Enhanced Methodology for Production-Education Integration and Quality Evaluation of Rural Vocational Education Under Rural Revitalization with 2-Tuple Linguistic Neutrosophic Numbers

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Abstract—Rural vocational education (RVE) plays a crucial role in nurturing practical talents for the development of rural economy and society in the new era, as well as cultivating future generations of agricultural successors. Quality evaluation serves as an essential means of ensuring educational excellence. It acts as a key element for supervision, assurance, and enhancement of educational quality. For the production-education integration (PEI) in RVE, establishing a quality evaluation system that aligns with national and rural conditions, caters to the needs of modern agricultural industry development, and reflects the characteristics of RVE is crucial. Such a system plays a vital role in leading and promoting the deep integration of industry and education in rural vocational colleges. The PEI quality evaluation in RVE under rural revitalization involves MAGDM. Currently, Exponential TODIM (ExpTODIM) approach and grey relational analysis (GRA) approach has been utilized to address MAGDM challenges. To handle uncertain information in PEI quality evaluation in RVE under rural revitalization, 2-tuple linguistic neutrosophic sets (2TLNSs) are conducted as a valuable tool. This paper introduces the implementation of the 2-tuple linguistic neutrosophic number Exponential TODIM-GRA (2TLNN-ExpTODIM-GRA) approach to effectively manage MAGDM problems using 2TLNSs. Additionally, a numerical study is conducted to validate the application of this approach for PEI quality evaluation in RVE under rural revitalization.

Keywords—Multiple-attribute group decision-making (MAGDM); 2TLNSs; ExpTODIM approach; GRA approach; PEI quality evaluation

I. INTRODUCTION

The adaptability of RVE to the needs of rural economic and social development in the new era is a prerequisite for RVE to fully play its inherent role and effectively serve rural economic and social development [1-3]. "Adaptability" means changing with the external environment. Under the rural revitalization strategy, development of the rural economy and society in the new era, agricultural modernization, and new urbanization construction, external development environment of RVE has undergone tremendous changes [4, 5]. This poses new tasks and requirements for RVE, requiring it to adapt to external environment changes and times demands, and timely adjust and optimize its positioning, development ideas, program offerings, and talent development goals to meet the changing external environment and the requirements of the times [6, 7]. Therefore,

to assess and evaluate the adaptability of RVE to the needs of rural economic development and social development for a new era, it is necessary to examine the positioning of agricultural colleges and the positioning of talent development goals from the perspective of external demand. It is important to monitor the compatibility between the development plans of agricultural colleges and rural revitalization strategy, rural economic and social development, agricultural modernization, urbanization construction, and local agricultural development and rural development plans [8, 9]. It is necessary to observe whether the development plans are synchronized with the new era's rural economic and social development plans [10, 11]. It is crucial to observe the alignment between the professional development plans of agricultural colleges and local industries, especially the relationship between the school's advantages, distinctive programs, and leading majors, and the layout and structure of local modern agricultural industries [12, 13]. The construction of programs should closely follow the dynamic adjustment mechanism of local agricultural industry transformation and upgrading. It is essential to observe the alignment between the talent development goals of colleges and the needs of various types of rural talents and their qualifications and structure required for rural revitalization [14, 15]. The talent development goals and specifications of each major should reflect the need for compound and innovative rural talents with systematic knowledge structures and comprehensive abilities to adapt to new industries development, new formats, and new modes for rural areas [16-18].

Decision-making is a deliberate and selective process employed by humans to accomplish specific objectives [19-22]. Essentially, it involves utilizing available decision information, employing a particular method to evaluate various alternative solutions, and selecting the best solution that aligns with the decision-maker's expectations [23-27]. In our daily lives, decision-making is intertwined with various aspects such as food, clothing, housing, and transportation, where we constantly make choices [28-35]. The PEI quality evaluation of RVE under rural revitalization involves MAGDM. To address MAGDM challenges, recent approaches have utilized the ExpTODIM approach [36, 37] and GRA approach [38] approach [37]. Additionally, 2TLNSs [39] have been administrated to represent uncertain information during the PEI quality evaluation of RVE under rural revitalization. This paper introduces ExpTODIM-GRA approach

specifically to manage MAGDM problems using 2TLNSs. Firstly, the basic concepts of 2TLNSs are reviewed. Then, ExpTODIM-GRA approach is administrated to address MAGDM under 2TLNSs. Finally, numerical study is administrated to validate the effectiveness of 2TLNN-ExpTODIM-GRA in the PEI quality evaluation of RVE under rural revitalization. In summary, the motivation behind this study is administrated: (1) the utilization of Entropy [39] to obtain attribute weights with 2TLNSs; (2) the implementation of the 2TLNN-ExpTODIM-GRA approach to manage the MAGDM with 2TLNSs; (3) the construction of numerical study to demonstrate the application of the administrated approach in the PEI quality evaluation of RVE under rural revitalization; and (4) the provision of comparative analyses with existing approaches.

The organization of this work is conducted: Section II introduces 2TLNSs. Section III describes the implementation of 2TLNN-ExpTODIM-GRA approach under the framework of 2TLNSs, incorporating the entropy approach. In Section IV, numerical example is presented to showcase the PEI quality evaluation in RVE under rural revitalization, accompanied by a comparative analysis. Finally, Section V offers concluding remarks to wrap up the study.

II. Preliminaries

Wang et al. [39] administrated the 2TLNSs.

