Spherical Fuzzy Z-Numbers-based CRITIC CRADIAS and MARCOS Approaches for Evaluating English Teacher Performance

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***Abstract*—Consider combining quantitative and qualitative data for these case studies, such as interviews with English teachers, student evaluations, classroom observations, and surveys. Contextual elements, including community support, resources, and school demographics, should also be taken into consideration. The assessment process in English teaching performance evaluation is very complicated and diverse, making it a perfect fit for use in the Multi-Attribute Group Decision Making (MAGDM) framework. The utilization of Spherical Fuzzy $\breve{Z}$-Number Sets ($SF\breve{Z}N$S) is essential in Multi-Attribute Group Decision Making (MAGDM) to handle intricate problems. These sets are significantly more capable of handling higher levels of uncertainty than the fuzzy set designs used today. Here, we provide a method, Compromise Ranking of Alternatives from Distance to Ideal Solution (CRADIAS), designed to address MAGDM problems in $SF\breve{Z}N$S, particularly in cases when attribute weights are opaque. Attribute weights may be found by applying the CRITIC technique. The first section of the research covers the examination of spherical fuzzy Z numbers, their accuracy and scoring functions, and the main concept behind their functioning. We then propose the use of spherical fuzzy $\breve{Z}$-Number data to handle MAGDM cases in a decision-making process. This work strengthens the topic's theoretical underpinnings as well as its practical applicability. By conducting a comparison study, we apply the MARCOS approach to validate and illustrate the validity of our findings. This methodical approach guarantees a thorough evaluation of the suggested method's effectiveness and adds to the current discussion on how to make wise decisions in difficult and uncertain situations.**

***Keywords*—$SF\breve{Z}NS$; CRITRIC technique; CRADIAS method; MARCOS method**

I. Introduction

At its essence, this case study is propelled by an unwavering conviction, asserting that effective English teaching transcends a mere static concept; rather, it is a dynamic and ever-evolving tapestry woven with intricate threads of innovation, empathy, and adaptability. While traditional metrics undeniably offer valuable insights, their limitations are evident in the confined shadows they cast on the comprehensive impact of English teaching. It is against this backdrop that the imperative to explore diverse evaluation methods, including but not limited to classroom observations, self-assessment, student feedback, peer reviews, and performance data, emerges. By purposefully weaving together these diverse strands of evaluation, this study endeavors not only to uncover the symphony of English teaching effectiveness but also to delve into the nuanced notes within. It is within these subtleties that the potential for targeted support and development lies, poised to bring about transformative harmonies that enrich the educational experience for both English teachers and students alike. In embracing the multifaceted nature of effective English teaching, this study aims to contribute to the ongoing dialogue surrounding pedagogical excellence and the continuous refinement of educational practices. The application of Multi-Attribute Group Decision Making in the context of English teaching performance evaluation presents a promising avenue for creating a comprehensive and inclusive assessment framework. By considering diverse criteria, involving multiple stakeholders, and utilizing decision support systems, MAGDM can contribute to a more nuanced and robust evaluation process, ultimately fostering continuous improvement in English teaching practices and enhancing the overall quality of education.

A. Literature Review

A mathematical foundation for handling ambiguity and imprecision in decision-making processes is offered by fuzzy set theory. Fuzzy set theory permits partial membership, enabling things to belong to a set to variable degrees, in contrast to classical set theory, which classifies components as either belonging to or not belonging to a set. Fuzzy set theory is especially useful in situations involving decision-making when ambiguity and uncertainty are common because of its versatility. The notion of fuzzy sets (FS) was first presented by Zadeh [1] in 1965 as a ground-breaking method for managing the complexity of ambiguity in decision-making. Fuzzy sets offer a more nuanced view of membership by enabling the attribution of degrees of satisfaction between 0 and 1. Fuzzy set theory was first introduced and has since become well known as an important concept with a wide range of applications in various scientific and industrial fields. To satisfy these strict requirements, Atanasov [2] developed the idea of "intuitionistic fuzzy sets (IFS)". It has the formula $0 \leq \epsilon(\psi) + \varsigma(\psi) \leq 1$, in which the variables $\varphi(\psi)$ and $\epsilon(\psi)$ denote different levels of pleasure and discontent. IFS and fuzzy sets (FS) are related instruments for dealing with complex issues resulting from uncertainty, which frequently originate from flaws in parameter estimate methods. It can be more difficult to arrive at a suitable result under the IFS model when combining membership degrees in situations where the total might be more than one. This strategy, however, has drawbacks since it includes traits that are inherent to humans, such as constraint and refuse. Cuong and Kreinovich (2013) [3] introduced the idea of picture fuzzy sets (PFS), which was an important innovation. Three components, $\epsilon(\psi)$, $\nu(\psi)$, and $\varphi(\psi)$, which stand for different degrees of neutrality, displeasure, and satisfaction, define these PFS. $0 \leq \epsilon(\psi) + \nu(\psi) + \varsigma(\psi) \leq 1$ is a critical condition for PFS.

A Russian professor named Molodtsov [4] established the notion of soft sets (SS) in 1999 as a practical answer to a common problem. This novel idea presents a unique categorization strategy that is useful in several domains, including decision-making and function smoothness evaluation. There are many different fields in which soft sets find practical use. An important extension was the incorporation of entropy into intuitionistic fuzzy soft sets (IFSS) by Jiang et al. [5]. Although the distinction between "degrees of abstention" (ς) and "degrees of contentment" (ϵ) seems clear-cut, Fuzzy Sets (FSS) and Intuitionistic Fuzzy Sets (IFSS) struggle with errors and uncertainty. When decision-makers choose values of 0.5 for degrees of satisfaction (MG) and 0.7 for degrees of abstention (NMG) in the IFSS framework, this presents a problem because it goes against the constraint $0.8 + 0.9 > 1$. Yager [6] developed the idea of pythagorean fuzzy sets (PFS) to overcome this restriction, rewriting the fundamental constraints as $0 \leq \epsilon^2 + \varsigma^2 \leq 1$ instead of $0 \leq \epsilon + \varsigma \leq 1$. Peng et al. [7] cleverly integrated the idea of pythagorean fuzzy sets (PFS) with Soft Sets (SS), Novel information measures for Fermatean fuzzy sets introduced by [8], Ashraf work on Spherical q-linear Diophantine fuzzy aggregation information [9] and whereas Yager [10] suggested q-Rung Orthopair fuzzy sets as an expansion of IFSS. Remarkably, given their structural underpinnings, FSS and IFSS are both special instances in the q-ROFS paradigm. But even while the q-ROFS framework has been helpful in addressing a number of issues with multi-attribute decision-making (MAGDM) [11], it is not without limitations.

Spherical fuzzy sets (SFSs), first proposed by Ashraf [12], express membership, neutrality, and degrees of abstentions, and increase the dimensionality of membership gradations, such as $\epsilon(\psi)$, $\nu(\psi)$, and $\varsigma(\psi)$. The requirement $0 \leq \epsilon^2(\psi) + \nu^2(\psi) + \varsigma^2(\psi) \leq 1$ is rigorously followed by SFSs.

B. Motivation

As part of an ongoing effort to improve fuzzy set theory, Zadeh presented the ground-breaking concept of $\breve{z}$-numbers in 2011 [13]. By combining ordered pairs with fuzzy numbers, these $\breve{z}$ numbers outperform traditional fuzzy numbers. Ashraf [14] presented the idea of sets of spherical fuzzy $\breve{z}$-numbers ($SF\breve{Z}N$S) in a different line of inquiry. $0 \leq \tau_{\epsilon^2(\psi)} + \tau_{\nu^2(\psi)} + \tau_{\ni^2(\psi)} \leq 1$ and $0 \leq \tau_{\epsilon^2(\psi)} + \tau_{\nu^2(\psi)} + \tau_{\ni^2(\psi)} \leq 1$ are the two requirements that these sets meet. The three values in this context are $\epsilon(\psi)$, $\nu(\psi)$, and $\ni(\psi)$, which represent satisfaction, abstinence, and dissatisfaction; the indicators, on the other hand, are $\tau_{\epsilon(\psi)}$, $\tau_{\nu(\psi)}$, and $\tau_{\ni(\psi)}$, which represent the dependability of these levels.

Ashraf [15] introduced the pythagorean fuzzy Z-numbers, Ashraf [16] introduced Sugeno Weber Model under Spherical Fuzzy Z-numbers. Information Sciences, 120428.Notable applications of pythagorean fuzzy sets [17], A new Pythagorean fuzzy based decision framework for assessing healthcare waste treatment [18], Novel Distance Measure and CRADIS Method in Picture Fuzzy Environment [19], and Market assessment of pear varieties in Serbia using fuzzy CRADIS and CRITIC methods [20]. Application of fuzzy TRUST CRADIS method for selection of sustainable suppliers in agribusiness [21], A complex spherical fuzzy CRADIS method based Fine-Kinney framework for occupational risk evaluation in natural gas pipeline construction [22], Fuzzy multi-criteria analyses on green supplier selection in an agri-food company [23], A Hybrid Improved Fuzzy SWARA and Fuzzy CRADIS Approach [24], and An Integrated Spherical Fuzzy Multi-criterion Group Decision-Making Approach and Its Application in Digital Marketing Technology Assessment [25]. A new fuzzy MARCOS method for road traffic risk analysis [26], MCDM under the MARCOS method [27], Evaluation software of project management by using (MARCOS) method. [28], MARCOS method [29], Supplier selection for steelmaking company by using combined Grey MARCOS methods [30], CRITIC MARCOS method with spherical fuzzy information [31], Spherical fuzzy SWARA MARCOS approach for green supplier selection [32], and Road safety assessment and risks prioritization using an integrated SWARA and MARCOS approach under SFS environment [33]. Extension of WASPAS with spherical fuzzy sets [34], multiple attribute group decision making (MAGDM) [35], and Market assessment of pear varieties in Serbia using fuzzy CRADIS and CRITIC methods [36]. Attributes' weight using CRITRIC method [37] resolves numerical problems by employing compromise ranking of alternatives from distance to ideal solution (CRADIAS) [38], and for comparative analysis, measurement of alternatives and ranking according to compromise solution MARCOS method is utilized [39].

