Robust Stability Analysis of Switched Neutral Delayed Systems with Parameter Uncertainties

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***Abstract*—A time-delay neural system is an accurate class of neural system that exposes delays in both the state values and their derivatives. In this case, it is critical to maintain the system stability. Here, the stability investigation on uncertain switched-neutral systems with state-time delays is the focus of this paper. In fact, a novel adequate condition in terms of the feasibility of Linear Matrix Inequalities (LMIs) is offered to guarantee the global asymptotically stability of this category of systems with parameter uncertainties, based on the Lyapunov-Krasovskii functional method. Additionally, resistance against errors and disturbances can be ensured using the Multiple Quadratic Lyapunov Functions (MQLFs). Through a numerical example, the designed method's effectiveness is proven.**

Keywords*—Switched neutral systems; parameter uncertainties; delay-dependent; robust stability; multiple quadratic Lyapunov-Krasovskii; LMI technique

I. INTRODUCTION

Researchers have become more intrigued by neural systems in recent years due to their ability to be practice to numerous real-dynamical systems in different domains of knowledge. Engineering encompasses information science, combinatorial optimization, automatic control, signal processing, and fault diagnosis [1], [2], [3], [4].

Indeed, a time-delay neural is a specific type of neural system that exhibits lags (retards or delays) in the state values along with their derivatives. In fact, time delays can widely arise during the electronic realization of neural networks, by dint of the finite switching speed of amplifiers and the time needed for communication. For this reason, there has been a significant amount of interest in delayed neural networks. It is crucial to maintain stability when using neural networks for tasks such as designing associative memory and pattern recognition...etc. The hardware realization of neural networks can cause delays in signal transmission, which can result in undesirable dynamical behaviors, such as oscillation and instability [5], [6].

Hence, when studying the stability of the neural system, it's important to take into account time delays. Numerous valuable stability criteria have been developed due to the extensive research on the stability analysis of delayed neural networks in the past several decades [7], [8]. The stabilization of delayed dynamical systems has further appealed to a lot of interest, and many feedback stabilization control techniques have been proposed [9], [10], [11].

Despite that, switched systems, which are an important subclass of hybrid systems, feature a logic rule that governs the switching between a finite number of subsystems [12]. During the beyond few decades, switched structures had been investigated due to their fulfillment in real-global applications [13], [14], [15]. Exponential stabilisation and L2-gain for uncertain switched nonlinear systems with interval time-varying delays have been discussed by Dong et al. [16]. Moreover, the average dwell time approach has been used by Liu et al. Robust stability requirements for discrete-time switched neural networks with different activation functions has been provided through Arunkumar et al. [17]. Ma et al.'s study [18] looked into the use of an asynchronous switching delay system to stabilize networked switched linear systems. After that, global exponential stability for switched stochastic neural networks with time-varying delays was examined by Wu et al. [19].

Referring to [20], stability study for uncertain switched systems with time-varying latency has been examined. In [21], the semi-tensor product of matrices was used to study the stabilization analysis and stabilizing switching signal design of switched Boolean networks. The conversation above demonstrates the need to research switched neural networks with parametric uncertainty and time delay.

The robust stability issue for uncertain switched neutral time-delay systems has not roughly been studied. The focus of this paper is on analysing the stability of switched neutral delayed systems with parameter uncertainties. To offer updated stability conditions in the occurrence of faults and disturbances within the investigated system, the research work introduces a novel criteria for ensuring robust asymptotic stability by using (MQLF) approach.

Moreover, LMI is used for optimization, problem verification, and deriving feasibility conditions. Lastly, numerical example is provided to show the efficiency of the proposed theorems.

The remainder of this work is demonstrated below: Section II contains the problem formulation. In Section III, the robust stability study for like systems with affected by faults and disturbances is clarified, as well as the suggested theorems are hereafter shown in details. The simulation results of the developed stability method is provided in Section IV. At last, Section V gives a conclusion.

