrix} SV\left(\left[FA_{ij}^{N}, FB_{ij}^{N}\right], \left[FC_{ij}^{N}, FD_{ij}^{N}\right]\right) \\ +AV\left(\left[FA_{ij}^{N}, FB_{ij}^{N}\right], \left[FC_{ij}^{N}, FD_{ij}^{N}\right]\right) + 0.5 \end{pmatrix}}{\sum_{i=1}^{m}\left(\begin{pmatrix} SV\left(\left[FA_{ij}^{N}, FB_{ij}^{N}\right], \left[FC_{ij}^{N}, FD_{ij}^{N}\right]\right) \\ +AV\left(\left[FA_{ij}^{N}, FB_{ij}^{N}\right], \left[FC_{ij}^{N}, FD_{ij}^{N}\right]\right) + 0.5 \end{pmatrix}\right)}, \quad (18)$$

The IVIFN Shannon entropy (IVIFNSE) is produced in Eq. (19):

$$IVIFNSE_j = -\frac{1}{\ln m}\sum_{i=1}^{m} NDM_{ij} \ln NDM_{ij} \quad (19)$$

and $NDM_{ij} \ln NDM_{ij} = 0$ if $TIVIFNDM_{ij} = 0$.

The weight numbers are performed in Eq. (20):

$$f\omega_j = \frac{1 - IVIFNSE_j}{\sum_{j=1}^{n}\left(1 - IVIFNSE_j\right)} \quad (20)$$

C. IVIFN-ExpTODIM-MABAC Approach for MAGDM

The IVIFN-ExpTODIM-MABAC is performed for MAGDM.

Step 4. Perform relative weight in Eq. (21):

$$rf\omega_j = f\omega_j \big/ \max_j f\omega_j, \quad (21)$$

Step 5. The IVIFN dominance degree (IVIFNDD) is performed in Eq. (22) based on two kinds of distances measures:

$$IVIFNDD_j\left(FA_i, FA_t\right) = \begin{cases} \frac{1}{2}\left(\begin{matrix} \dfrac{rf\omega_j \times \left(1 - 10^{-\rho IVIFNED\left(NFR_{ij}, NFR_{tj}\right)}\right)}{\sum_{j=1}^{n} rf\omega_j} \\ + \dfrac{rf\omega_j \times \left(1 - 10^{-\rho IVIFNED\left(NFR_{ij}, NFR_{tj}\right)}\right)}{\sum_{j=1}^{n} rf\omega_j} \end{matrix}\right) & \text{if } SV\left(NFR_{ij}\right) > SV\left(NFR_{tj}\right) \\ 0 & \text{if } SV\left(NFR_{ij}\right) = SV\left(NFR_{tj}\right) \\ \frac{1}{2}\left(\begin{matrix} -\dfrac{1}{\theta}\dfrac{\sum_{j=1}^{n} rt\omega_j \times \left(1 - 10^{-\rho IVIFNED\left(NTR_{ij}, NTR_{tj}\right)}\right)}{rt\omega_j} \\ -\dfrac{1}{\theta}\dfrac{\sum_{j=1}^{n} rt\omega_j \times \left(1 - 10^{-\rho IVIFNED\left(NTR_{ij}, NTR_{tj}\right)}\right)}{rt\omega_j} \end{matrix}\right) & \text{if } SV\left(NFR_{ij}\right) < SV\left(NFR_{tj}\right) \end{cases} \quad (22)$$

where θ is from [65] and $\rho \in [1,5]$ [66].