The principal motivation for the creation and implementation of CRADIAS in the context of Spherical Fuzzy $\breve{Z}$-Numbers is its capacity to manage intricate situations involving several criteria. Multiple factors must be taken into consideration while making decisions in real-world circumstances, as opposed to relying just on one criterion. When faced with several criteria, CRADIAS offers a methodical way to assess and prioritize possibilities. By combining criteria using the weighted sum product method, CRADIAS helps decision-makers get a clear picture of how well options perform overall in a variety of areas. The systematic and transparent integration of many aspects in the decision-making process is facilitated by this aggregation strategy.

C. Significance of the Study

The research proposal delineates the core aims as follows:

- Analyze the applicability and performance of CRADIAS for spherical fuzzy $\breve{Z}$-Numbers.
- Handle decision making tasks that require weighing several factors or criteria that are considered while analyzing possibilities in their whole.
- By properly combining the contributions of each criterion, the weighted sum product method employed in CRADIAS offers a thorough evaluation of the options.
- To improve the way that uncertainty is represented in decision-making by using Spherical Fuzzy Z Numbers ($SF\breve{Z}N$). This goal acknowledges $SF\breve{Z}N$'s exceptional capacity to manage uncertainty in both direction and magnitude, giving decision-makers a more realistic representation of the inherent imprecision and ambiguity in choice criteria.
- In order to guarantee that the decision model is in line with the complexities of spherical fuzzy information, this entails giving decision-makers an organized method that takes into consideration the spherical representation of uncertainties.

D. Organization of the Study

The article is structured as follows: Section II introduces fundamental preliminary operations, encompassing related operators, scoring and accuracy functions, SF$\breve{Z}$N distance measure and SF$\breve{Z}$N CRITRIC method to calculate the attributes weights.. Section III provides an overview of the methodology of CRADIAS method in $SF\breve{Z}N$ environment for Multiple Attributes Group Decision Making (MAGDM). Section IV delves into numerical aspects related to Evaluating Teaching Performance in a Secondary School Setting. Section V conducts a comparative analysis between CRADIAS and MARCOS method based on $SF\breve{Z}N$ environment. Finally, in Section VI, we offer concluding remarks and present the study's findings.

II. Preliminaries

This section introduces several fundamental definitions and operations that played a crucial role in developing the proposed tasks.

Definition II.1. [1] The fuzzy set Identified under the Entire Set Ξ is

$$\widetilde{\wp} = \left\{ (\daleth, \varphi_{\widetilde{\wp}}(\psi) | \daleth \in \Xi) \right\}$$

where $\varphi_{\widetilde{\nu}}(\psi)$ degrees of contentment,of ς in $\widetilde{\widetilde{\Xi}}$ and $\varphi_{\widetilde{\widetilde{\Xi}}} : \Xi \to [0, 1]$.

Definition II.2. [13] A $\breve{z}$-numbers is an ordered pair of fuzzy number embodied by $Z = (\imath, \tau\imath)$ the $\imath$ component is the contentment While $\tau\imath$ is the reliability of the $\imath$.

Definition II.3. [12] The spherical fuzzy set is Identified under the Entire Set Ξ :

$$\widetilde{\nu} = \left\{ \left(\daleth, \left(\varphi(\psi), \tau(\psi), o(\psi) \right) \right) | \daleth \in \Xi \right\}$$

such that $\varphi : \Xi \to [0, 1]$ and $\tau : \Xi \to [0, 1]$ are degrees of contentment and abstention respectively in a set Ξ. Where,

$$0 \leq (\varphi(\psi))^2 + (\tau(\psi))^2 + (o(\psi))^2 \leq 1$$

Definition II.4. [14] Suppose Ξ is the Entire Set then $SF\breve{Z}N$s is Identified as:

$$\pounds_\diamond = \{\varsigma, (\epsilon, \tau_\epsilon)(\psi), (\nu, \tau_\nu)(\psi), (\ni, \tau_\ni)(\psi) | \varsigma \in \Xi\}$$

such that $(\epsilon, \tau_\epsilon)) : \Xi \longrightarrow [0, 1]$,$(\nu, \tau_\nu) : \Xi \longrightarrow [0, 1]$ and $(\ni, \tau_\ni) : \Xi \longrightarrow [0, 1]$ are the order pair of degrees of contentment, and abstention respectively in a set ν and second component is spherical measure of intergrity for first component along all the conditions.

$$0 \leq \epsilon^2(\psi) + \nu^2(\psi) + \ni^2 (\psi) \leq 1$$

and

$$0 \leq \tau_\epsilon^2(\psi) + \tau_\nu^2(\psi) + \tau_\ni^2(\psi) \leq 1$$

Definition II.5. [14]

Suppose $\pounds_{\diamond_1} = \left\{ (\epsilon_1, \tau_{\epsilon_1}), (\nu_1, \tau_{\nu_1}), (\ni_1, \tau_{\ni_1}) \right\}$ and $\pounds_{\diamond_2} = \left\{ (\epsilon_2, \tau_{\epsilon_2}), (\nu_2, \tau_{\nu_2}), (\ni_2, \tau_{\ni_2}) \right\}$ be any two $SF\breve{Z}N$s and $\lambda \geq 0$ then the following operation Identified as:

1) $\pounds_{\diamond_1} \supseteq \pounds_{\diamond_2} \Leftrightarrow \epsilon_2 \geq \epsilon_1, \tau_{\epsilon_2} \geq \tau_{\epsilon_1}, \nu_2 \leq \nu_1, \tau_{\nu_2} \leq \tau_{\nu_1}, \ni_2 \leq \ni_1, \tau_{\ni_2} \leq \tau_{\ni_1}$.
2) $\pounds_{\diamond_1} = \pounds_{\diamond_2} \Leftrightarrow \pounds_{\diamond_1} \supseteq \pounds_{\diamond_2}$ and $\pounds_{\diamond_1} \subseteq \pounds_{\diamond_2}$.
3) $\pounds_{\diamond_1} \cup \pounds_{\diamond_2} = \Big\{(\epsilon_1 \vee \epsilon_2, \tau_{\epsilon_1} \vee \tau_{\epsilon_2}), (\nu_1 \wedge \nu_2, \tau_{\nu_1} \wedge \tau_{\nu_2}), (\ni_1 \wedge \ni_2, \tau_{\ni_1} \wedge \tau_{\ni_2})\Big\}$.
4) $\pounds_{\diamond_1} \cap \pounds_{\diamond_2} = \Big\{(\epsilon_1 \wedge \epsilon_2, \tau_{\epsilon_1} \wedge \tau_{\epsilon_2}), (\nu_1 \wedge \nu_2, \tau_{\nu_1} \wedge \tau_{\nu_2}), (\ni_1 \vee \ni_2, \tau_{\ni_1} \vee \tau_{\ni_2})\Big\}$.
5) $(\pounds_{\diamond_1})^c = \Big\{(\epsilon_1, \tau_{\epsilon_1}), (\nu_1, \tau_{\nu_1}), (\ni_1, \tau_{\ni_1})\Big\}^c = \Big\{(\ni_1, \tau_{\ni_1}), (\nu_1, \tau_{\nu_1}), (\epsilon_1, \tau_{\epsilon_1})\Big\}$.
6) $\pounds_{\diamond_1} \oplus \pounds_{\diamond_2} = \left\{ \begin{array}{c} \left(\sqrt{\epsilon_1^2 + \epsilon_2^2 - \epsilon_1^2\epsilon_2^2}, \sqrt{\tau_{\epsilon_1}^2 + \tau_{\epsilon_2}^2 - \tau_{\epsilon_1}^2\tau_{\epsilon_2}^2}\right), \\ \left(\nu_1\nu_2, \tau_{\nu_1}\tau_{\nu_2}\right), \left(\ni_1\ni_2, \tau_{\ni_1}\tau_{\ni_2}\right). \end{array} \right\}$.
7) $\pounds_{\diamond_1} \otimes \pounds_{\diamond_2} = \left\{ \begin{array}{c} (\epsilon_1\epsilon_2, \tau_{\epsilon_1}\tau_{\epsilon_2}), , (\nu_1\nu_2, \tau_{\nu_1}\tau_{\nu_2}), \\ \left(\sqrt{{\ni_1}^2 + {\ni_2}^2 - {\ni_1}^2{\ni_2}^2}, \sqrt{\tau_{\ni_1}^2 + \tau_{\ni_2}^2 - \tau_{\ni_1}^2\tau_{\ni_2}^2}\right) \end{array} \right\}$.
8) $\lambda\pounds_{\diamond_1} = \left\{ \begin{array}{c} \left(\sqrt{1-(1-\epsilon_1^2)^\lambda}, \sqrt{1-(1-\tau_{\epsilon_1}^2)^\lambda}\right), \\ \left(\nu_1^\lambda \tau_{\nu_1}^\lambda\right), \left(\ni_1^\lambda\ \tau_{\ni_1}^\lambda\right) \end{array} \right\}$.
9) $(\pounds_{\diamond_1})^\lambda = \left\{ \begin{array}{c} \left(\epsilon_1^\lambda \tau_{\epsilon_1}^\lambda\right), \left(\nu_1^\lambda \tau_{\nu_1}^\lambda\right), \\ \left(\sqrt{1-(1-{\ni_1}^2)^\lambda}, \sqrt{1-(1-\tau_{\ni_1}^2)^\lambda}\right) \end{array} \right\}$.