Definition 1[28-29]. Let $ds_1, ds_2, ..., ds_{\phi}$ be linguistic information and ds is administrated:

$$ds = \begin{cases} ds_0 = extremely \ poor, ds_1 = very \ poor, \\ ds_2 = poor, ds_3 = medium, ds_4 = good, \\ ds_5 = very \ good, ds_6 = extremely \ good. \end{cases}$$

Definition 2[6-7]. The SVNSs is administrated:

$$d\eta = \left\{ \left(\theta, \phi_{\eta}(\theta), \varphi_{\eta}(\theta), \gamma_{\eta}(\theta)\right) \middle| \theta \in \Theta \right\} \tag{1}$$

where $\phi_{\eta}\left(\theta\right), \varphi_{\eta}\left(\theta\right), \gamma_{\eta}\left(\theta\right) \in \left[0,1\right]$ is truth-membership (TM), indeterminacy-membership (IM) and falsity-membership (FM), $0 \le \phi_{\eta}\left(\theta\right) + \varphi_{\eta}\left(\theta\right) + \gamma_{\eta}\left(\theta\right) \le 3$.

Definition 3[39]. Let $d\delta_j$ (j=1,2,3,...,k) be 2TLSs. If $d\delta = \left\langle (ds_t, d\xi), (ds_i, d\psi), (ds_f, d\zeta) \right\rangle$ is administrated for $ds_t, ds_i, ds_f \in ds$, $d\xi, d\psi, d\zeta \in \left[0,0.5\right)$, where $(ds_t, d\xi), (ds_i, d\psi), (ds_f, d\zeta)$ is administrated for TM,

IM and FM by employing 2TLSs, the 2TLNSs is administrated:

$$d\delta_{j} = \left\langle \left(ds_{t_{j}}, d\xi_{j} \right), \left(ds_{i_{j}}, d\psi_{j} \right), \left(ds_{f_{j}}, d\zeta_{j} \right) \right\rangle$$
(2)

where

$$\begin{split} &0 \leq \Delta^{-1}\left(ds_{t_{j}}, d\xi_{j}\right) \leq \phi, 0 \leq \Delta^{-1}\left(ds_{i_{j}}, d\psi_{j}\right) \leq \phi, \\ &0 \leq \Delta^{-1}\left(ds_{f_{j}}, d\zeta_{j}\right) \leq \phi \\ &+ \Delta^{-1}\left(ds_{i_{j}}, d\psi_{j}\right) + \Delta^{-1}\left(ds_{f_{j}}, d\zeta_{j}\right) \leq 3\phi \end{split}$$

Definition 4[27]. Let $d\delta = \left\langle \left(ds_t, d\xi\right), \left(ds_i, d\psi\right), \left(ds_f, d\zeta\right) \right\rangle$. The uncertain score function (USF) and uncertain accuracy function (UAF) are administrated:

$$USF(d\delta) = \frac{\begin{pmatrix} 2\phi + \Delta^{-1}(ds_t, d\xi) \\ -\Delta^{-1}(ds_i, d\psi) - \Delta^{-1}(ds_f, d\zeta) \end{pmatrix}}{3\phi},$$

$$USF(d\delta) \in [0,1]$$
(3)

$$UAF(d\delta) = \frac{\phi + \Delta^{-1}(ds_t, d\xi) - \Delta^{-1}(ds_f, d\zeta)}{2\phi},$$

$$UAF(d\delta) \in [0,1]$$
(4)

Definition 5[27]. Let
$$d\delta_1 = \langle (ds_{t_1}, d\xi_1), (ds_{i_1}, d\psi_1), (ds_{f_1}, d\zeta_1) \rangle$$
 and $d\delta_2 = \langle (ds_{t_2}, d\xi_2), (ds_{i_2}, d\psi_2), (ds_{f_2}, d\zeta_2) \rangle$, then

(1) if
$$USF(d\delta_1) \prec USF(d\delta_2)$$
, $d\delta_1 \prec d\delta_2$;

(2) if
$$USF(d\delta_1) = USF(d\delta_2), UAF(d\delta_1) \prec UAF(d\delta_2)$$
, $d\delta_1 \prec d\delta_2$;

(3) if
$$USF(d\delta_1) = USF(d\delta_2)$$
, $UAF(d\delta_1) = UAF(d\delta_2)$, $d\delta_1 = d\delta_2$;

Definition 6[27]. Let
$$d\delta_1 = \left\langle \left(ds_{t_1}, d\xi_1 \right), \left(ds_{t_1}, d\psi_1 \right), \left(ds_{f_1}, d\zeta_1 \right) \right\rangle ,$$
$$d\delta_2 = \left\langle \left(ds_{t_2}, d\xi_2 \right), \left(ds_{t_2}, d\psi_2 \right), \left(ds_{f_2}, d\zeta_2 \right) \right\rangle$$
 and
$$d\delta = \left\langle \left(ds_t, d\xi \right), \left(ds_t, d\psi \right), \left(ds_f, d\zeta \right) \right\rangle$$
 be three 2TLNNs, then

$$d\delta_{1} \oplus d\delta_{2} = \begin{cases} \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{t_{1}}, d\xi_{1} \right)}{\phi} + \frac{\Delta^{-1} \left(ds_{t_{2}}, d\xi_{2} \right)}{\phi} - \frac{\Delta^{-1} \left(ds_{t_{1}}, d\xi_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{t_{2}}, d\xi_{2} \right)}{\phi} \right) \right), \\ \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{t_{1}}, d\psi_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{t_{2}}, d\psi_{2} \right)}{\phi} \right) \right), \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{f_{1}}, d\zeta_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{f_{2}}, d\zeta_{2} \right)}{\phi} \right) \right) \right) \end{cases};$$

$$(1)$$

$$d\delta_{1} \otimes d\delta_{2} = \begin{cases} \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{i_{1}}, d\xi_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{i_{2}}, d\xi_{2} \right)}{\phi} \right) \right), \\ \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{i_{1}}, d\psi_{1} \right)}{\phi} + \frac{\Delta^{-1} \left(ds_{i_{2}}, d\psi_{2} \right)}{\phi} - \frac{\Delta^{-1} \left(ds_{i_{1}}, d\psi_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{i_{2}}, d\psi_{2} \right)}{\phi} \right) \right), \\ \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{f_{1}}, d\zeta_{1} \right)}{\phi} + \frac{\Delta^{-1} \left(ds_{f_{2}}, d\zeta_{2} \right)}{\phi} - \frac{\Delta^{-1} \left(ds_{f_{1}}, d\zeta_{1} \right)}{\phi} \cdot \frac{\Delta^{-1} \left(ds_{f_{2}}, d\zeta_{2} \right)}{\phi} \right) \right) \right) \end{cases}$$