The IVIFNDD under FG_j is performed:

$$IVIFNDD_j\left(FA_i\right)=\left[IVIFNDD_j\left(FA_i,FA_t\right)\right]_{m\times m}$$

$$=\begin{matrix} & FA_1 & FA_2 & \cdots & FA_m \\ \begin{matrix} FA_1 \\ FA_2 \\ \vdots \\ FA_m \end{matrix} & \left[\begin{matrix} 0 \\ IVIFNDD_j\left(FA_2,FA_1\right) \\ \vdots \\ IVIFNDD_j\left(FA_m,FA_1\right) \end{matrix}\right. & \begin{matrix} IVIFNDD_j\left(FA_1,FA_2\right) \\ 0 \\ \vdots \\ IVIFNDD_j\left(FA_m,FA_2\right) \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} & \left.\begin{matrix} IVIFNDD_j\left(FA_1,FA_m\right) \\ IVIFNDD_j\left(FA_2,FA_m\right) \\ \vdots \\ 0 \end{matrix}\right] \end{matrix}$$

(3) Perform the IVIFNDD of FA_i under FG_j :

$$IVIFNDD_j\left(FA_i\right)=\sum_{t=1}^{m}IVIFNDD_j\left(FA_i,FA_t\right) \tag{23}$$

The IVIFNDD is performed:

$$IVIFNDD=\left(IVIFNDD_{ij}\right)_{m\times n}$$

$$=\begin{bmatrix} & FG_1 & FG_2 & \ldots & FG_n \\ FA_1 & \sum_{t=1}^{m}IVIFNDD_1\left(FA_1,FA_t\right) & \sum_{t=1}^{m}IVIFNDD_2\left(FA_1,FA_t\right) & \ldots & \sum_{t=1}^{m}IVIFNDD_n\left(FA_1,FA_t\right) \\ FA_2 & \sum_{t=1}^{m}IVIFNDD_1\left(FA_2,FA_t\right) & \sum_{t=1}^{m}IVIFNDD_2\left(FA_2,FA_t\right) & \ldots & \sum_{t=1}^{m}IVIFNDD_n\left(FA_2,FA_t\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ FA_m & \sum_{t=1}^{m}IVIFNDD_1\left(FA_m,FA_t\right) & \sum_{t=1}^{m}IVIFNDD_2\left(FA_m,FA_t\right) & \ldots & \sum_{t=1}^{m}IVIFNDD_n\left(FA_m,FA_t\right) \end{bmatrix}$$

Step 6. Perform the IVIFNBAA (IVIFN Border Approximation Area).

$$IVIFNPIA_j=\max_{j=1}^{n}IVIFNDD_{ij},$$
$$IVIFNNIA_j=\min_{j=1}^{n}IVIFNDD_{ij} \tag{24}$$

$$NIVIFNDD_{ij}=1+\frac{IVIFNDD_{ij}-IVIFNNIA_j}{IVIFNPIA_j-IVIFNNIA_j}, \tag{25}$$

$$IVIFNBAA=\left[IVIFNBAA_j\right]_{1\times n} \tag{26}$$

$$IVIFNBAA_j=\prod_{j=1}^{n}\left(NIVIFNDD_{ij}\right)^{1/m}$$
$$=\prod_{j=1}^{n}\left(1+\frac{IVIFNDD_{ij}-IVIFNNIA_j}{IVIFNPIA_j-IVIFNNIA_j}\right)^{1/m} \tag{27}$$

Step 7. Perform the IVIFNDM (IVIFN distance measures) from IVIFNBAA:

$$IVIFNDM_{ij}=NIVIFNDD_{ij}-IVIFNBAA_j \tag{28}$$

Step 8. Perform the IVIFNWDM (IVIFN weighted distance measures) from IVIFNBAA.

$$IVIFNWDM_i=\sum_{j=1}^{n}\left(fw_j\times IVIFNDM_{ij}\right)$$
$$=\sum_{j=1}^{n}\left(fw_j\times\left(NIVIFNDD_{ij}-IVIFNBAA_j\right)\right) \tag{29}$$

Step 9. From the IVIFNWDM, the largest IVIFNWDM is the optimal choice.