Definition II.6. [14]

Suppose $\pounds_{\diamond_i} = \Big\{(\epsilon_i, \tau_{\epsilon_i}), (\nu_i, \tau_{\nu_i}), (\ni_i, \tau_{\ni_i})\Big\}$ be $SF\breve{Z}N$s and then the algebraic and geometric aggregation operator Identified as:

$$SF\breve{Z}NWA(\pounds_{\diamond_1}, \pounds_{\diamond_2}, \pounds_{\diamond 3}..., \pounds_{\diamond n}) = \sum_{i=1}^{n} \breve{\Omega}_i \pounds_{\diamond_i}$$

where $\sum_{i=1}^{n} \breve{\Omega}_i = 1$, $\breve{\Omega}_i \in [0,1]$

$$= \left\{ \begin{array}{c} \left(\sqrt{1-\prod_{i=1}^{n}(1-(\epsilon_i)^2)^{\breve{\Omega}_i}}, \right. \\ \left. \sqrt{1-\prod_{i=1}^{n}(1-(\tau_{\epsilon_i})^2)^{\breve{\Omega}_i}}\right) \\ \left(\prod_{i=1}^{n}((\nu_i))^{\breve{\Omega}_i}, \prod_{i=1}^{n}(\tau_{\nu_i})^{\breve{\Omega}_i}\right) \\ \left(\prod_{i=1}^{n}((\ni_i))^{\breve{\Omega}_i}, \prod_{i=1}^{n}(\tau_{\ni_i})^{\breve{\Omega}_i}\right) \end{array} \right\}.$$

$$SF\breve{Z}NWG(\pounds_{\diamond_1}, \pounds_{\diamond_2}, \pounds_{\diamond 3}..., \pounds_{\diamond n}) = \prod_{i=1}^{n} {\pounds_{\diamond_i}}^{\breve{\Omega}_i}$$

where $\sum_{i=1}^{n} \breve{\Omega}_i = 1$, $\breve{\Omega}_i \in [0,1]$

$$= \left\{ \begin{array}{c} \left(\prod_{i=1}^{n}((\epsilon_i))^{\breve{\Omega}_i}, \prod_{i=1}^{n}(\tau_{\epsilon_i})^{\breve{\Omega}_i}\right) \\ \left(\prod_{i=1}^{n}((\nu_i))^{\breve{\Omega}_i}, \prod_{i=1}^{n}(\tau_{\nu_i})^{\breve{\Omega}_i}\right) \\ \left(\sqrt{1-\prod_{i=1}^{n}(1-(\ni_i)^2)^{\breve{\Omega}_i}}, \right. \\ \left. \sqrt{1-\prod_{i=1}^{n}(1-(\tau_{\ni_i})^2)^{\breve{\Omega}_i}}\right) \end{array} \right\}.$$

Definition II.7. To compare $SF\breve{Z}N$ $\pounds_{\diamond_i} = \Big\{(\epsilon_i, \tau_{\epsilon_i}), (\nu_i, \tau_{\nu_i}), (\ni_i, \tau_{\ni_i})\Big\}$ we introduced the score function as

$$\Im(\pounds_{\diamond_i}) = \frac{2 + ((\epsilon_i)(\tau_{\epsilon_i})) - ((\nu_i)(\tau_{\nu_i})) - ((\ni_i)(\tau_{\ni_i}))}{3}$$

$\Im(\pounds_{\diamond_i}) \in [-1, 1]$
if $\Im(\pounds_{\diamond_i}) = \Im(\pounds'_{\diamond_i})$ then calculate the accuracy function

$$\Re(\pounds_{\diamond_i}) = \frac{((\epsilon_i)(\tau_{\epsilon_i})) - ((\nu_i)(\tau_{\nu_i})) - ((\ni_i)(\tau_{\ni_i}))}{3}$$

$\Re(\pounds_{\diamond_i}) \in [0, 1]$

Definition II.8. Suppose $\pounds_{\diamond_1} = \Big\{(\epsilon_1, \tau_{\epsilon_1}), (\nu_1, \tau_{\nu_1}), (\ni_1, \tau_{\ni_1})\Big\}$ and $\pounds_{\diamond_2} = \Big\{(\epsilon_2, \tau_{\epsilon_2}), (\nu_2, \tau_{\nu_2}), (\ni_2, \tau_{\ni_2})\Big\}$ be any two $SF\breve{Z}N$s then the Euclidean distance between them as follows:
$d(\pounds_{\diamond_1}, \pounds_{\diamond_2}) =$

$$\left(\begin{array}{c} \Big((\epsilon_1.\tau_{\epsilon_1}) - (\epsilon_2.\tau_{\epsilon_2})\Big)^2 + \Big((\nu_1.\tau_{\nu_1}) - (\nu_2.\tau_{\nu_2})\Big)^2 + \\ \Big((\ni_1.\tau_{\ni_1}) - (\ni_2.\tau_{\ni_2})\Big)^2 \end{array} \right)^{\frac{1}{2}}.$$

III. CRADIAS [38] Method under $SF\breve{Z}N$ for Multi Attributes Group Decision Making

In this section, we have formulated an algorithm to tackle the Multiple Attributes Decision Making (MAGDM) problem using Compromise Ranking of Alternatives from Distance to Ideal Solution (CRADIAS) [38]. Additionally, we provided an MAGDM example to illustrate the application of these operators. Let's assume we have a collection of alternatives represented as $\top = \{\top_1, \top_2, \top_3, ...\top_n\}$, and a collection of attributes represented as $¥ = \{¥_1, ¥_2, ¥_3, ...¥_n\}$, with their respective weight vectors $\breve{\Omega} = \{\breve{\Omega}_1, \breve{\Omega}_2, ...\breve{\Omega}_n\}$. The weight vectors must satisfy the requirement that the weights belong to a closed unit interval (i.e., ranging from 0 to 1) and that their sum must be equal to 1, ensuring a valid weighting scheme. Suppose the spherical Fuzzy spherical Fuzzy $\breve{Z}$- Number decision matrix denoted by $\pounds_{\diamond}^k = [\pounds_{\diamond_{ij}}^k]_{m\times n}$.

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1) Decision matrices by the experts.
2) Using aggregation operators to to aggregate all individuals spherical fuzzy $\breve{z}$-numbers decision matrices into collective spherical fuzzy $\breve{z}$-numbers decision matrix $\pounds_{\diamond_{ij}} = [\pounds_{\diamond_{ij}}]_{m\times n}$
3) The attribute weights are calculated by CRITRIC method. calculate attributes weights by using following equation: Compute the score values of decision matrix.

$$\Im(\pounds_{\diamond_{ij}}) = \frac{2+((\epsilon_{ij})(\tau_{\epsilon_{ij}}))-((\nu_{ij})(\tau_{\nu_{ij}}))-((\ni_{ij})(\tau_{\ni_{ij}}))}{3} \quad \forall i,j. \ .$$

$$\overbrace{\Im(\pounds_{\diamond_{ij}})} = \begin{cases} \frac{\Im(\pounds_{\diamond_{ij}}) - \Im(\pounds_{\diamond_{ij}})^-}{\Im(\pounds_{\diamond_{ij}})^+ - \Im(\pounds_{\diamond_{ij}})^-}, & j \in \mathrm{R_b}. \\ \frac{\Im(\pounds_{\diamond_{ij}})^+ - \Im(\pounds_{\diamond_{ij}})}{\Im(\pounds_{\diamond_{ij}})^+ - \Im(\pounds_{\diamond_{ij}})^-}, & j \in \mathrm{R_c}. \end{cases}$$

where $\mathrm{R_b}$ and $\mathrm{R_c}$ are the benefit and cost type of criteria sets respectively. $\Im(\pounds_{\diamond_{ij}})^- = min_i \Im(\pounds_{\diamond_{ij}})$ and $\Im(\pounds_{\diamond_{ij}})^+ = max_i \Im(\pounds_{\diamond_{ij}})$

Calculate the standard deviation by using the following equation:

$$\S_j = \sqrt{\frac{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)^2}{m}}.$$

Where $\overline{\Im(\pounds_{\diamond_{ij}})} = \frac{\overbrace{\Im(\pounds_{\diamond_{ij}})}}{m}$.

Calculate the correlation between criteria pairs by using the following equation:

$$\Upsilon_{jl} = \frac{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)\left(\overbrace{\Im(\pounds_{\diamond_{il}})} - \overline{\Im(\pounds_{\diamond_{il}})}\right)}{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)^2 \sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{il}})} - \overline{\Im(\pounds_{\diamond_{il}})}\right)^2}. \ .$$

Calculate each criterion's information amount using the formula below:

$$\Gamma_j = \sum_{l=1}^{n}(1 - \Upsilon_{jl})$$

Calculated the weight of each attribute b using following equation:

$$\breve{\Omega} = \frac{\Gamma_j}{\sum_{j=1}^{n}\Gamma_j}$$

4) Normalize the decision matrices.
Normalize by using following equation:

$$\pounds_{\diamond_{ij}} = \begin{cases} \pounds_{\diamond_{ij}} = \\ \left\{(\epsilon_{ij}, \tau_{\epsilon_{ij}}), (\nu_{ij}, \tau_{\nu_{ij}}), (\ni_{ij}, \tau_{\ni_{ij}})\right\}, & j \in \mathrm{R_b}, \\ {\pounds_{\diamond_{ij}}}^c = \\ \left\{(\ni_{ij}, \tau_{\ni_{ij}}), (\nu_{ij}, \tau_{\nu_{ij}}), (\epsilon_{ij}, \tau_{\epsilon_{ig}})\right\}, & j \in \mathrm{R_c}, \end{cases}$$

where $\mathrm{R_b}$ and $\mathrm{R_c}$ are the benefit and cost type of criteria set respectively.

5) Calculate weighted form of normalized $SF\check{Z}N$ decision matrix.
The weighted form of normalized $SF\breve{Z}N$ decision matrix is estimated as below:

$$\breve{\pounds}_{\diamond_{ij}} = \sum_{i=1}^{n}\breve{\Omega}_j \pounds_{\diamond_{ij}} = \left\{\begin{array}{c} \left(\sqrt{1-\prod_{i=1}^{n}(1-(\epsilon_{ij})^2)^{\breve{\Omega}_j}}, \right. \\ \left. \sqrt{1-\prod_{i=1}^{n}(1-(\tau_{\epsilon_{ij}})^2)^{\breve{\Omega}_j}}\right) \\ \left(\prod_{i=1}^{n}((\nu_{ij}))^{\breve{\Omega}_j}, \prod_{i=1}^{n}(\tau_{\nu_{ij}})^{\breve{\Omega}_j}\right) \\ \left(\prod_{i=1}^{n}((\ni_{ij}))^{\breve{\Omega}_j}, \prod_{i=1}^{n}(\tau_{\ni_{ij}})^{\breve{\Omega}_j}\right) \end{array}\right\}.$$

6) Compute the ideal t_j^+ and anti ideal t_j^- solution.