II. PROBLEM FORMULATION

The following describes a class of uncertain switched linear neutral systems with state delays:

$$\begin{cases} \dot{x}(t) - \bar{J}_\sigma \dot{x}(t-\varepsilon_2) = \bar{R}_\sigma x(t) + \bar{D}_\sigma x(t-\varepsilon_1) \\ \qquad + \bar{B}_\sigma u(t) + \mathrm{F}_\sigma d(t) + \mathrm{E}_\sigma f(t) \\ y(t) = C_\sigma x(t) + K_{d_\sigma} d(t) + K_{f_\sigma} f(t) \\ x(t) = \theta(t) \quad ; \forall t \in [-\gamma, 0] \end{cases} \tag{1}$$

The state vector of the system, denoted as $x(t) \in R^n$, is influenced by an input vector for control, $u(t) \in R^m$, while the output vector is represented by $y(t) \in R^p$. A switching signal, $\sigma : [0, \infty[\to N = \{1, 2, 3, ..., n\}$, manages the switching of subsystems $i \in N$. The constant matrices $\bar{R}_i$, $\bar{D}_i$, $\bar{J}_i$, $\bar{B}_i$, and C_i are confirmed to have appropriate dimensions. The disturbance input is denoted by $d(t) \in L_2^P [0, \infty[$, and the fault vector is represented as $f(t) \in R^l$. Each subsystem is characterized by known real matrices F_{f_i}, E_{d_i}, K_{f_i}, and K_{d_i} for every i. The state's derivative and delay time are specified by $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, with $\gamma = \max\{\varepsilon_1, \varepsilon_2\}$, and $\theta(t)$ is an initial continuous vector-valued function.

III. MULTIPLE QUADRATIC LYAPUNOV FUNCTIONS

In system theory, developing Lyapunov functions is essential, especially when determining whether the system under study is internally stable. Stability is indicated by the existence of a suitable Lyapunov function. A common option is the Common Quadratic Lyapunov Function, which acts as a total Lyapunov candidate function for all of the modes that comprise the switched dynamical system. On the other hand, by connecting several quadratic Lyapunov functions, MQLFs provide an unorthodox method. Every function is maximized in the area that it is assigned.

In fact, MQLFs are preferred over CQLFs due to their less conservative nature, even though the global function may allow discontinuities and exhibit non-decreasing behavior over state trajectories. Relevant literature has emphasized the usefulness of MQLFs and their intuitive results, as discussed in [11]. It is noteworthy that MQLFs show a decrease in each active mode, as shown in [12], with their values post-switching instances staying lower than beforehand.

A. New Stability Criterion

The subsequent paper investigates the stability study of the switched neutral system in linear form with state delay-dependent (1) behavior. From this, choose a Lyapunov functional candidate using the following criteria:

$$V_i(x,t) = V_{1_i}(x,t) + V_{2_i}(x,t) + V_{3_i}(x,t) \tag{2}$$

When given positive constants P_i, Q_i, and H_i, the following theorem holds for system (Eq. 1) with the Lyapunov functional candidate given by (Eq. 3).
The parameter uncertainties are expressed through the following formulations:

$$\bar{J} = J_\sigma + \Delta J_\sigma,\ \bar{R} = R_\sigma + \Delta R_\sigma,\ \bar{D} = D_\sigma + \Delta D_\sigma, \text{ and } \bar{B} = B_\sigma + \Delta B_\sigma.$$

These uncertain matrices, denoted by the symbol Δ, are time-dependent, with ΔJ_σ, ΔR_σ, ΔD_σ,and ΔB_σ varying with time t.
Furthermore, the parameter uncertainties are subject to norm-bounded terms: As well, the norm-bounded parameter uncertainty terms are given as

$\Delta J_i = Z_{i_1} \sum_{i_1} W_{1_i}, \Delta R_i = Z_{i_2} \sum_{i_2} W_{2_i}$, $\Delta D_i = Z_{i_3} \sum_{i_3} W_{3_i}$,

$\Delta B_i = Z_{i_4} \sum_{i_4} W_{4_i}$ where Z_{i_1}, Z_{i_2}, Z_{i_3}, Z_{i_4}, W_{i_1}, W_{i_2}, W_{i_2} and W_{4_i} are known constant matrices. After that, $\sum\limits_{1_i}^{T} \sum < I_i$, $\sum\limits_{2_i}^{T} \sum < I_i$, $\sum\limits_{3_i}^{T} \sum < I_i$ and $\sum\limits_{4_i}^{T} \sum < I_i$