IV. Numerical Example and Comparative Analysis

A. *Numerical Example for College English Teaching Quality Evaluation*

The evaluation of English teaching quality is a systematic process aimed at assessing the effectiveness of teaching and student learning outcomes to ensure that teaching activities effectively enhance students' English abilities. This process involves multiple aspects, including curriculum content, teaching methods, student engagement, the efficiency of learning resource use, and the teaching environment. Firstly, an essential aspect of the evaluation is the appropriateness of the curriculum content. This includes the rationality of the syllabus, the clarity of course objectives, and whether the depth and breadth of the content meet students' learning needs. Effective course design should cover all necessary language skills, such as listening, speaking, reading, and writing, and progressively enhance students' abilities through appropriate difficulty gradients. Secondly, the innovation and diversity of teaching methods are key to the evaluation. Teachers should use various teaching strategies according to the specific needs and learning styles of students, such as cooperative learning, project-based learning, and multimedia teaching, to enhance the interactivity and interest of learning. The teacher's teaching style and abilities, such as clear expression, effective classroom management, and the timeliness and appropriateness of feedback to students, are important indicators for evaluation. Student engagement is another crucial metric for assessing teaching quality. Students' interaction in class, enthusiasm for participating in discussions, the quality and attitude toward completing assignments, and their mastery of learning materials all reflect the effectiveness of teaching. Additionally, the evaluation should also focus on students' learning motivation and self-development, including their self-assessment abilities and the achievement of learning objectives. During the evaluation process, it is also essential to fully utilize various assessment tools and technologies, such as classroom observations, student evaluations, standardized tests, and performance displays. These tools can provide direct and indirect evidence of teaching effectiveness, helping teachers and educational administrators make corresponding adjustments and improvements. Finally, the support of teaching resources and the environment is also crucial for ensuring teaching quality. Schools should provide sufficient learning materials, modern teaching facilities, and an environment conducive to learning. Moreover, continuous professional development support for teachers and resource updates are key to enhancing teaching quality. In summary, the evaluation of English teaching quality is a multi-faceted assessment process that not only focuses on teachers' teaching behaviors and students' learning outcomes but also involves curriculum design, teaching methods, student engagement, and teaching resources and environments. Through this comprehensive evaluation system, teaching quality can be effectively enhanced, ensuring students achieve the best outcomes in their English learning. The college English teaching quality evaluation is MAGDM process. Five potential English colleges $FA_i\left(i=1,2,3,4,5\right)$ are assessed with four attributes (Table I):

Five potential English colleges $FA_i\left(i=1,2,3,4,5\right)$ are assessed through linguistic scales (see Table II [67]) under experts $FE_t\left(t=1,2,3\right)$ with expert's weight $fw=\left(1/3,1/3,1/3\right)$.

TABLE I. FOUR ATTRIBUTES FOR SUSTAINABLE DEVELOPMENT EVALUATION IN HIGHER EDUCATION MANAGEMENT

Attribute	Attribute description
Student Learning Outcomes-FG$_1$	This indicator focuses on the specific learning achievements students gain through English instruction. It can be assessed by examining improvements in students' language skills, exam scores, and performance in simulated applications. This indicator reflects whether teaching activities can effectively enhance students' English capabilities.
Cost-effectiveness of Teaching-FG$_2$	The cost-related indicator primarily evaluates the relationship between input and output. In English teaching, this includes calculating the input of teaching resources (such as textbook costs, equipment investment, teacher training expenses, etc.) and comparing it with teaching outcomes. For example, the cost-effectiveness of teaching can be evaluated by analyzing the average improvement in students' scores or progression in language skills per unit cost.
Teacher Performance-FG$_3$	Return on educational investment is a crucial indicator of a university's economic efficiency. By assessing the returns on educational investments, universities can optimize resource allocation, enhance education quality, and improve students' employability, thereby increasing competitiveness and social impact.
Student Engagement and Feedback-FG$_4$	Active student participation is one of the crucial factors for successful teaching. This indicator measures through assessing students' interaction in class, homework completion, the level of activity in class discussions, and students' feedback on the teaching content. Student satisfaction with the teaching process and their suggestions can be obtained through surveys, interviews, or observations, reflecting the attractiveness and effectiveness of teaching activities.