$$t_j^+ = \left\{\begin{array}{c} (\max_{i=1,\dots m}\epsilon_j, \max_{i=1,\dots m}\tau_{\epsilon_j}), \\ (\min_{i=1,\dots m}\nu_j, \min_{i=1,\dots m}\tau_{\nu_{ij}}), \\ (\min_{i=1,\dots m}\ni_{ij}, \min_{i=1,\dots m}\tau_{\ni_{ij}}) \end{array}\right\}.$$

$$t_j^- = \left\{\begin{array}{c} (\min_{i=1,\dots m}\epsilon_j, \min_{i=1,\dots m}\tau_{\epsilon_j}), \\ (\min_{i=1,\dots m}\nu_j, \min_{i=1,\dots m}\tau_{\nu_{ij}}), \\ (\max_{i=1,\dots m}\ni_{ij}, \max_{i=1,\dots m}\tau_{\ni_{ij}}) \end{array}\right\}.$$

7) Compute distance between weighted normalized decision matrix and Ideal solution d_{ij}^+ and weighted normalized decision matrix and Anti Ideal solution d_{ij}^+

$$d_{ij}^+ = d^+(\breve{\pounds}_{\diamond_{ij}}, t_j^+)$$

$$= \left\{\begin{array}{c} \left(\left((\epsilon_{ij}.\tau_{\epsilon_{ij}}) - (\epsilon_j^+.\tau_{\epsilon_j}^+)\right)^2 + \right. \\ \left((\nu_{ij}.\tau_{\nu_{ij}}) - (\nu_j^+.\tau_{\nu_j}^+)\right)^2 + \\ \left.\left((\ni_{ij}.\tau_{\ni_{ij}}) - (\ni_j^+.\tau_{\ni_j}^+)\right)^2\right)^{\frac{1}{2}} \end{array}\right\}.$$

$$d_{ij}^- = d^-(\breve{\pounds}_{\diamond_{ij}}, t_j^-)$$

$$= \left\{\begin{array}{c} \left(\left((\epsilon_{ij}.\tau_{\epsilon_{ij}}) - (\epsilon_j^-.\tau_{\epsilon_j}^-)\right)^2 + \right. \\ \left((\nu_{ij}.\tau_{\nu_{ij}}) - (\nu_j^-.\tau_{\nu_j}^-)\right)^2 + \\ \left.\left((\ni_{ij}.\tau_{\ni_{ij}}) - (\ni_j^-.\tau_{\ni_j}^-)\right)^2\right)^{\frac{1}{2}} \end{array}\right\}.$$

8) Compute the degree of deviation of every option from ideal and anti ideal solution.

$$\mathfrak{S}_{\mathrm{i}}^+ = \sum_{j=1}^{n} d_{ij}^+$$

$$\mathfrak{S}_{\mathrm{i}}^- = \sum_{j=1}^{n} d_{ij}^-$$

9) Compute the utility function of each alternative. The utility function of each alternative is estimated as:

$$\mathrm{K_i}^+ = \frac{\mathfrak{S}_{\diamond}^+}{\mathfrak{S}_{\mathrm{i}}^+}$$

$$\mathrm{K_i}^- = \frac{\mathfrak{S}_{\mathrm{i}}^-}{\mathfrak{S}_{\diamond}^-}$$

Where $\mathfrak{S}_{\diamond}^-$ is the best option that is the furthest away from the anti ideal solution and $\mathfrak{S}_{\diamond}^+$ is the best option that is the closest to the ideal solution.

10) Calculate the average departure of the options. The average departure of the options is computed as:

$$\mathrm{Q_i} = \frac{\mathrm{K_i}^+ + \mathrm{K_i}^-}{2}$$

11) To rank all alternatives in descending order and choose the best one.

The flow chart of algorithm is given in Fig. 1

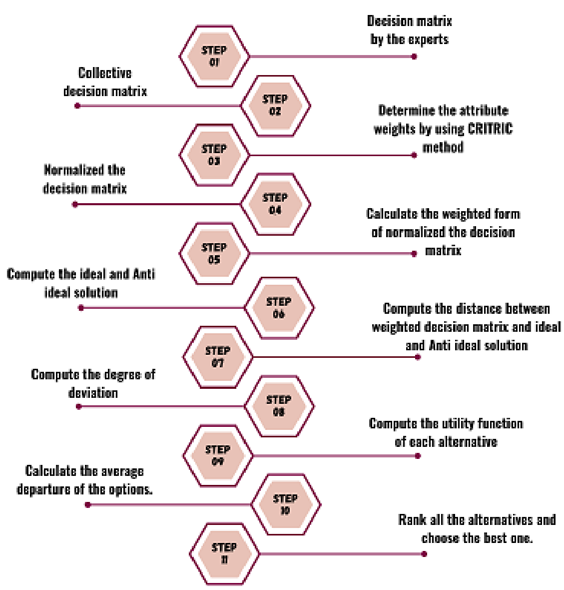

Fig. 1. Flow chart of algorithm of CRADIAS method.

IV. Case Study

In this segment of the article, we present a Multi-Attributes Decision Group Making (MAGDM) problem to demonstrate the applicability and efficacy of this approach in tackling complex decision-making challenges. To exemplify this, we present a scenario of Evaluating Teaching Performance in a Secondary School Setting. Within this context, we have deliberately selected four distinct attributes for evaluating the performance of these operations: Effectiveness of Instructional Strategies, Classroom Management and Learning Environment, Student Assessment and Feedback, and Professional Development Engagement. We have identified four potential alternatives: Peer Mentoring and Collaborative Learning, Student-Led Assessments and Portfolios, Degree Evaluation, and Innovative Professional Development Formats.

In the tapestry of education, English teachers stand as architects, sculptors, and mentors, shaping the intellectual, practical, and ethical dimensions of students. The heartbeat of this transformative process lies in the nuanced artistry of English teaching. Evaluating English teaching performance emerges not merely as an administrative exercise but as a compass guiding educators to refine their pedagogical artistry, adapt to the diverse and evolving needs of learners, and ultimately elevate the quality of education. In the crucible of secondary education, where students stand at the crossroads of their academic journey, the significance of effective English teaching takes center stage. This case study seeks to embark on an intricate exploration of the multifaceted process of evaluating English teaching performance, unveiling that the essence of excellence in English teaching transcends traditional metrics, embracing a holistic and student-centric paradigm. At its core, this case study is animated by the unwavering belief that effective English teaching is an ever-evolving masterpiece, intricately woven with threads of innovation, empathy, and adaptability. While traditional metrics offer a glimpse into the multifaceted world of English teaching, they often cast a confined shadow on the profound and holistic impact of educators. Hence, the imperative exploration of a myriad of evaluation methods, including but not limited to classroom observations, self-assessment, student feedback, peer reviews, and performance data.

In this symphony of methodologies, the study seeks not merely to uncover the melodies of English teaching effectiveness but to discern the subtle notes where targeted support and development can orchestrate transformative harmonies, enriching the educational experience for both English teachers and students alike. Classroom observations serve as a key lens through which the study gains insights into the daily practices of educators. By immersing itself in the classroom environment, it captures the dynamic interplay between English teachers and students, the strategies employed, and the overall atmosphere conducive to learning. However, the study does not stop at mere observation; it extends its reach to the introspective domain of self-assessment. Encouraging educators to reflect on their own practices, strengths, and areas for improvement, self-assessment becomes a reflective tool. It fosters a culture of continuous improvement, empowering English teachers to refine their approaches and pedagogical strategies. In this process, the study seeks to unearth the inherent potential for growth and development that lies within each educator. Moreover, the symphony of evaluation methods resonates with the harmonious notes of student feedback. Acknowledging the unique perspective students bring, the study values their voices as integral components in the evaluation process. Students, as active participants in their own education, offer invaluable insights into the effectiveness of English teaching methods, communication styles, and the overall impact on their learning journey.

Peer reviews add another layer to this melodic exploration. They bring a collaborative dimension, fostering a community of practice among educators. The insights shared among peers create a supportive network for professional development, allowing English teachers to learn from each other's experiences and expertise. Lastly, the study recognizes the significance of performance data as a quantitative measure. It acknowledges the role of data in providing tangible evidence of English teaching effectiveness, adding a quantitative dimension to the qualitative aspects explored through other methods. The significance of this case study reverberates through the ethos of education as it grapples with dynamic changes. It recognizes English teaching as an art form where practitioners continuously evolve, adapting to the ever-changing needs of learners and the broader educational landscape. In challenging the limitations of traditional metrics, the study aspires to illuminate the path toward a more nuanced understanding of English teaching impact. Through this, it seeks to contribute to the narrative of education as a living, breathing entity, shaped by the innovative spirit, compassionate heart, and resilient adaptability of educators.

Within the expansive canvas of secondary education, this study casts its gaze upon a tapestry of educators, each weaving their unique narrative into the educational fabric. Through the interplay of qualitative and quantitative research methods, the study endeavors not only to unravel the patterns, strengths, and areas for improvement in English teaching performance but to illuminate the individual brushstrokes that form the broader masterpiece. The ultimate aspiration is to craft recommendations that transcend the ordinary, guiding educators through a continuous journey of professional growth, fostering collaborative environments that resonate with the harmonies of effective English teaching, and inspiring the evolution of institutional policies that acknowledge and nurture the diverse facets of English teacher evaluation. In the ever-evolving landscape of education, where tradition meets innovation, and where learners bring diverse perspectives into the classroom, this case study unfolds. It acknowledges that effective English teaching is a dynamic dance, where the rhythm is set by the pulse of innovation and the melody by the empathetic understanding of diverse learner needs. The study recognizes that the pursuit of excellence in English teaching requires an intricate balance, where tradition provides the foundation, and innovation propels educators into uncharted territories of pedagogical exploration.

There are four Attributes

Effectiveness of Instructional Strategies ($¥_1$):

Assess the impact and efficiency of instructional methods employed by english teachers. Evaluate the alignment of instructional strategies with diverse learning styles and educational objectives. Measure the engagement and participation levels of students during various instructional activities. Examine the integration of technology and other innovative approaches in enhancing the overall learning experience.

Classroom Management and Learning Environment ($¥_2$):

Evaluate the effectiveness of classroom management strategies in maintaining a positive and inclusive learning environment. Assess the organization and physical layout of the classroom to optimize student engagement. Consider the implementation of behavior management techniques and their impact on student behavior and focus. Explore the incorporation of culturally responsive practices in creating an inclusive classroom atmosphere.

Student Assessment and Feedback ($¥_3$):

Examine the design and implementation of assessments to measure student understanding and progress. Evaluate the timeliness and quality of feedback provided to students to support their learning. Analyze the alignment between assessments and learning objectives. Explore the use of formative assessments as tools for ongoing evaluation and adjustment of instructional strategies.