$$(2)$$

$$\xi d\delta = \left\{ \Delta \left(\phi \left(1 - \left(1 - \frac{\Delta^{-1} (ds_t, d\xi)}{\phi} \right)^{\xi} \right) \right), \Delta \left(\phi \left(\frac{\Delta^{-1} (ds_i, d\psi)}{\phi} \right)^{\xi} \right), \Delta \left(D \left(\frac{\Delta^{-1} (ds_f, d\zeta)}{\phi} \right)^{\xi} \right) \right\}, \xi > 0;$$
(3)

$$d\delta^{\xi} = \left\{ \Delta \left(\phi \left(\frac{\Delta^{-1} \left(ds_{t}, d\xi \right)}{\phi} \right)^{\xi} \right), \Delta \left(\phi \left(1 - \left(1 - \frac{\Delta^{-1} \left(ds_{i}, d\psi \right)}{\phi} \right)^{\xi} \right) \right), \Delta \left(\phi \left(1 - \left(1 - \frac{\Delta^{-1} \left(ds_{f}, d\zeta \right)}{\phi} \right)^{\xi} \right) \right) \right\}, \xi > 0.$$

$$(4)$$

Definition 7[40]. Let $d\delta_1 = \left\langle \left(ds_{t_1}, d\xi_1\right), \left(ds_{i_1}, d\psi_1\right), \left(ds_{f_1}, d\zeta_1\right) \right\rangle$, $d\delta_2 = \left\langle \left(ds_{t_2}, d\xi_2\right), \left(ds_{i_2}, d\psi_2\right), \left(ds_{f_2}, d\zeta_2\right) \right\rangle$, then the 2TLNN Hamming distance (2TLNNHD) is constructed:

 $2TLNNHD(d\delta_1, d\delta_2)$

$$= \frac{1}{3} \left| \frac{\Delta^{-1}(ds_{t_{1}}, d\xi_{1}) - \Delta^{-1}(ds_{t_{2}}, d\xi_{2})}{\phi} \right| + \frac{\Delta^{-1}(ds_{t_{1}}, d\psi_{1}) - \Delta^{-1}(ds_{t_{2}}, d\psi_{2})}{\phi} \right| + \frac{\Delta^{-1}(ds_{f_{1}}, d\zeta_{1}) - \Delta^{-1}(ds_{f_{2}}, d\zeta_{2})}{\phi} \right|$$
(5)

The 2TLNNWA approach is administrated:

Definition

7[39].

Let

$$d\delta_{j} = \langle (ds_{t_{j}}, d\xi_{j}), (ds_{i_{j}}, d\psi_{j}), (ds_{f_{j}}, d\zeta_{j}) \rangle$$
, the 2TLNNWA approach is administrated:

2TLNNWA
$$(d\delta_1, d\delta_2, ..., d\delta_n) = \bigoplus_{j=1}^n dw_j d\delta_j$$

$$\Delta \left(\phi \left[1 - \prod_{j=1}^{n} \left(1 - \frac{\Delta^{-1} \left(ds_{t_{j}}, d\alpha_{j} \right)}{\phi} \right)^{w_{j}} \right) \right),$$

$$= \Delta \left(\phi \prod_{j=1}^{n} \left(\frac{\Delta^{-1} \left(ds_{t_{j}}, d\beta_{j} \right)}{\phi} \right)^{w_{j}} \right),$$

$$\Delta \left(\phi \prod_{j=1}^{n} \left(\frac{\Delta^{-1} \left(ds_{f_{j}}, d\chi_{j} \right)}{\phi} \right)^{w_{j}} \right)$$

$$\Delta \left(\phi \prod_{j=1}^{n} \left(\frac{\Delta^{-1} \left(ds_{f_{j}}, d\chi_{j} \right)}{\phi} \right)^{w_{j}} \right)$$
(6)

IV. 2TLNN-EXPTODIM-GRA APPROACH FOR MAGDM WITH ENTROPY

A. 2TLNN-MAGDM Data

The 2TLNN-ExpTODIM-GRA approach is administrated MAGDM. Let $DA = \{DA_1, DA_2, \dots, DA_m\}$ $DG = \{DG_1, DG_2, \dots, DG_n\}$ be alternatives, attributes with weight numbers $dw = (dw_1, dw_2, \dots, dw_n)$, $dw_j \in [0,1]$, $\sum_{i=1}^n dw_j = 1$ and experts $DE = \left\{ DE_1, DE_2, \dots, DE_q \right\}^{J=1}$ with weight numbers $d\omega = (d\omega_1, d\omega_2, \dots, d\omega_q)$ $d\omega_j \in [0,1], \sum_{i=1}^q d\omega_j = 1.$

Then, 2TLNN-ExpTODIM-GRA approach is administrated for MAGDM.