TABLE II. LINGUISTIC SCALES AND IVIFNS

Linguistic information scale	IVIFNs
Exceedingly Bad-FEB	<[0.05,0.10], [0.85, 0.90]>
Very Bad-FVB	<[0.10,0.15], [0.75, 0.85]>
Bad-FB	<[0.15,0.20], [0.60, 0.70]>
Medium-FM	<[0.50,0.50], [0.50, 0.50]>
Good-FG	<[0.60,0.70], [0.15, 0.20]>
Very Good-FVG	<[0.75,0.85], [0.10, 0.15]>
Exceedingly Good-FEG	<[0.85,0.90], [0.05, 0.10]>

The IVIFN-ExpTODIM-MABAC is performed for college English teaching quality evaluation.

Step 1. Describe the IVIFN-matrix $FR = \left[FR_{ij}^{(t)} \right]_{5\times4} \left(t = 1,2,3 \right)$ (Tables III to V).

TABLE III. EVALUATION VALUES FOR FE_1

	FG$_1$	FG$_2$	FG$_3$	FG$_4$
FA$_1$	FM	FB	FVG	FVB
FA$_2$	FVB	FVG	FG	FM
FA$_3$	FVG	FG	FM	FB
FA$_4$	FVB	FM	FG	FVG
FA$_5$	FM	FVB	FVB	FVG

TABLE IV. EVALUATION INFORMATION FOR FE_2

	FG$_1$	FG$_2$	FG$_3$	FG$_4$
FA$_1$	FG	FVB	FVB	FM
FA$_2$	FG	FM	FB	FVG
FA$_3$	FB	FG	FVG	FVB
FA$_4$	FM	FM	FG	FB
FA$_5$	FM	FG	FVG	FB

TABLE V. EVALUATION INFORMATION FOR FE_3

	FG$_1$	FG$_2$	FG$_3$	FG$_4$
FA$_1$	FVG	FG	FM	FB
FA$_2$	FVG	FB	FVB	FM
FA$_3$	FM	FVG	FB	FG
FA$_4$	FVB	FM	FVG	FG
FA$_5$	FVB	FVG	FM	FG

Then, employing IVIFWG approach, the $FR = \left[FR_{ij} \right]_{5\times4}$ is performed (see Table VI).

TABLE VI. THE $FR = \left[FR_{ij} \right]_{5\times4}$

Alternatives	FG$_1$	FG$_2$
FA$_1$	([0.2431, 0.3410], [0.4571, 0.5589])	([0.4301, 0.4899], [0.3170, 0.3689])
FA$_2$	([0.1305, 0.2297], [0.7680, 0.8664])	([0.1470, 0.2459], [0.6521, 0.7503])
FA$_3$	([0.2540, 0.3498], [0.5460, 0.6452])	([0.3286, 0.4275], [0.4721, 0.5709])
FA$_4$	([0.1012, 0.1990], [0.7009, 0.7991])	([0.2017, 0.2975], [0.6007, 0.6985])
FA$_5$	([0.2233, 0.3196], [0.5778, 0.6754])	([0.3034, 0.4012], [0.4954, 0.5932])
Alternatives	FG$_3$	FG$_5$
FA$_1$	([0.2021, 0.3049], [0.5813, 0.6810])	([0.2568, 0.3502], [0.5350, 0.6348])
FA$_2$	([0.3698, 0.4691], [0.4190, 0.5172])	([0.0945, 0.1907], [0.7082, 0.8056])
FA$_3$	([0.0593, 0.1551], [0.7435, 0.8407])	([0.1124, 0.2086], [0.6881, 0.7879])
FA$_4$	([0.3925, 0.4928], [0.4057, 0.5045])	([0.0032, 0.1070], [0.8003, 0.8965])
FA$_5$	([0.0117, 0.1095], [0.7889, 0.8867])	([0.3058, 0.4026], [0.4942, 0.5900])

Step 2. Normalize the $FR = \left[FR_{ij} \right]_{5\times4}$ to $NFR = \left[NFR_{ij} \right]_{5\times4}$ (Table VII).