Theorem 1:
The stability of the switched neutral system together with state-time delays (Eq. 1) is established for a fixed value $\varepsilon > 0, \gamma > 0$, under the condition that there exist positive definite symmetric matrices X_i, T_i, and Y_i, along with scalar λ_i. This stability is satisfied by the satisfaction of the following LMI.

$$\begin{bmatrix} N(X_i) & 0 & \bar{J}_i Y_i & \bar{B}_i & E_i + C_i{}^T K_{f_i} & F_i + C_i{}^T K_{d_i} & X_i \bar{R}_i^T & X_i C_i{}^T & X_i & E_i Y_i \\ * & -T_i & 0 & 0 & 0 & 0 & T_i \bar{D}_i^T & 0 & 0 & 0 \\ * & * & -Y_i & 0 & 0 & 0 & Y_i \bar{J}_i^T & 0 & 0 & 0 \\ * & * & * & I_i & 0 & 0 & \bar{B}_i^T & 0 & 0 & 0 \\ * & * & * & * & -\lambda_i{}^2 I_i + K_{f_i}^T K_{d_i} & 0 & E_i^T & 0 & 0 & 0 \\ * & * & * & * & * & K_{d_i}^T K_{d_i} & F_i^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{1+\gamma} Y_i & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I_i & 0 & 0 \\ * & * & * & * & * & * & * & * & -T_i & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{\gamma} Y_i \end{bmatrix} < 0 \tag{3}$$

The expression $N(X_i)$ is given by
$N(X_i) = (\bar{R}_i + \bar{D}_i) X_i + X_i (\bar{R}_i + \bar{D}_i)^T$,

where $(^T)$ represents the transposition operation applied symmetrically and I stands for the identity matrix.

Proof:
In essence, express $x = x(t)$, $x_{\varepsilon_1} = x(t-\varepsilon_1)$, $x_{\varepsilon_2} = x(t-\varepsilon_2)$, $f_1 = f(t)$ $d_1 = d(t)$ and $\alpha_1 = (t+\alpha)$.
Additionally, denote $\omega = (1+\varepsilon_1)$ in the subsequent demonstration. The (MQLF) functional (2)is introduced.
with

- $V_{1_i}(x,t) = x^T(t) P_i x(t)$
- $V_{2_i}(x,t) = \int_{t-\varepsilon_1}^{t} x^T(s) Q_i x(s) ds$
- $V_{3_i}(x,t) = \int_{t-\varepsilon_2}^{t} \dot{x}^T(s) H_i \dot{x}(s) ds + \int_{-\varepsilon_1}^{0} \left(\int_{t+\alpha}^{t} \dot{x}^T(s) H_i \dot{x}(s) ds \right) d\alpha$

The (MQLF) function is fulfilled when the matrices P_i, Q_i, and H_i are symmetric positive definite.