Step 1. Build the 2TLNN-matrix
$$DM = \left[DM_{ij}^{(t)}\right]_{m \times n} = \begin{cases} \left(ds_{t_{ij}}^{(t)}, d\alpha_{ij}^{(t)}\right), \\ \left(ds_{i_{ij}}^{(t)}, d\beta_{ij}^{(t)}\right), \left(ds_{f_{ij}}^{(t)}, d\chi_{ij}^{(t)}\right) \end{cases}_{m \times n}$$

and manage the average matrix $DM = [DM_{ij}]_{max}$:

$$DG_{1} \quad DG_{2} \quad \dots \quad DG_{n}$$
 For beneficial attributes:
$$DA_{1} \begin{bmatrix} DM_{11}^{(t)} & DM_{12}^{(t)} & \dots & DM_{1n}^{(t)} \\ DM_{21}^{(t)} & DM_{22}^{(t)} & \dots & DM_{2n}^{(t)} \\ \vdots & \vdots & \vdots & \vdots \\ DA_{m}^{(t)} & DM_{m1}^{(t)} & DM_{m2}^{(t)} & \dots & DM_{mm}^{(t)} \end{bmatrix}$$

$$NDM_{ij} = DM_{ij} = \begin{cases} (ds) \\ (ds) \\$$

$$DM = \begin{bmatrix} DM_{ij} \end{bmatrix}_{m \times n} = \begin{bmatrix} DA_1 \\ DA_2 \\ \vdots \\ DA_m \end{bmatrix} \begin{bmatrix} DM_{11} & DM_{12} & \dots & DM_{1n} \\ DM_{21} & DM_{22} & \dots & DM_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ DM_{m1} & DM_{m2} & \dots & DM_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\alpha_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\beta_{ij} \right) \right), \\ \Delta$$

 $DM = \left[DM_{ij}\right]_{m \times n} = \begin{cases} \left(ds_{t_{ij}}, d\alpha_{ij}\right), \\ \left(ds_{i}, d\beta_{ii}\right), \left(ds_{i}, d\gamma_{ii}\right) \end{cases}$ is:

$$DM_{ij} = d\omega_{1}DM_{ij}^{1} \oplus d\omega_{2}DM_{ij}^{2} \oplus \cdots \oplus d\omega_{t}DM_{ij}^{t}$$

$$= \left\{ \Delta \left(\phi \prod_{t=1}^{q} \left(1 - \frac{\Delta^{-1} \left(ds_{t_{ij}}^{(t)}, d\alpha_{ij}^{(t)} \right)}{\phi} \right)^{dw_{t}} \right) \right\},$$

$$= \left\{ \Delta \left(\phi \prod_{t=1}^{q} \left(\frac{\Delta^{-1} \left(ds_{t_{ij}}^{(t)}, d\beta_{ij}^{(t)} \right)}{\phi} \right)^{dw_{t}} \right),$$

$$\Delta \left(\phi \prod_{t=1}^{q} \left(\frac{\Delta^{-1} \left(ds_{t_{ij}}^{(t)}, d\chi_{ij}^{(t)} \right)}{\phi} \right)^{dw_{t}} \right) \right\}$$

$$(9)$$

Administrate $NDM = \left[NDM_{ij} \right]_{m \times n} = \left\{ \begin{pmatrix} \left(ds_{t_{ij}}, d\alpha_{ij} \right), \\ \left(ds_{i_{ij}}, d\beta_{ij} \right), \left(ds_{f_{ij}}, d\chi_{ij} \right) \end{pmatrix} \right\}_{m \times n}$ $DM = \left[DM_{ij}\right]_{m \times n} = \begin{cases} \left(ds_{t_{ij}}, d\alpha_{ij}\right), \\ \left(ds_{i_{ii}}, d\beta_{ij}\right), \left(ds_{f_{ii}}, d\chi_{ij}\right) \end{cases}.$

For beneficial attributes:

$$NDM_{ij} = DM_{ij} = \left\{ \begin{pmatrix} ds_{t_{ij}}, d\alpha_{ij} \end{pmatrix}, \\ \left(ds_{i_{ij}}, d\beta_{ij} \right), \left(ds_{f_{ij}}, d\chi_{ij} \right) \right\}$$

$$(10)$$

$$NDM_{ij} = \left\{ \left(ds_{t_{ij}}, d\alpha_{ij} \right), \left(ds_{i_{ij}}, d\beta_{ij} \right), \left(ds_{f_{ij}}, d\chi_{ij} \right) \right\}$$

$$= \left\{ \Delta \left(\phi - \Delta^{-1} \left(ds_{t_{ij}}, d\alpha_{ij} \right) \right), \Delta \left(\phi - \Delta^{-1} \left(ds_{f_{ij}}, d\chi_{ij} \right) \right) \right\}$$

$$\left\{ \Delta \left(\phi - \Delta^{-1} \left(ds_{i_{ij}}, d\beta_{ij} \right) \right), \Delta \left(\phi - \Delta^{-1} \left(ds_{f_{ij}}, d\chi_{ij} \right) \right) \right\}$$

$$(11)$$

B. Administrate the Weight Numbers with Entropy

Step 3. Normalized hybrid decision matrix *NHDM*; is administrated through Entropy [41]:

 $NHDM_{ii}$

$$=\frac{UAF\left\{\left(ds_{t_{ij}},d\alpha_{ij}\right),\left(ds_{i_{ij}},d\beta_{ij}\right),\left(ds_{f_{ij}},d\chi_{ij}\right)\right\}}{USF\left\{\left(ds_{t_{ij}},d\alpha_{ij}\right),\left(ds_{i_{ij}},d\beta_{ij}\right),\left(ds_{f_{ij}},d\chi_{ij}\right)\right\}}}{\sum_{i=1}^{m}\left\{\frac{UAF\left\{\left(ds_{t_{ij}},d\alpha_{ij}\right),\left(ds_{i_{ij}},d\beta_{ij}\right),\left(ds_{f_{ij}},d\chi_{ij}\right)\right\}}{USF\left\{\left(ds_{t_{ij}},d\alpha_{ij}\right),\left(ds_{i_{ij}},d\beta_{ij}\right),\left(ds_{f_{ij}},d\chi_{ij}\right)\right\}}},$$

$$(12)$$

Then, the uncertain hybrid information Shannon entropy (UHISE) is administrated:

$$UHISE_{j} = -\frac{1}{\ln m} \sum_{i=1}^{m} NHDM_{ij} \ln NHDM_{ij}$$
(13)

and $NHDM_{ij} \ln NHDM_{ij} = 0$ if $NHDM_{ij} = 0$.