TABLE VII. THE $NFR = \left[NFR_{ij} \right]_{5\times4}$

Alternatives	FG_1	FG_2
FA_1	([0.2431, 0.3410], [0.4571, 0.5589])	([0.3170, 0.3689], [0.4301, 0.4899])
FA_2	([0.1305, 0.2297], [0.7680, 0.8664])	([0.6521, 0.7503], [0.1470, 0.2459])
FA_3	([0.2540, 0.3498], [0.5460, 0.6452])	([0.4721, 0.5709], [0.3286, 0.4275])
FA_4	([0.1012, 0.1990], [0.7009, 0.7991])	([0.6007, 0.6985], [0.2017, 0.2975])
FA_5	([0.2233, 0.3196], [0.5778, 0.6754])	([0.4954, 0.5932], [0.3034, 0.4012])
Alternatives	FG_3	FG_5
FA_1	([0.2021, 0.3049], [0.5813, 0.6810])	([0.2568, 0.3502], [0.5350, 0.6348])
FA_2	([0.3698, 0.4691], [0.4190, 0.5172])	([0.0945, 0.1907], [0.7082, 0.8056])
FA_3	([0.0593, 0.1551], [0.7435, 0.8407])	([0.1124, 0.2086], [0.6881, 0.7879])
FA_4	([0.3925, 0.4928], [0.4057, 0.5045])	([0.0032, 0.1070], [0.8003, 0.8965])
FA_5	([0.0117, 0.1095], [0.7889, 0.8867])	([0.3058, 0.4026], [0.4942, 0.5900])

Step 3. Perform the weight numbers (Table VIII):

TABLE VIII. THE WEIGHT NUMBERS

	FG_1	FG_2	FG_3	FG_4
weight numbers	0.2599	0.3620	0.1912	0.1869

Step 4. Perform the relative weight numbers (Table IX):

TABLE IX. THE RELATIVE WEIGHT NUMBERS

	FG_1	FG_2	FG_3	FG_4
relative weight	0.7180	1.0000	0.5282	0.5163

Step 5. Perform the $IVIFNDD = \left(IVIFNDD_{ij} \right)_{5\times4}$ (see Table X):

TABLE X. THE $IVIFNDD = \left(IVIFNDD_{ij} \right)_{5\times4}$

	FG_1	FG_2	FG_3	FG_4
FA_1	1.6044	-2.5732	0.6424	-1.1950
FA_2	-0.9564	2.0414	-3.5905	0.9154
FA_3	-1.1400	-0.1256	-0.3628	1.6101
FA_4	1.1026	-0.7552	0.3485	0.5389
FA_5	-1.4750	-2.1363	-1.6255	-1.3453

Step 6. Perform the IVIFNBAA (see Tables XI to XIII).

TABLE XI. THE IVIFNPIA AND IVIFNNIA

	FG_1	FG_2	FG_3	FG_4
IVIFNPIA	1.6044	2.0414	0.6424	1.6101
IVIFNNIA	-1.4750	-2.5732	-3.5905	-1.3453

TABLE XII. THE NIVIFNDD

	FG_1	FG_2	FG_3	FG_4
FA_1	1.0000	1.2824	1.0000	1.2093
FA_2	1.3743	2.0000	2.0000	1.3730
FA_3	1.3112	1.5068	1.5253	2.0000
FA_4	1.0417	1.7260	1.6653	1.7196
FA_5	2.0000	1.0000	1.6899	1.0000

TABLE XIII. THE IVIFNBAA

	FG_1	FG_2	FG_3	FG_4
IVIFNBAA	1.3662	1.3611	1.5838	1.4345

Step 7. Perform the IVIFNDM (see Tables XIV and XV):

TABLE XIV. THE IVIFNDM

	FG_1	FG_2	FG_3	FG_4
FA_1	0.6338	-0.3611	0.4162	-0.3836
FA_2	-0.1978	0.6389	-0.5838	0.3305
FA_3	-0.2574	0.1694	0.1788	0.5655
FA_4	0.4708	0.0329	0.3468	0.2031
FA_5	-0.3662	-0.2664	-0.1195	-0.4345