Professional Development Engagement ($¥_4$):

Assess english teachers' participation in professional development activities related to pedagogy, technology, and content knowledge. Evaluate the impact of professional development on english teaching practices and student outcomes. Explore english teachers' proactive engagement in seeking continuous learning opportunities. Consider the alignment between professional development activities and identified areas for improvement.

There are four Alternatives

Peer Mentoring and Collaborative Learning ($\top_1$):

Implement a peer mentoring program where english teachers collaborate and share successful english teaching practices. Encourage collaborative lesson planning and team english teaching among educators. Facilitate regular forums for english teachers to discuss challenges and successes in a supportive community.

Student-Led Assessments and Portfolios ($\top_2$):

Explore alternative assessment methods, such as student-led conferences or portfolios, to capture a more comprehensive view of student progress. Encourage students to actively participate in setting learning goals and self-assessing their performance. Incorporate reflective exercises where students assess their own learning journey.

Degree Evaluation ($\top_3$):

Expand the evaluation process to include input from students, parents, and colleagues through degree feedback. Implement student and parent surveys to gather perspectives on english teaching effectiveness. Encourage collaborative evaluations where english teachers receive feedback from their peers, administrators, and students.

Innovative Professional Development Formats ($\top_4$):

Introduce alternative professional development formats, such as workshops, webinars, and online courses, to cater to diverse learning preferences. Provide english teachers with opportunities to attend conferences, engage in action research, or participate in collaborative projects. Foster a culture of continuous improvement by integrating professional development into regular team meetings and planning sessions.

We have a collection of alternatives denoted as $\top = \{\top_1, \top_2, \top_3, \top_4\}$,. We also have a set of attributes denoted as ¥$=\{$¥$_1$, ¥$_2$, ¥$_3$, ¥$_4\}$. We have experts weight vector $\breve{\Omega} = \{0.47, 0.38, 0.15\}$.

Step 1 Decision matrices by the expert1, expert2 and expert3 in Table I, II and III respectively.

TABLE I. Decision Matrix by the Expert 1

$\pounds_{\diamond_{ij}}$	¥$_1$
$\top_1$	$((0.5, 0.6), (0.6, 0.2), (0.6, 0.5))$
$\top_2$	$((0.4, 0.5), (0.4, 0.3), (0.7, 0.4))$
$\top_3$	$((0.8, 0.4), (0.5, 0.4), (0.3, 0.6))$
$\top_4$	$((0.4, 0.3), (0.4, 0.6), (0.2, 0.7))$
$\pounds_{\diamond_{ij}}$	¥$_2$
$\top_1$	$((0.4, 0.5), (0.4, 0.4), (0.6, 0.6))$
$\top_2$	$((0.3, 0.4), (0.5, 0.3), (0.5, 0.5))$
$\top_3$	$((0.5, 0.6), (0.7, 0.2), (0.4, 0.7))$
$\top_4$	$((0.6, 0.4), (0.5, 0.4), (0.4, 0.5))$
$\pounds_{\diamond_{ij}}$	¥$_3$
$\top_1$	$((0.6, 0.8), (0.7, 0.3), (0.3, 0.3))$
$\top_2$	$((0.4, 0.4), (0.3, 0.6), (0.6, 0.5))$
$\top_3$	$((0.7, 0.6), (0.4, 0.5), (0.4, 0.6))$
$\top_4$	$((0.6, 0.3), (0.4, 0.7), (0.6, 0.4))$
$\pounds_{\diamond_{ij}}$	¥$_4$
$\top_1$	$((0.4, 0.5), (0.6, 0.4), (0.6, 0.7))$
$\top_2$	$((0.4, 0.5), (0.3, 0.5), (0.7, 0.3))$
$\top_3$	$((0.6, 0.3), (0.2, 0.5), (0.6, 0.4))$
$\top_4$	$((0.1, 0.2), (0.4, 0.2), (0.3, 0.6))$

Step 2 In Table IV by using $SF\breve{Z}NWA$ aggregation operator aggregate all individuals Spherical fuzzy $\breve{z}$-numbers decision matrices into collective spherical fuzzy $\breve{z}$-numbers decision matrix

Step 3 The weights of attribute by using CRITRIC method is given in Table V .

Step 4 The normalized decision matrix is calculated In Table VI.

TABLE II. Decision Matrix by the Expert 2

$\pounds_{\diamond_{ij}}$	¥$_1$
$\top_1$	$((0.6, 0.2), (0.3, 0.4), (0.4, 0.5))$
$\top_2$	$((0.4, 0.3), (0.3, 0.5), (0.2, 0.3))$
$\top_3$	$((0.6, 0.3), (0.2, 0.6), (0.2, 0.4))$
$\top_4$	$((0.7, 0.3), (0.4, 0.6), (0.5, 0.4))$
$\pounds_{\diamond_{ij}}$	¥$_2$
$\top_1$	$((0.7, 0.4), (0.4, 0.4), (0.2, 0.3))$
$\top_2$	$((0.6, 0.7), (0.1, 0.3), (0.2, 0.5))$
$\top_3$	$((0.7, 0.3), (0.2, 0.5), (0.3, 0.4))$
$\top_4$	$((0.6, 0.4), (0.3, 0.4), (0.4, 0.6))$
$\pounds_{\diamond_{ij}}$	¥$_3$
$\top_1$	$((0.4, 0.4), (0.2, 0.6), (0.2, 0.4))$
$\top_2$	$((0.4, 0.5), (0.4, 0.6), (0.2, 0.4))$
$\top_3$	$((0.2, 0.5), (0.3, 0.4), (0.6, 0.3))$
$\top_4$	$((0.4, 0.3), (0.2, 0.7), (0.3, 0.5))$
$\pounds_{\diamond_{ij}}$	¥$_4$
$\top_1$	$((0.4, 0.5), (0.4, 0.4), (0.8, 0.2))$
$\top_2$	$((0.3, 0.3), (0.3, 0.2), (0.2, 0.3))$
$\top_3$	$((0.2, 0.4), (0.2, 0.1), (0.1, 0.4))$
$\top_4$	$((0.1, 0.2), (0.6, 0.6), (0.3, 0.5))$

TABLE III. Decision Matrix by the Expert 3

$\pounds_{\diamond_{ij}}$	¥$_1$	¥$_2$
$\top_1$	$((0.2, 0.3), (0.6, 0.4), (0.2, 0.6))$	$((0.6, 0.4), (0.3, 0.6), (0.5, 0.4))$
$\top_2$	$((0.2, 0.5), (0.4, 0.5), (0.4, 0.6))$	$((0.5, 0.2), (0.2, 0.4), (0.3, 0.3))$
$\top_3$	$((0.3, 0.4), (0.2, 0.5), (0.3, 0.4))$	$((0.4, 0.1), (0.7, 0.2), (0.4, 0.2))$
$\top_4$	$((0.4, 0.6), (0.5, 0.3), (0.2, 0.7))$	$((0.3, 0.7), (0.1, 0.1), (0.2, 0.6))$
$\pounds_{\diamond_{ij}}$	¥$_3$	¥$_4$
$\top_1$	$((0.4, 0.8), (0.2, 0.3), (0.1, 0.3))$	$((0.2, 0.1), (0.6, 0.5), (0.6, 0.3))$
$\top_2$	$((0.2, 0.2), (0.3, 0.2), (0.2, 0.5))$	$((0.4, 0.5), (0.6, 0.3), (0.4, 0.5))$
$\top_3$	$((0.1, 0.1), (0.4, 0.1), (0.3, 0.6))$	$((0.6, 0.3), (0.5, 0.5), (0.6, 0.5))$
$\top_4$	$((0.6, 0.5), (0.5, 0.5), (0.4, 0.4))$	$((0.7, 0.6), (0.4, 0.2), (0.4, 0.6))$

TABLE IV. Aggregate Decision Matrices by use the $SF\breve{Z}NWA$ Operator

$\pounds_{\diamond_{ij}}$	¥$_1$
$\top_1$	$((0.5169, 0.4614), (0.4610, 0.2887), (0.4361, 0.5138))$
$\top_2$	$((0.3781, 0.4391), (0.3585, 0.3932), (0.3998, 0.3810))$
$\top_3$	$((0.6965, 0.3661), (0.3076, 0.4825), (0.2571, 0.4839))$
$\top_4$	$((0.5523, 0.3698), (0.4136, 0.5407), (0.2833, 0.5659))$
$\pounds_{\diamond_{ij}}$	¥$_2$
$\top_1$	$((0.5769, 0.4511), (0.3831, 0.4250), (0.38450.4338))$
$\top_2$	$((0.4761, 0.5394), (0.2364, 0.3132), (0.32690.4631))$
$\top_3$	$((0.5840, 0.4679), (0.4348, 0.2833), (0.35850.4689))$
$\top_4$	$((0.5703, 0.4696), (0.3234, 0.3249), (0.36050.5507))$
$\pounds_{\diamond_{ij}}$	¥$_3$
$\top_1$	$((0.5106, 0.7094), (0.3603, 0.3904), (0.2180, 0.3346))$
$\top_2$	$((0.3781, 0.4232), (0.3346, 0.5088), (0.3351, 0.4593))$
$\top_3$	$((0.5325, 0.5237), (0.3585, 0.3608), (0.4469, 0.4610))$
$\top_4$	$((0.5388, 0.3406), (0.3178, 0.6655), (0.4338, 0.4354))$
$\pounds_{\diamond_{ij}}$	¥$_4$
$\top_1$	$((0.3781, 0.4670), (0.5143, 0.4136), (0.6693, 0.3829))$
$\top_2$	$((0.3661, 0.4391), (0.3328, 0.3269), (0.3998, 0.3238))$
$\top_3$	$((0.5033, 0.3424), (0.2294, 0.2712), (0.3037, 0.4136))$
$\top_4$	$((0.3221, 0.3108), (0.4666, 0.3036), (0.3132, 0.5598))$

TABLE V. WEIGHTS OF THE ATTRIBUTES

$\breve{\Omega}_1$	$\breve{\Omega}_2$	$\breve{\Omega}_3$	$\breve{\Omega}_4$
0.18	0.26	0.36	0.20