Then, the weights values $dw = (dw_1, dw_2, \dots, dw_n)$ is managed:

$$dw_{j} = \frac{1 - UHISE_{j}}{\sum_{j=1}^{n} \left(1 - UHISE_{j}\right)}$$
(14)

C. 2TLNN-ExpTODIM-GRA Approach for MAGDM

2TLNN-ExpTODIM-GRA approach is administrated for MAGDM.

Step 4. Administrate relative weight:

$$rdw_{j} = dw_{j} / \max_{j} dw_{j}, \qquad (15)$$

Step 5. 2TLNN dominance degree (2TLNNDD) of DA_i for DA_i under DG_i is administrated:

$$2TLNNDD_{j}\left(DA_{i},DA_{i}\right) = \begin{cases} \frac{rdw_{j} \times \left(1-10^{-\rho 2TLNNHDHD\left(NDM_{ij},NDM_{ij}\right)}\right)}{\sum_{j=1}^{n} rdw_{j}} & \text{if } SF\left(NDM_{ij}\right) > SF\left(NDM_{ij}\right) \\ 0 & \text{if } SF\left(NDM_{ij}\right) = SF\left(NDM_{ij}\right) \\ -\frac{1}{\theta} \frac{\sum_{j=1}^{n} rdw_{j} \times \left(1-10^{-\rho 2TLNNHDHD\left(NDM_{ij},NDM_{ij}\right)}\right)}{rdw_{j}} & \text{if } SF\left(NDM_{ij}\right) < SF\left(NDM_{ij}\right) \end{cases}$$

$$(16)$$

where θ is administrated [42] and $\rho \in [1,5]$ [36].

The 2TLNNDD under DG_j is administrated:

$$\begin{aligned} 2TLNNDD_{j}\left(DA_{i}\right) &= \begin{bmatrix} 2TLNNDD_{j}\left(DA_{i},DA_{i}\right) \end{bmatrix}_{m\times m} \\ DA_{1} & DA_{2} & \cdots & DA_{m} \\ DA_{1} & 0 & 2TLNNDD_{j}\left(DA_{1},DA_{2}\right) & \cdots & 2TLNNDD_{j}\left(DA_{1},DA_{m}\right) \\ &= DA_{2} & 2TLNNDD_{j}\left(DA_{2},DA_{1}\right) & 0 & \cdots & 2TLNNDD_{j}\left(DA_{2},DA_{m}\right) \\ &\vdots & \vdots & & \vdots & \cdots & \vdots \\ DA_{m} & 2TLNNDD_{j}\left(DA_{m},DA_{1}\right) & 2TLNNDD_{j}\left(DA_{m},DA_{2}\right) & \cdots & 0 \end{aligned}$$

(3) Administrate the overall 2TLNNDD of DA_i for other alternatives under DG_i :

$$2TLNNDD_{j}(DA_{i}) = \sum_{t=1}^{m} 2TLNNDD_{j}(DA_{i}, DA_{t})$$
(17)

The overall 2TLNNDD is administrated:

$$2TLNNDD = (2TLNNDD_{ij})_{m \times n}$$

$$\begin{bmatrix}
DG_{1} & DG_{2} & \dots & DG_{n} \\
DA_{1} & \sum_{t=1}^{m} 2TLNNDD_{1}(DA_{1}, DA_{t}) & \sum_{t=1}^{m} 2TLNNDD_{2}(DA_{1}, DA_{t}) & \dots & \sum_{t=1}^{m} 2TLNNDD_{n}(DA_{1}, DA_{t}) \\
DA_{2} & \sum_{t=1}^{m} 2TLNNDD_{1}(DA_{2}, DA_{t}) & \sum_{t=1}^{m} 2TLNNDD_{2}(DA_{2}, DA_{t}) & \dots & \sum_{t=1}^{m} 2TLNNDD_{n}(DA_{2}, DA_{t}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
DA_{m} & \sum_{t=1}^{m} 2TLNNDD_{1}(DA_{m}, DA_{t}) & \sum_{t=1}^{m} 2TLNNDD_{2}(DA_{m}, DA_{t}) & \dots & \sum_{t=1}^{m} 2TLNNDD_{n}(DA_{m}, DA_{t})
\end{bmatrix}$$
(18)

Step 6. Administrate the 2TLNNPIDS (2TLNN positive ideal decision solution) and 2TLNNNIDS (2TLNN negative ideal decision solution):

$$= (2TLNNPIDS_1, 2TLNNPIDS_1, \cdots, 2TLNNPIDS_n)$$
(19)

2TLNNNIDS

$$= (2TLNNNIDS_1, 2TLNNNIDS_1, \cdots, 2TLNNNIDS_n)$$
(20)

$$2TLNNPIDS_{j} = \max_{i} \left(2TLNNDD_{ij}\right)$$
 (21-a)

$$2TLNNNIDS_{j} = \min_{i} \left(2TLNNDD_{ij} \right) \tag{21-b}$$

Step 7. Administrate the 2TLNNGRC (2TLNN grey rational coefficients) from 2TLNNPIDS and 2TLNNNIDS.

$$2TLNNGRC(\zeta_{ij}^+)$$

$$= \frac{\left(\underset{1 \le i \le m}{\min} \underset{1 \le j \le n}{\min} \left| 2TLNNDD_{ij} - 2TLNNPIDS_{j} \right| + \rho \underset{1 \le i \le m}{\max} \underset{1 \le j \le n}{\max} \left| 2TLNNDD_{ij} - 2TLNNPIDS_{j} \right| \right)}{\left(\left| 2TLNNDD_{ij} - 2TLNNPIDS_{j} \right| + \rho \underset{1 \le i \le m}{\max} \underset{1 \le j \le n}{\max} \left| 2TLNNDD_{ij} - 2TLNNPIDS_{j} \right| \right)}$$

$$(22)$$

$2TLNNGRC(\zeta_{ij}^{-})$

$$= \frac{\left(\underset{1 \leq i \leq m}{\min} \underset{1 \leq j \leq n}{\min} \left| 2TLNNDD_{ij} - 2TLNNNIDS_{j} \right| + \rho \underset{1 \leq i \leq m}{\max} \underset{1 \leq j \leq n}{\max} \left| 2TLNNDD_{ij} - 2TLNNNIDS_{j} \right|}{\left(\left| 2TLNNDD_{ij} - 2TLNNNIDS_{j} \right| + \rho \underset{1 \leq i \leq m}{\max} \underset{1 \leq j \leq n}{\max} \left| 2TLNNDD_{ij} - 2TLNNNIDS_{j} \right| \right)} (23)$$