TABLE XV. THE WEIGHTED IVIFNDM

	FG_1	FG_2	FG_3	FG_4
FA_1	0.1647	-0.1307	0.0796	-0.0717
FA_2	-0.0514	0.2313	-0.1116	0.0618
FA_3	-0.0669	0.0613	0.0342	0.1057
FA_4	0.1224	0.0119	0.0663	0.0380
FA_5	-0.0952	-0.0964	-0.0229	-0.0812

Step 8. Perform the IVIFNWDM (see Table XVI).

TABLE XVI. THE IVIFNWDM

	IVIFNWDM	Order
FA_1	0.0419	4
FA_2	0.1300	3
FA_3	0.1343	2
FA_4	0.2385	1
FA_5	-0.2957	5

Step 9. From the IVIFNWDIM, the order is performed: $FA_4 > FA_3 > FA_2 > FA_1 > FA_5$ and TA_4 is the best English college.

B. Comparative Analysis

Then, the IVIFN-ExpTODIM-MABAC approach is compared with IVIFWA approach [60], IVIFWG approach [61], IVIFPWA approach [68], IVIFPWG approach [68], IVIFN-MABAC approach [69], IVIFN-Taxonomy approach [70] and IVIFN-TODIM approach [71]. The comparative results are performed in Table XVII.

TABLE XVII. ORDER FOR DIFFERENT APPROACHES

Approaches	Order
IVIFWA approach [60]	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$
IVIFWG approach [61]	$FA_4 > FA_3 > FA_1 > FA_2 > FA_5$
IVIFPWA approach [68]	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$
IVIFPWG approach [68],	$FA_4 > FA_3 > FA_1 > FA_2 > FA_5$
IVIFN-MABAC approach [69]	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$
IVIFN-Taxonomy approach [70]	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$
IVIFN-TODIM approach [71]	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$
IVIFN-ExpTODIM-MABAC approach	$FA_4 > FA_3 > FA_2 > FA_1 > FA_5$

In light with RW coefficients [72], the RW coefficient between IVIFWA approach [60], IVIFWG approach [61], IVIFPWA approach [68], IVIFPWG approach [68], IVIFN-MABAC approach [69], IVIFN-Taxonomy approach [70], IVIFN-TODIM approach [71] and IVIFN-ExpTODIM-MABAC approach is 1.0000, 0.9246, 1.0000, 0. 9246, 1.0000, 1.0000, 1.0000. This verifies the IVIFN-ExpTODIM-MABAC approach is effective.

V. CONCLUSION

The purpose of evaluating the quality of English teaching is to ensure the effectiveness of teaching methods, content, and resources, and to improve the teaching skills of educators and the learning outcomes of students. Through evaluation, strengths and weaknesses in teaching can be identified, promoting the professional development of teachers and innovation in teaching methods. Moreover, this process helps in designing courses that better meet the needs of students, enhancing the overall level of teaching, and ultimately aiming to improve students' English proficiency. The evaluation of college English teaching quality involves MAGDM. Currently, the TODIM-MABAC method is utilized in MAGDM contexts, with IVIFSs used to represent fuzzy decision data during the assessment. This research introduces the IVIFN-ExpTODIM-MABAC model for MAGDM, incorporating IVIFSs. To validate the effectiveness of the IVIFN-ExpTODIM-MABAC approach, a numerical example focusing on the evaluation of college English teaching quality is presented, alongside several comparative analyses. The key contributions of this study are: (1) The expansion of the ExpTODIM-MABAC method to include IVIFSs and an entropy model. (2) The use of entropy to assign weights within the context of IVIFSs. (3) The introduction of the IVIFN-ExpTODIM-MABAC method for MAGDM using IVIFSs. (4) The provision of a numerical example and execution of various comparative analyses to confirm the applicability of the IVIFN-ExpTODIM-MABAC model in assessing college English teaching quality.

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