TABLE VI. THE NORMALIZED DECISION MATRIX

$\pounds_{\diamond_{ij}}$	¥$_1$
$\top_1$	$((0.5169, 0.4614), (0.4610, 0.2887), (0.4361, 0.5138))$
$\top_2$	$((0.3781, 0.4391), (0.3585, 0.3932), (0.3998, 0.3810))$
$\top_3$	$((0.6965, 0.3661), (0.3076, 0.4825), (0.2571, 0.4839))$
$\top_4$	$((0.5523, 0.3698), (0.4136, 0.5407), (0.2833, 0.5659))$
$\pounds_{\diamond_{ij}}$	¥$_2$
$\top_1$	$((0.5769, 0.4511), (0.3831, 0.4250), (0.38450.4338))$
$\top_2$	$((0.4761, 0.5394), (0.2364, 0.3132), (0.32690.4631))$
$\top_3$	$((0.5840, 0.4679), (0.4348, 0.2833), (0.35850.4689))$
$\top_4$	$((0.5703, 0.4696), (0.3234, 0.3249), (0.36050.5507))$
$\pounds_{\diamond_{ij}}$	¥$_3$
$\top_1$	$((0.5106, 0.7094), (0.3603, 0.3904), (0.2180, 0.3346))$
$\top_2$	$((0.3781, 0.4232), (0.3346, 0.5088), (0.3351, 0.4593))$
$\top_3$	$((0.5325, 0.5237), (0.3585, 0.3608), (0.4469, 0.4610))$
$\top_4$	$((0.5388, 0.3406), (0.3178, 0.6655), (0.4338, 0.4354))$
$\pounds_{\diamond_{ij}}$	¥$_4$
$\top_1$	$((0.3781, 0.4670), (0.5143, 0.4136), (0.6693, 0.3829))$
$\top_2$	$((0.3661, 0.4391), (0.3328, 0.3269), (0.3998, 0.3238))$
$\top_3$	$((0.5033, 0.3424), (0.2294, 0.2712), (0.3037, 0.4136))$
$\top_4$	$((0.3221, 0.3108), (0.4666, 0.3036), (0.3132, 0.5598))$

Step 5 The weighted form of normalized decision matrix is calculated in Table VII.

TABLE VII. THE WEIGHTED FORM OF NORMALIZED DECISION MATRIX

$\pounds_{\diamond_{ij}}$	¥$_1$
$\top_1$	$((0.2301, 0.2025), (0.8732, 0.8046), (0.8648, 0.8900))$
$\top_2$	$((0.1632, 0.1918), (0.8357, 0.8493), (0.8517, 0.8446))$
$\top_3$	$((0.3312, 0.1577), (0.8135, 0.8802), (0.7884, 0.8807))$
$\top_4$	$((0.2484, 0.1594), (0.8568, 0.8979), (0.8019, 0.9051))$
$\pounds_{\diamond_{ij}}$	¥$_2$
$\top_1$	$((0.3148, 0.2388), (0.7807, 0.8019), (0.7815, 0.8062))$
$\top_2$	$((0.2533, 0.2913), (0.6893, 0.7412), (0.7494, 0.8199))$
$\top_3$	$((0.3194, 0.2485), (0.8067, 0.7222), (0.7675, 0.8225))$
$\top_4$	$((0.3106, 0.2495), (0.7474, 0.7482), (0.7686, 0.8573))$
$\pounds_{\diamond_{ij}}$	¥$_3$
$\top_1$	$((0.3229, 0.4745), (0.6892, 0.7096), (0.5739, 0.6709))$
$\top_2$	$((0.2339, 0.2635), (0.6709, 0.7816), (0.6712, 0.7530))$
$\top_3$	$((0.3383, 0.3321), (0.6880, 0.6895), (0.7455, 0.7540))$
$\top_4$	$((0.3428, 0.2096), (0.6584, 0.8620), (0.7375, 0.7384))$
$\pounds_{\diamond_{ij}}$	¥$_4$
$\top_1$	$((0.1753, 0.2204), (0.8740, 0.8363), (0.9219, 0.8234))$
$\top_2$	$((0.1694, 0.2060), (0.8003, 0.7974), (0.8306, 0.7959))$
$\top_3$	$((0.2396, 0.1579), (0.7423, 0.7678), (0.7856, 0.8363))$
$\top_4$	$((0.1481, 0.1427), (0.8570, 0.7856), (0.7905, 0.8892))$

Step 6 The ideal and anti ideal solution are estimated in Table VIII and in Table IX respectively.

TABLE VIII. THE IDEAL SOLUTION

$\pounds_{\diamond_{ij}}$	¥$_1$
t^+	$((0.3312, 0.2025), (0.8135, 0.8046), (0.7884, 0.8446))$
$\pounds_{\diamond_{ij}}$	¥$_2$
t^+	$((0.3194, 0.2913), (0.7769, 0.8019), (0.8223, 0.8640))$
$\pounds_{\diamond_{ij}}$	¥$_3$
t^+	$((0.3428, 0.4745), (0.8182, 0.8366), (0.7661, 0.8256))$
$\pounds_{\diamond_{ij}}$	¥$_4$
t^+	$((0.2396, 0.2204), (0.7729, 0.7958), (0.81170.8209))$

TABLE IX. THE ANTI IDEAL SOLUTION

$\pounds_{\diamond_{ij}}$	¥$_1$
t^-	$((0.1632, 0.1577), (0.8135, 0.8046), (0.8648, 0.9051))$
$\pounds_{\diamond_{ij}}$	¥$_2$
t^-	$((0.2533, 0.2388), (0.7769, 0.8019), (0.8459, 0.9008))$
$\pounds_{\diamond_{ij}}$	¥$_3$
t^-	$((0.2339, 0.2096), (0.8182, 0.8366), (0.8685, 0.8732))$
$\pounds_{\diamond_{ij}}$	¥$_4$
t^-	$((0.1481, 0.1427), (0.7729, 0.7958), (0.9321, 0.9034))$

Step 7 The distance between weighted normalized decision matrix and Ideal solution d_{ij}^+ and weighted normalized decision matrix and Anti Ideal solution d_{ij}^+ in Table X and in Table XI respectively.

TABLE X. DISTANCE BETWEEN WEIGHTED NORMALIZED DECISION MATRIX AND IDEAL SOLUTION d_{ij}^+

$\pounds_{\diamond_{ij}}$	¥$_1$	¥$_2$	¥$_3$	¥$_4$
$\top_1$	0.11615	0.10829	0.02665	0.19177
$\top_2$	0.08473	0.02366	0.14285	0.07344
$\top_3$	0.06940	0.07461	0.13644	0.03276
$\top_4$	0.13238	0.06862	0.16539	0.12292

TABLE XI. DISTANCE BETWEEN WEIGHTED NORMALIZED DECISION MATRIX AND ANTI IDEAL SOLUTION d_{ij}^-

$\pounds_{\diamond_{ij}}$	¥$_1$	¥$_2$	¥$_3$	¥$_4$
$\top_1$	0.05398	0.11034	0.16539	0.15820
$\top_2$	0.08419	0.04679	0.06313	0.15684
$\top_3$	0.11092	0.07864	0.06500	0.14756
$\top_4$	0.12889	0.05455	0.08152	0.14160

Step 8 The degree of deviation of every option from ideal and anti ideal solution are given in Table XII

TABLE XII. DEGREE OF DEVIATION OF EVERY OPTION FROM IDEAL AND ANTI IDEAL SOLUTION

	$\mathfrak{S}_i^+$	$\mathfrak{S}_i^-$
$\top_1$	0.44285	0.48791
$\top_2$	0.32469	0.35094
$\top_3$	0.31321	0.40212
$\top_4$	0.48930	0.40657
$\mathfrak{S}_\diamond$	0.31321	0.48791

Step 9 The utility function of each alternative is computed In Table XIII

TABLE XIII. THE UTILITY FUNCTION OF EACH ALTERNATIVE

	K_i^+	K_i^-
$\top_1$	0.70726	1.00000
$\top_2$	0.96466	0.71928
$\top_3$	1.00000	0.82417
$\top_4$	0.64012	0.83330

Step 10 The average departure of the options is computed In Table XIV

TABLE XIV. THE AVERAGE DEPARTURE OF THE OPTIONS Q_i

Q_i
0.85363
0.84197
0.91208
0.73671

Step 11 Ranking all possibilities in descending order in Table XV.

TABLE XV. RANKING OF NUMERICAL PROBLEM

method	scoring
CRADIAS Method	$\top_3 \geq \top_1 \geq \top_2 \geq \top_4$

As a result, we determine that option $\top_3$ is the best optimal solution. The graphical representation of CRADIAS method given in Fig. 2

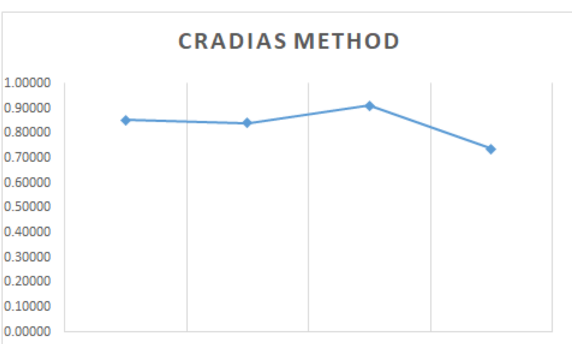

Fig. 2. Graphical Representation of Ranking

V. COMPARISON ANALYSIS

In this section, we compare the CRADIAS method to the developed MAGDM technique, highlighting the advantages of the established methodology. The characteristics of the CRADIAS method provided in this study are compared to the MARCOS method [39]. We demonstrate how the suggested approach efficiently solves real-life decision-making problems (DMPs) with uncertainty through this extensive comparison, stressing its efficacy and robustness.