Step 8. Administrate the 2TLNNGRD (2TLNN grey relation degree) from 2TLNNPIDS and 2TLNNNIDS.

$$2TLNNGRD\left(\zeta_{i}^{+}\right) = \sum_{j=1}^{n} dw_{j} 2TLNNGRC\left(\zeta_{ij}^{+}\right)$$
(24)

$$2TLNNGRD\left(\zeta_{i}^{-}\right) = \sum_{j=1}^{n} dw_{j} 2TLNNGRC\left(\zeta_{ij}^{-}\right)$$
(25)

Step 9. Administrate the 2TLNNRRD (2TLNN relative relational degree) from the 2TLNNPIDS.

$$2TLNNRRD(\zeta_{i}) = \frac{2TLNNGRD(\zeta_{i}^{+})}{2TLNNGRD(\zeta_{i}^{-}) + 2TLNNGRD(\zeta_{i}^{+})}$$

$$= \frac{\sum_{j=1}^{n} dw_{j} 2TLNNGRC(\zeta_{ij}^{+})}{\sum_{j=1}^{n} dw_{j} 2TLNNGRC(\zeta_{ij}^{+}) + \sum_{j=1}^{n} dw_{j} 2TLNNGRC(\zeta_{ij}^{-})}$$
(26)

Step 10. Sort and choose the optimal alternative in line with largest 2TLNNRRD.

V. NUMERICAL EXAMPLE AND COMPARATIVE ANALYSIS

A. Numerical Example

Under the background of rural revitalization strategy and development of rural economy and society in the new era, construction of quality evaluation for PIE in rural areas needs to break free from the shackles of traditional concepts in terms of evaluation standards, evaluation orientation, evaluation mechanisms, evaluation methods, and more. It requires establishing a distinctive view of quality and a new perspective on evaluation, focusing on student-centered and comprehensive development, public governance and collaborative education, diagnostic improvement, and giving importance to the process. It is necessary to prioritize laws and policies, base standards on the highest level, emphasize individual characteristics, and prioritize systematic norms. The primary focus lies in the core elements and essential links of integrated vocational education and industry development, which aim to lead and promote deep

integration and high-quality advancement of PEI in rural areas, thereby contributing to rural revitalization. The PEI quality evaluation in RVE under rural revitalization falls under the framework of MAGDM. In this evaluation, there are five potential vocational and technical colleges $DA_i \ (i=1,2,3,4,5)$ to choose through four attributes [43]: DG1: Social benefit for PEI. DG2: management cost for PEI.

DG1: Social benefit for PEI. DG2: management cost for PEI. DG3: Economic benefits for PEI. DG4: Production-education

integration achievements. Five vocational and technical colleges DA_i (i=1,2,3,4,5) are assessed using linguistic scales (refer to Table I [40]) under four criteria. The evaluation involves three experts DE_t (t=1,2,3), each with their respective weight $d\omega = (1/3,1/3,1/3)$ in the assessment process.

TABLE I. LINGUISTIC INFORMATION SCALES AND 2TLNNS

Linguistic Terms←	2TLNNs← ³
Exceedingly Terrible-DET←	$\{(ds_0,0), (ds_5,0), (ds_6,0)\} \in$
Very Terrible-DVT←	$\{(ds_1,0), (ds_4,0), (ds_5,0)\} \subset$
Terrible-DT←	$\{(ds_2,0), (ds_3,0), (ds_4,0)\}$
Medium-DM←	$\{(ds_3,0), (ds_3,0), (ds_3,0)\}$
Well-DW←	$\{(ds_4,0), (ds_3,0), (ds_2,0)\}$
Very Well-DVW←	$\{(ds_5,0), (ds_2,0), (ds_1,0)\}$
Exceedingly Well-DEW←	$\{(ds_6,0), (ds_1,0), (ds_0,0)\}$

The PEI quality evaluation in RVE under rural revitalization is managed using the 2TLNN-ExpTODIM-GRA approach.

Step 1. Construct the 2TLNN-matrix
$$DM = \left\lceil DM_{ij}^{(t)} \right\rceil_{S \times A} (t = 1, 2, 3)$$
 (Table II - IV).

Then, from 2TLNNWA approach, the $DM = [DM_{ij}]_{5\times4}$ is constructed (Table V).

Step 2. Normalize the
$$DM = [DM_{ij}]_{5\times4}$$
 into $NDM = [NDM_{ij}]_{5\times4}$ (see Table VI).

Step 3. Administrate the weight (Table VII):

Step 4. Administrate the relative weight (see Table VIII):

Step 5. Administrate the
$$2TLNNDD = \left(2TLNNDD_{ij}\right)_{5\times4}$$
 (Table IX):

Step 6. Administrate the 2TLNNPIDS and 2TLNNNIDS (Table X).

Step 7. Administrate the $2TLNNGRC\left(\zeta_{ij}^{+}\right)$ and $2TLNNGRC\left(\zeta_{ij}^{-}\right)$ (Table XI-XII).

Step 8. Establish the $2TLNNGRD\left(\zeta_{i}^{+}\right)$ and $2TLNNGRD\left(\zeta_{i}^{-}\right)$ (Table XIII).

Step 9. Establish the 2TLNNRRD (Table XIV).