$$\begin{aligned}
&\dot{V}_i(x,t) = 2\dot{x}^T P_i x + x^T L_i x - x_{\varepsilon_1}^T Q_i x_{\varepsilon_1} + \dot{x}^T H_i \dot{x} \\
&-\dot{x}_{\varepsilon_2}^T H_i \dot{x}_{\varepsilon_2} + \int_{-\varepsilon_1}^{0} \left[\dot{x}^T H_i \dot{x} \; -\dot{x}^T \alpha_1 H_i \dot{x} \alpha_1\right] d\alpha \\
&\qquad = 2\dot{x}^T P_i x + x^T Q_i x - x_{\varepsilon_1}^T Q_i x_{\varepsilon_1} + \eta \dot{x}^T H_i \dot{x} \\
&\qquad -\dot{x}_{\varepsilon_2}^T M_i \dot{x}_{\varepsilon_2} - \int_{-\varepsilon_1}^{0} \dot{x}^T \alpha_1 H_i \dot{x} \alpha_1 d\alpha \\
&= x^T \left(P_i(\bar{R}_i + \bar{D}_i) + (\bar{R}_i + \bar{D}_i)^T P_i + Q_i \right) x \\
&-2x^T P_i \int_{-\varepsilon_1}^{0} \bar{D}_i \dot{x} \alpha_1 d\alpha + 2x^T P_i \bar{J}_i \dot{x}_{\varepsilon_2} \\
&+2x^T P_i \bar{B}_i u(t) + 2x^T P_i E_i f_1 + 2x^T(t) P_i F_i d_1 \\
&-x_{\varepsilon_1}^T Q_i x_{\varepsilon_1}^T + \eta \dot{x}^T H_i \dot{x} - \dot{x}_{\varepsilon_2}^T H_i \dot{x}_{\varepsilon_2} \\
&- \int_{-\varepsilon_1}^{0} \dot{x}^T \alpha_1 H_i \dot{x} \alpha_1 d\alpha \\
&-2x^T P_i \int_{-\varepsilon_1}^{0} \bar{D}_i \dot{x} \alpha_1 d\alpha = - \int_{-\varepsilon_1}^{0} 2x^T P_i \bar{D}_i \dot{x} \alpha_1 d\alpha \\
&\leq \int_{-\varepsilon_1}^{0} x^T P_i \bar{D}_i H_i^{-1} \bar{D}_i^T P_i x + \dot{x}^T \alpha_1 H_i \dot{x} \alpha_1 d\alpha \\
&\leq \varepsilon_1 x^T P_i \bar{D}_i H_i^{-1} \bar{D}_i^T P_i x + \int_{-\varepsilon_1}^{0} \dot{x}^T \alpha_1 H_i \dot{x} \alpha_1 d\alpha
\end{aligned} \tag{4}$$

Additionally substituting,

$$\begin{aligned}
&\dot{x}^T (H_i + \varepsilon_1 H_i) \dot{x} = x^T \bar{R}_i^T \omega H_i \bar{R}_i x + 2x^T \bar{R}_i^T \omega H_i \bar{D}_i x_{\varepsilon_1} \\
&+2x^T \bar{R}_i^T \omega M_i \bar{J}_i \dot{x}_{\varepsilon_2} + 2x^T \bar{R}_i^T \omega H_i \bar{B}_i u(t) \\
&+2x^T \bar{R}_i^T \omega H_i E_i f_1 + 2x^T \bar{R}_i^T \omega H_i F_i d_1 \\
&+2x_{\varepsilon_2}^T \bar{D}_i^T \omega H_i \bar{J}_i \dot{x}_{\varepsilon_2} + 2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i \bar{B}_i u(t) \\
&+2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i E_i f_1 + 2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i F_i d_1 \\
&+x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i \bar{D}_i x_{\varepsilon_1} + 2\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i \bar{B}_i u(t) \\
&+2\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i E_i f_1 + 2\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i F_i d_1 \\
&+\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i \bar{J}_i \dot{x}_{\varepsilon_2} + u^T(t) \bar{B}_i^T \omega H_i \bar{B}_i u(t) \\
&+u^T(t) \bar{B}_i^T \omega H_i E_i f_1 + u^T(t) F_i^T \omega H_i F_i d_1 \\
&+d_1^T F_i \omega H_i \bar{B}_i u(t) + d_1^T F_i \omega H_i E_i f_1 \\
&+d_1^T F_i \omega H_i F_i d_1 + f^T(t) E_i^T \omega H_i \bar{B}_i u(t) \\
&+f_1^T E_i^T \omega H_i E_i f_1 + f_1^T E_i^T \omega H_i F_i d_1
\end{aligned} \tag{5}$$