A. *MARCOS approach for* $SF\acute{Z}N$

Algorithm

1) Decision matrices by the experts.
2) Using aggregation operators to to aggregate all individuals spherical fuzzy $\breve{z}$-numbers decision matrices into collective spherical fuzzy $\breve{z}$-numbers decision matrix $\pounds_{\diamond_{ij}} = [\pounds_{\diamond_{ij}}]_{m\times n}$
3) The attribute weights are calculated by CRITRIC method. calculate attributes weights by using following equation: Compute the score values of decision matrix.
$$\Im(\pounds_{\diamond_{ij}}) = \frac{2+((\epsilon_{ij})(\tau_{\epsilon_{ij}}))-((\nu_{ij})(\tau_{\nu_{ij}}))-((\ni_{ij})(\tau_{\ni_{ij}}))}{3} \quad \forall i,j. \; .$$
$$\overbrace{\Im(\pounds_{\diamond_{ij}})} = \begin{cases} \frac{\Im(\pounds_{\diamond_{ij}}) \; - \; \Im(\pounds_{\diamond_{ij}})^-}{\Im(\pounds_{\diamond_{ij}})^+ \; - \; \Im(\pounds_{\diamond_{ij}})^-}, & j \in R_b. \\ \\ \frac{\Im(\pounds_{\diamond_{ij}})^+ \; - \Im(\pounds_{\diamond_{ij}})}{\Im(\pounds_{\diamond_{ij}})^+ \; - \; \Im(\pounds_{\diamond_{ij}})^-}, & j \in R_c. \end{cases}$$
where R_b and R_c are the benefit and cost type of criteria sets respectively. $\Im(\pounds_{\diamond_{ij}})^- = min_i \Im(\pounds_{\diamond_{ij}})$ and $\Im(\pounds_{\diamond_{ij}})^+ = max_i \Im(\pounds_{\diamond_{ij}})$
Calculate the standard deviation by using the following equation:
$$\S_j = \sqrt{\frac{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)^2}{m}}.$$
Where $\overline{\Im(\pounds_{\diamond_{ij}})} = \frac{\overbrace{\Im(\pounds_{\diamond_{ij}})}}{m}$.
Calculate the correlation between criteria pairs by using the following equation:
$$\Upsilon_{jl} = \frac{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)\left(\overbrace{\Im(\pounds_{\diamond_{il}})} - \overline{\Im(\pounds_{\diamond_{il}})}\right)}{\sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{ij}})} - \overline{\Im(\pounds_{\diamond_{ij}})}\right)^2 \sum_{i=1}^{n}\left(\overbrace{\Im(\pounds_{\diamond_{il}})} - \overline{\Im(\pounds_{\diamond_{il}})}\right)^2}.$$
Calculate each criterion's information amount using the formula below:
$$\Gamma_j = \sum_{l=1}^{n}(1 - \Upsilon_{jl})$$
Calculated the weight of each attribute b using following equation:
$$\breve{\Omega} = \frac{\Gamma_j}{\sum_{j=1}^{n}\Gamma_j}$$
4) Normalize the decision matrices.
Normalize by using following equation:

$$\pounds_{\diamond_{ij}} = \begin{cases} \pounds_{\diamond_{ij}} = \\ \Big\{(\epsilon_{ij}, \tau_{\epsilon_{ij}}), (\nu_{ij}, \tau_{\nu_{ij}}), (\ni_{ij}, \tau_{\ni_{ij}})\Big\}, & j \in \mathrm{R_b}, \\ {\pounds_{\diamond_{ij}}}^{c} = \\ \Big\{(\ni_{ij}, \tau_{\ni_{ij}}), (\nu_{ij}, \tau_{\nu_{ij}}), (\epsilon_{ij}, \tau_{\epsilon_{ig}})\Big\}, & j \in \mathrm{R_c}, \end{cases}$$

where $\mathrm{R_b}$ and $\mathrm{R_c}$ are the benefit and cost type of criteria set respectively.

5) The positive ideal solution (PIS) P_j^+ and negative ideal solution (NIS) N_j^-

$$P_j^+ = \left\{ \begin{array}{l} (\max_{i=1,\dots m}\epsilon_j, \max_{i=1,\dots m}\tau_{\epsilon_j}), \\ (\min_{i=1,\dots m}\nu_j, \min_{i=1,\dots m}\tau_{\nu_{ij}}), \\ (\min_{i=1,\dots m}\ni_{ij}, \min_{i=1,\dots m}\tau_{\ni_{ij}}) \end{array} \right\}.$$

$$N_j^- = \left\{ \begin{array}{l} (\min_{i=1,\dots m}\epsilon_j, \min_{i=1,\dots m}\tau_{\epsilon_j}), \\ (\min_{i=1,\dots m}\nu_j, \min_{i=1,\dots m}\tau_{\nu_{ij}}), \\ (\max_{i=1,\dots m}\ni_{ij}, \max_{i=1,\dots m}\tau_{\ni_{ij}}) \end{array} \right\}.$$

6) Calculate the distance between normalized decision matrix and PIS $\ddot{d}_{ij}^+$ and NIS $\ddot{d}_{ij}^-$ by using following equations:

$$\ddot{d}_i^+ = (\breve{\pounds}_{\diamond_{ij}}, P_j^+)$$

$$\ddot{d}_i^- = d(\breve{\pounds}_{\diamond_{ij}}, N_j^-)$$

where,

$$d(\breve{\pounds}_{\diamond_{ij}}, P_j^+) = \left\{ \begin{array}{l} \Big(\Big((\epsilon_{ij}.\tau_{\epsilon_{ij}}) - (\epsilon_j^+.\tau_{\epsilon_j}^+)\Big)^2 + \\ \Big((\nu_{ij}.\tau_{\nu_{ij}}) - (\nu_j^+.\tau_{\nu_j}^+)\Big)^2 + \\ \Big((\ni_{ij}.\tau_{\ni_{ij}}) - (\ni_j^+.\tau_{\ni_j}^+)\Big)^2\Big)^{\frac{1}{2}} \end{array} \right\}.$$

$$d(\breve{\pounds}_{\diamond_{ij}}, N_j^-) = \left\{ \begin{array}{c} \Big(\Big((\epsilon_{ij}.\tau_{\epsilon_{ij}}) - (\epsilon_j^-.\tau_{\epsilon_j}^-)\Big)^2 + \\ \Big((\nu_{ij}.\tau_{\nu_{ij}}) - (\nu_j^-.\tau_{\nu_j}^-)\Big)^2 + \\ \Big((\ni_{ij}.\tau_{\ni_{ij}}) - (\ni_j^-.\tau_{\ni_j}^-)\Big)^2\Big)^2\Big)^{\frac{1}{2}} \end{array} \right\}.$$

7) Calculate the closeness coefficient. Utilizing $\ddot{d}_{ij}^+$ and $\ddot{d}_{ij}^-$, determine the closeness coefficient as follows:

$$\mathfrak{C}ij = \frac{\ddot{d}_{ij}^-}{\ddot{d}_{ij}^+ + \ddot{d}_{ij}^-}.$$

8) Calculate the extended decision matrix. Make the extended decision matrix by insertion of $\mathfrak{C}_{ij}$, and the anti-ideal ($\mathfrak{A}^- = \{\mathfrak{C}i1^-, \mathfrak{C}i2^-, ..., \mathfrak{C}in^-\}$) and ideal ($\mathfrak{A}^+ = \{\mathfrak{C}ij^+; j = 1, 2, ..., n\}$) solution.

$$\mathfrak{A} = \begin{pmatrix} \mathfrak{C}i1^- & \mathfrak{C}i2^- & \dots & \mathfrak{C}_{in}^- \\ \mathfrak{C}11 & \mathfrak{C}12 & \dots & \mathfrak{C}_{1n} \\ \mathfrak{C}21 & \mathfrak{C}22 & \dots & \mathfrak{C}_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \mathfrak{C}m1 & \mathfrak{C}m2 & \dots & \mathfrak{C}_{mn} \\ \mathfrak{C}i1^+ & \mathfrak{C}i2^+ & \dots & \mathfrak{C}_{in}^+ \end{pmatrix}$$

Here
For benefit type criteria

$$\mathfrak{C}ij^- = min\mathfrak{C}ij$$

and

$$\mathfrak{C}ij^+ = max\mathfrak{C}ij$$

For cost type criteria

$$\mathfrak{C}ij^- = max\mathfrak{C}ij$$

and

$$\mathfrak{C}ij^+ = min\mathfrak{C}ij$$

9) Convert the extended decision matrix $\mathfrak{A}$ into normalized form $E = [n_{ij}]_{(m+2)\times n}$,based on the following equation:
For benefit type criteria

$$n_{ij} = \frac{\mathfrak{C}ij}{\mathfrak{C}ij^+}$$

For cost type criteria

$$n_{ij} = \frac{\mathfrak{C}ij^+}{\mathfrak{C}ij}$$

where, $\pounds_{\diamond_{ij}}$ and $\mathfrak{C}ij^+$ are the elements in the E matrix.

10) Calculate the weighted decision matrix. Build up the final weighted decision matrix $F = [f_{ij}]_{(m+2)\times n}$ by the following equation

$$f_{ij} = n_{ij} \times \breve{\Omega}_j$$

where, n_{ij} is an element of the matrix E' and $\breve{\Omega}_j$ is the weight of jth criteria.

11) Determine the utility degree of alternatives $\mathfrak{U}_i$ by employing following equations:

$$\mathfrak{U}i^- = \frac{\mathfrak{S}i}{\mathfrak{S}^-},$$

$$\mathfrak{U}i^+ = \frac{\mathfrak{S}i}{\mathfrak{S}^+},$$

where, $\mathfrak{S}i = \sum j = 1^n f_{(i+1)j} (i = 1, 2, ..., m)$, $\mathfrak{S}^- = \sum_{j=1}^{n} f_{1j}$ and $\mathfrak{S}^+ = \sum_{j=1}^{n} f_{(m+2)j}$.

12) Compute the utility function of alternatives $F(\mathfrak{U}_i)$ based on the following equation:

$$F(\mathfrak{U}i) = \frac{\mathfrak{U}i^+ + \mathfrak{U}i^-}{1 + \frac{1 - F(\mathfrak{U}i^+)}{F(\mathfrak{U}i^+)} + \frac{1 - F(\mathfrak{U}i^-)}{F(\mathfrak{U}_i^-)}},$$

where the utility function with respect to the ideal $F(\mathfrak{U}i^+)$ and anti-ideal $F(\mathfrak{U}i^-)$ are given, respectively, by the following formulas:

$$F(\mathfrak{U}i^+) = \frac{\mathfrak{U}i^-}{\mathfrak{U}i^+ + \mathfrak{U}i^-},$$

$$F(\mathfrak{U}i^-) = \frac{\mathfrak{U}i^+}{\mathfrak{U}i^+ + \mathfrak{U}i^-}.$$

13) Rank all alternatives in descending order and choose the best one.