TABLE II. DECISION DATA FROM DE_1

	DG_1	DG_2	DG_3	DG_4
DA_1	DM	DW	DT	DM
DA_2	DW	DVW	DT	DM
DA ₃	DVT	DVT	DM	DW
DA ₄	DM	DT	DVW	DW
DA ₅	DW	DVW	DVT	DT

TABLE III. DECISION DATA FROM DE_2

	DG_1	DG_2	DG_3	DG_4
DA ₁	DVW	DT	DW	DM
DA_2	DM	DVW	DW	DVT
DA ₃	DVW	DM	DW	DVT
DA ₄	DW	DM	DT	DVW
DA ₅	DT	DVT	DM	DVW

Table IV. Decision Data From DE_3

	DG_1	DG_2	DG_3	DG_4
DA_1	DVT	DVT	DVW	DM
DA_2	DT	DVW	DVT	DM
DA ₃	DVW	DW	DM	DVT
DA ₄	DW	DM	DT	DVW
DA ₅	DM	DW	DVW	DVT

TABLE V. THE
$$DM = [DM_{ij}]_{5\times4}$$

	TIBLE 1. THE
	DG_1
DA_1	$\{(ds_4, -0.1294), (ds_3, -0.4493), (ds_2, 0.1175)\}$
DA_2	$\{(ds_4, -0.1576), (ds_3, -0.3438), (ds_2, 0.1072)\}$
DA ₃	$\{(ds_3, 0.3672), (ds_3, 0.2714), (ds_3, -0.3098)\}$
DA ₄	$\{(ds_3, 0.1296), (ds_3, 0.2128), (ds_3, -0.0394)\}$
DA ₅	$\{(ds_4, -0.4564), (ds_3, -0.2296), (ds_2, 0.4565)\}$
	DG_2
DA_1	$\{(ds_2, -0.2769), (ds_3, 0.2714), (ds_4, 0.2335)\}$
DA ₂	$\{(ds_4, 0.1435), (ds_3, 0.1342), (ds_2, 0.1657)\}$
DA ₃	$\{(ds_4, -0.3523), (ds_3, -0.3436), (ds_2, 0.3127)\}$
DA ₄	$\{(ds_5, -0.3195), (ds_2, 0.3524), (ds_1, 0.3139)\}$
DA ₅	$\{(ds_4, 0.1654), (ds_3, -0.3436), (ds_2, -0.1763)\}$
	DG_3
DA_1	$\{(ds_2, -0.0327), (ds_3, 0.3659), (ds_4, 0.1298)\}$
DA ₂	$\{(ds_1, 0.3457), (ds_4, -0.3523), (ds_5, -0.3452)\}$
DA ₃	$\{(ds_4, -0.1572), (ds_3, -0.3438), (ds_2, 0.1576)\}$
DA ₄	$\{(ds_4, 0.3793), (ds_2, 0.4625), (ds_2, -0.2697)\}$
DA ₅	$\{(ds_3, -0.4968), (ds_3, 0.2714), (ds_3, 0.4975)\}$
	DG_4
DA_1	$\{(ds_3, -0.4019), (ds_3, 0.2762), (ds_3, 0.4954)\}$

DA ₂	$\{(ds_5, -0.3914), (ds_2, 0.2589), (ds_1, 0.3207)\}$
DA ₃	$\{(ds_3, -0.2584), (ds_3, 0.3655), (ds_3, 0.2582)\}$
DA ₄	$\{(cs_3, -0.4743), (cs_3, 0.2715), (hs_3, 0.4746)\}$
DA ₅	$\{(ds_4, 0.4843), (ds_3, -0.4492), (ds_2, -0.4102)\}$

Table VI. The
$$NDM = [NDM_{ij}]_{5\times4}$$

	DG_1
DA ₁	$\{(ds_4, -0.1294), (ds_3, -0.4493), (ds_2, 0.1175)\}$
DA ₂	$\{(ds_4, -0.1576), (ds_3, -0.3438), (ds_2, 0.1072)\}$
DA ₃	$\{(ds_3, 0.3672), (ds_3, 0.2714), (ds_3, -0.3098)\}$
DA ₄	$\{(ds_3, 0.1296), (ds_3, 0.2128), (ds_3, -0.0394)\}$
DA ₅	$\{(ds_4, -0.4564), (ds_3, -0.2296), (ds_2, 0.4565)\}$
	DG_2
DA ₁	$\{(ds_4, 0.2769), (ds_3, -0.2714), (ds_2, -0.2335)\}$
DA ₂	$\{(ds_2, -0.1435), (ds_3, -0.1342), (ds_4, -0.1657)\}$
DA ₃	$\{(ds_2, 0.3523), (ds_3, 0.3436), (ds_4, -0.3127)\}$
DA ₄	$\{(ds_1, 0.3195), (ds_4, -0.3524), (ds_5, -0.3139)\}$
DA ₅	$\{(ds_2, -0.1654), (ds_3, 0.3436), (ds_4, 0.1763)\}$
	DG ₃
DA ₁	$\{(ds_2, -0.0327), (ds_3, 0.3659), (ds_4, 0.1298)\}$
DA ₂	$\{(ds_1, 0.3457), (ds_4, -0.3523), (ds_5, -0.3452)\}$
DA ₃	$\{(ds_4, -0.1572), (ds_3, -0.3438), (ds_2, 0.1576)\}$
DA ₄	$\{(ds_4, 0.3793), (ds_2, 0.4625), (ds_2, -0.2697)\}$
DA ₅	$\{(ds_3, -0.4968), (ds_3, 0.2714), (ds_3, 0.4975)\}$
	DG_4
DA ₁	$\{(ds_3, -0.4019), (ds_3, 0.2762), (ds_3, 0.4954)\}$
DA ₂	$\{(ds_5, -0.3914), (ds_2, 0.2589), (ds_1, 0.3207)\}$
DA ₃	$\{(ds_3, -0.2584), (ds_3, 0.3655), (ds_3, 0.2582)\}$
DA ₄	$\{(cs_3, -0.4743), (cs_3, 0.2715), (hs_3, 0.4746)\}$
DA ₅	$\{(ds_4, 0.4843), (ds_3, -0.4492), (ds_2, -0.4102)\}$