The flow chart of algorithm of MARCOS method is given in Fig. 3

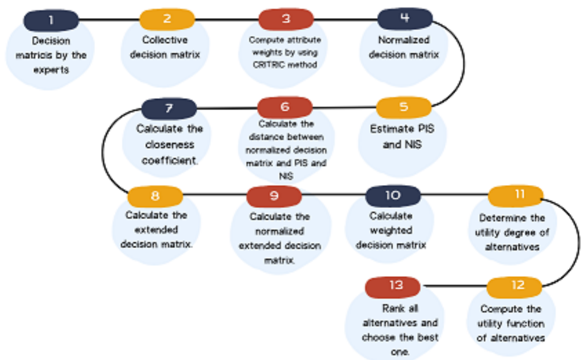

Fig. 3. Flow Chart of Algorithm of MARCOS method

B. Numerical Illustration

Step 1 Decision matrices by the expert1 ,expert2 and expert3 In Table I ,II and III respectively.

Step 2 In Table IV by using $SF\breve{Z}NWA$ aggregation operator aggregate all individuals Spherical fuzzy $\breve{z}$-numbers decision matrices into collective spherical fuzzy $\breve{z}$-numbers decision matrix

Step 3 The weights of attribute by using CRITRIC method given in Table V .

Step 4 The normalized decision matrix is calculated In Table VI.

Step 5 The positive ideal solution (PIS) P^+ and negative ideal solution (NIS) N^- are estimated in Table XVI and in Table XVII respectively.

TABLE XVI. The Positive Ideal Solution (PIS) P^+

$\pounds_{\diamond_{ij}}$	$\yen_1$
P^+	$((0.69654, 0.46142), (0.30765, 0.28879), (0.25716, 0.38106))$
$\pounds_{\diamond_{ij}}$	$\yen_1$
P^+	$((0.58407, 0.53947), (0.23641, 0.28330), (0.32695, 0.43386))$
$\pounds_{\diamond_{ij}}$	$\yen_3$
P^+	$((0.53882, 0.70940), (0.31784, 0.36083), (0.21809, 0.33466))$
$\pounds_{\diamond_{ij}}$	$\yen_4$
P^+	$((0.50338, 0.46702), (0.05266, 0.07357), (0.09224, 0.10491))$

TABLE XVII. The Negative Ideal Solution (NIS) N^-

$\pounds_{\diamond_{ij}}$	$\yen_1$
N^-	$((0.3781, 0.3661), (0.3076, 0.2887), (0.4361, 0.5659))$
$\pounds_{\diamond_{ij}}$	$\yen_2$
N^-	$((0.4761, 0.4511), (0.2364, 0.2833), (0.3845, 0.5507))$
$\pounds_{\diamond_{ij}}$	$\yen_3$
N^-	$((0.3781, 0.3406), (0.3178, 0.3608), (0.4469, 0.4610))$
$\pounds_{\diamond_{ij}}$	$\yen_4$
N^-	$((0.3221, 0.3108), (0.0526, 0.0735), (0.4479, 0.3134))$

Step 6 The distance between normalized decision matrix and PIS $\ddot{d}^+_{ij}$ and NIS $\ddot{d}^-_{ij}$ are computed in Table XVIII and in Table XIX.

TABLE XVIII. Distance between Normalized Decision Matrix and PIS $\ddot{d}^+_{ij}$

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_1$	0.15730	0.11321	0.03279	0.22587
$\top_2$	0.17266	0.05944	0.24294	0.09307
$\top_3$	0.09305	0.07483	0.16914	0.06838
$\top_4$	0.18914	0.08304	0.24961	0.17450

TABLE XIX. Distance between Normalized Decision Matrix and NIS $\ddot{d}^-_{ij}$

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_1$	0.11177	0.11525	0.26998	0.20618
$\top_2$	0.11139	0.07390	0.08235	0.25684
$\top_3$	0.17921	0.09211	0.15080	0.25935
$\top_4$	0.17319	0.06666	0.11256	0.21459

Step 7 The closeness coefficient are given in Table XX.

TABLE XX. closeness coefficient

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_1$	0.41539	0.50445	0.89171	0.47721
$\top_2$	0.39216	0.55423	0.25317	0.73401
$\top_3$	0.65825	0.55173	0.47135	0.79136
$\top_4$	0.47799	0.44527	0.31079	0.55153

Step 8 The extended decision matrix is given in Table XXI.

TABLE XXI. Extended Decision Matrix

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_i^-$	0.39216	0.44527	0.25317	0.47721
$\top_1$	0.41539	0.50445	0.89171	0.47721
$\top_2$	0.39216	0.55423	0.25317	0.73401
$\top_3$	0.65825	0.55173	0.47135	0.79136
$\top_4$	0.47799	0.44527	0.31079	0.55153
$\top_i^+$	0.65825	0.55423	0.89171	0.79136

Step 9 The normalized extended decision matrix are given in Table XXII.

TABLE XXII. NORMALIZED EXTENDED DECISION MATRIX

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_i^-$	0.59576	0.80340	0.28391	0.60303
$\top_1$	0.63106	0.91017	1.00000	0.60303
$\top_2$	0.59576	1.00000	0.28391	0.92753
$\top_3$	1.00000	0.99549	0.52859	1.00000
$\top_4$	0.72616	0.80340	0.34854	0.69694
$\top_i^+$	1.00000	1.000007	1.00000	1.00000

Step 10 The weighted normalized of extended decision matrix are given in Table XXIII.

TABLE XXIII. WEIGHTED NORMALIZED OF EXTENDED DECISION MATRIX

$\pounds_{\diamond_{ij}}$	$\yen_1$	$\yen_2$	$\yen_3$	$\yen_4$
$\top_i^-$	0.10426	0.20724	0.10352	0.12207
$\top_1$	0.11044	0.23478	0.36461	0.12207
$\top_2$	0.10426	0.25795	0.10352	0.18777
$\top_3$	0.17501	0.25679	0.19273	0.20244
$\top_4$	0.12708	0.20724	0.12708	0.14108
$\top_i^+$	0.17501	0.25795	0.36461	0.20244

Step 11 The utility degree of alternatives $\mathfrak{U}_i^-$ and $\mathfrak{U}_i^+$ are given in Table XXIV.

TABLE XXIV. UTILITY DEGREE OF ALTERNATIVES

$\mathfrak{U}_i^-$	$\mathfrak{U}_i^+$
1.54890	0.83190
1.21673	0.65350
1.53969	0.82696
1.12175	0.60248

Step 12 The utility function of alternatives $F(\mathfrak{U}_i)$ are given in Table XXV.

TABLE XXV. UTILITY FUNCTION OF ALTERNATIVES

$F(\mathfrak{U}_i)$
0.69628
0.55023
0.70045
0.50728

Step 13 Ranking all possibilities in descending order are given in Table XXVI .

TABLE XXVI. RANKING OF ALL POSSIBILITIES

method	scoring
MARCOS Method	$\top_3 \geq \top_1 \geq \top_2 \geq \top_4$

Ranking of comparison between CRADIAS method and MARCOS method are given in Table XXVII.

TABLE XXVII. RANKING OF COMPARISON BETWEEN CRADIAS METHOD AND MARCOS METHOD

sr.	methods	scoring
1	CRADIAS method	$\top_3 \geq \top_1 \geq \top_2 \geq \top_4$
2	MARCOS method	$\top_3 \geq \top_1 \geq \top_2 \geq \top_4$

Graphical Representation of comparison between CRADIAS and MARCOS method Ranking in Fig. 4.

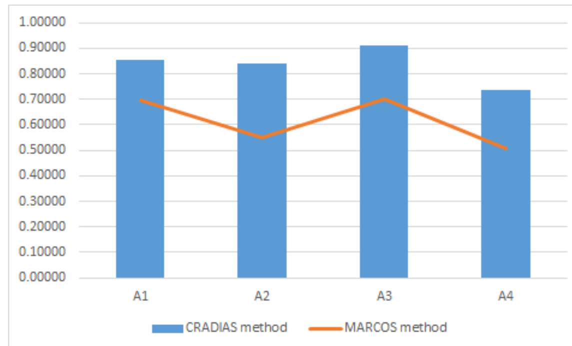

Fig. 4. Graphical Representation of Comparison between $SF\acute{Z}N_{SW}$ and MARCOS Method Ranking

VI. DISCUSSION

An experiment was conducted in the scenario of spherical fuzzy Z-numbers to assess the performance of the proposed algorithms with respect to existing metrics. All tactics eventually lead to the same optimal option, despite some variations in the ranking. A detailed comparison of rankings and graphical representations for the MARCOS approach and CRADIAS inside the $SF\breve{Z}N$ environment can be seen in Table XXVII and Fig. 4. The major objective of this study was to ascertain which method of decision-making was more effective in this specific circumstance. Throughout the investigation, it was found that the ranking order of the alternatives can exhibit slight variations based on the aggregating methods used. However, the optimal course of action was consistently determined by each strategy. Consequently, $\Im(\top_3)$ emerges as the optimal substitute option. The power and reliability of the recommended algorithms are demonstrated by the striking consistency in selecting the optimal solution. The fact that, despite minor ranking discrepancies, every participant chose the same optimal solution suggests how effective the recommended technique is in resolving issues brought on by spherical fuzzy $\breve{Z}$-numbers.

VII. CONCLUSION

This work attempts to offer basic operating principles for Spherical Fuzzy Z-numbers ($SF\breve{Z}N$) utilizing the CRADIAS technique. We address the inherent complexity of Multiple

Attributes Group Decision Making (MAGDM) scenarios by combining the strategies of these suggested operators in a novel decision-making approach. This innovative method adds a layer to the decision-making process that enables the assessment of both positive and negative factors. In summary, the empirical findings of our research demonstrate that the approach presented here is the most useful and realistic way to solve MAGDM difficulties. Following a thorough examination of situations related to the assessment of english teaching performance in a secondary school setting and comparisons with the MARCOS method, the recommended $SF\breve{Z}N$ Operators have been shown to be viable and valid. Furthermore, our work supports its results with a rigorous mathematical example. Ultimately, our findings demonstrate that the approach outlined in this paper is the most practical and effective means of resolving MAGDM issues. Further research endeavors will concentrate on developing innovative methods of decision-making that are particular to the $SF\breve{Z}N$ context. The ELECTRE technique, EDAS, TOPSIS, and other methods will be combined in these approaches to increase the effectiveness of decision-making.

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