TABLE VII. WEIGHT NUMBERS

	DG_1	DG_2	DG_3	DG_4
weight	0.1705	0.2726	0.2806	0.2763

TABLE VIII. RELATIVE WEIGHT

	DG_1	DG_2	DG ₃	DG_4
relative weight	0.6076	0.9715	1.0000	0.9847

TABLE IX. $2TLNNDD = \left(2TLNNDD_{ij}\right)_{5\times4}$

	DG_1	DG_2	DG_3	DG_4
DA_1	-0.2568	1.1397	0.4547	-0.8459
DA_2	0.2467	0.3815	-2.5416	0.6480
DA ₃	-1.1506	-0.9523	1.1357	-1.8215
DA ₄	0.7805	-0.5346	-0.6770	1.4450
DA ₅	-1.0441	-1.5122	-0.8070	-0.0889

TABLE X. 2TLNNPIDS AND 2TLNNNIDS

	DG_1	DG_2	DG_3	DG_4
2TLNNPIDS	0.7805	1.1397	1.1357	1.4450
2TLNNNIDS	-1.1506	-1.5122	-2.5416	-1.8215

Table XI. $2TLNNGRC\left(\zeta_{ij}^{\scriptscriptstyle +}\right)$

	DG_1	DG_2	DG_3	DG_4
DA ₁	0.6393	1.0000	0.7297	0.4452
DA_2	0.7750	0.7080	0.3333	0.6976
DA ₃	0.4877	0.4678	1.0000	0.3602
DA ₄	1.0000	0.5234	0.5035	1.0000
DA ₅	0.5019	0.4095	0.4862	0.5452

TABLE XII. $2TLNNGRCig(oldsymbol{\zeta}_{ij}^{-}ig)$

	DG_1	DG_2	DG_3	DG_4
DA_1	0.6729	0.4095	0.3803	0.6534
DA_2	0.5682	0.4926	1.0000	0.4268
DA ₃	1.0000	0.7666	0.3333	1.0000
DA ₄	0.4877	0.6529	0.4965	0.3602
DA ₅	0.9453	1.0000	0.5146	0.5149

Table XIII. The $2TLNNGRDig(oldsymbol{\zeta}_i^+ig)_{ ext{and}}2TLNNGRDig(oldsymbol{\zeta}_i^-ig)$

Alternative	$2TLNNGRDig(oldsymbol{\zeta}_i^+ig)$	$2TLNNGRDig(oldsymbol{\zeta}_i^-ig)$
DA_1	0.7094	0.5136
DA_2	0.6114	0.6297
DA_3	0.5908	0.7493
DA ₄	0.7308	0.5000
DA_5	0.4843	0.7204

TABLE XIV. 2TLNNRRD AND ORDER

Alternative	2TLNNRRD	Order
DA ₁	0.5801	2
DA_2	0.4926	3
DA ₃	0.4409	4
DA ₄	0.5938	1
DA ₅	0.4020	5

Thus, the order is administrated: $DA_4 > DA_1 > DA_2 > DA_3 > DA_5$ and the optimal vocational and technical college is DA_4 .

B. Comparative Analysis

Then, 2TLNN-ExpTODIM-GRA approach is fully compared with 2TLNNWA approach[39], 2TLNNWG approach [39], 2TLNN-CODAS approach [44], 2TLNN-GRA approach [45], 2TLNN-CLVA approach [46] and 2TLNN-TODIM approach [40]. The comparative result is administrated in Table XV.

TABLE XV. ORDER FOR DIFFERENT APPROACHES

Approaches	Order	
2TLNNWA approach [39]	$DA_4 > DA_1 > DA_2 > DA_3 > DA_5$	
2TLNNWG approach [39]	$DA_4 > DA_1 > DA_3 > DA_2 > DA_5$	
2TLNN-CODAS approach [44]	$DA_4 > DA_1 > DA_2 > DA_3 > DA_5$	
2TLNN-GRA approach [45]	$DA_4 > DA_1 > DA_2 > DA_3 > DA_5$	
2TLNN-TODIM approach [40]	$DA_4 > DA_1 > DA_2 > DA_3 > DA_5$	

The comparative analysis conducted in this study confirms the reasonableness and effectiveness of the 2TLNN-ExpTODIM-GRA approach. However, it is worth highlighting that main limitations of 2TLNN-ExpTODIM-GRA approach are the lack of emphasis on consensus for MAGDM.

VI. CONCLUSION

The rural revitalization strategy is significant decision made by Party and the state, guided by the ideology of socialism with Chinese characteristics. Within this strategy, RVE serves as a crucial force in promoting self-realization among adults, breaking the cycle of poverty across generations, fostering rural industrial development, and advancing the modernization of agriculture and rural areas. Despite its importance, RVE in our country faces practical challenges such as the need to align with accelerating agricultural modernization, underutilization of its functional role, the transition of the rural population into employment and the corresponding lack of momentum in its development, the persistence of the "jumping off the farm gate" mindset and its deviation from the goals of RVE training, and the increasing pace of rural population aging with a delayed response from RVE. The PEI quality evaluation in RVE under the rural revitalization context involves MAGDM. In recent times, ExpTODIM approach and GRA approach was employed to manage MAGDM. To address the PEI quality evaluation in RVE under rural revitalization, the utilization of 2TLNSs proves valuable for representing uncertain information. In this study, we propose the 2TLNN-ExpTODIM-GRA approach to effectively handle MAGDM within the framework of 2TLNSs. Additionally, a numerical study is conducted to validate the implementation of this approach for the PEI quality

evaluation in RVE under rural revitalization. The primary contribution of this paper lies in the development of ExpTODIM-GRA approach, specifically tailored to address MAGDM within the context of 2TLNSs, with a specific focus on the PEI quality evaluation in RVE under rural revitalization.

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