Finally, the candidate Lyapunov function is rewritten as:

$$\begin{aligned}
&\dot{V}_i(x,t) = x^T \Big(P_i(\bar{R}_i + \bar{D}_i) + (\bar{R}_i + \bar{D}_i)^T P_i + Q_i \\
&+\varepsilon_1 P_i \bar{R}_i M_i^{-1} \bar{R}_i^T P_i \Big) x + 2x^T P_i \bar{J}_i \dot{x}_{\varepsilon_2} + 2x^T P_i \bar{B}_i u(t) \\
&+2x^T P_i E_i f_1 + 2x^T(t) P_i F_i d_1 - x_{\varepsilon_1}^T Q_i x_{\varepsilon_1} \\
&+x^T \bar{R}_i^T \omega H_i \bar{R}_i x + 2x^T \bar{R}_i^T \omega H_i \bar{D}_i x_{\varepsilon_1} \\
&+2x^T \bar{R}_i^T \omega H_i \bar{J}_i \dot{x}_{\varepsilon_2} + 2x^T \bar{R}_i^T \omega H_i \bar{B}_i u(t) \\
&+2x^T \bar{R}_i^T \omega H_i E_i f_1 + 2x^T \bar{R}_i^T \omega H_i F_i d_1 \\
& \\
&+2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i \bar{J}_i \dot{x}_{\varepsilon_2} + 2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i \bar{B}_i u(t) \\
&+2x_{\varepsilon_1}^T \bar{D}_i^T \omega H_i E_i f_1 + 2x_{\varepsilon_1}^T E_i^T \omega H_i F_i d_1 \\
&+x_{\varepsilon_1}^T E_i^T \omega H_i E_i x_{\varepsilon_1} + 2\dot{x}_{\varepsilon_2}^T D_i^T \omega H_i \bar{B}_i u(t) \\
&+2\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i E_i f_1 + 2\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i F_i d_1 \\
&+\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i D_i \dot{x}_{\varepsilon_2} + u^T(t) \bar{B}_i^T \eta H_i \bar{B}_i u(t) \\
&+\dot{x}_{\varepsilon_2}^T \bar{J}_i^T \omega H_i \bar{J}_i \dot{x}_{\varepsilon_2} + u^T(t) \bar{B}_i^T \omega H_i \bar{B}_i u(t) \\
&+u^T(t) \bar{B}_i^T \omega H_i F_i f_1 + u^T(t) \bar{B}_i^T \omega H_i F_i d_1 \\
&+d_1^T \bar{B}_i \omega H_i \bar{B}_i u(t) + d_1^T F_i \omega H_i E_i f_1 \\
&+d_1^T F_i \omega H_i F_i d_1 + f_1^T E_i^T \omega H_i \bar{B}_i u(t) \\
&+f_1^T E_i^T \omega H_i E_i f_1 + f_1^T E_i^T \omega H_i F_i d_1 - \dot{x}_{\varepsilon_2}^T H_i \dot{x}_{\varepsilon_2}
\end{aligned} \tag{6}$$

The primary objective is to ensure the reduction of the impact of faults represented by the function f_1 and the output signal $y(t)$.

$$\psi_i = \sup_{f_1 \in \ L_2 - 0} \frac{\|y\|_2}{\|f_1\|_2} < \lambda_i \tag{7}$$

The criterion ψ_i will be used to minimise energy so that we can examine the stability of the system presented in the Eq. (1), as explained below.

Moreover, the objective is to reduce the criterion function in the manner shown below:

$$\begin{aligned}
\psi_i = \int_0^\infty y^T(t) y(t) - \lambda_i^2 f_1^T f_1 + \dot{V}_i(x,t) dt \\
+V_i(x,t)|_{t=0} - V_i(x,t)|_{t=\infty}
\end{aligned} \tag{8}$$

with

$$\begin{aligned}
&y^T(t) y(t) = \\
&[C_i x(t) + K_{f_i} f_1 + K_{d_i} d_1]^T [C_i x(t) + K_{f_i} f_1 + K_{d_i} d_1] \\
&= x^T(t) C_i^T C_i x(t) + x^T(t) C_i^T K_{f_i} f_1 + x^T(t) C_i^T K_{d_i} d_1 \\
&+f_1^T K_{f_i}^T C_i x(t) + f_1^T K_{f_i}^T K_{f_i} f_1 + f_1^T K_{f_i}^T K_{d_i} d_1 \\
&+d_1^T K_{d_i}^T C_i x(t) + d_1^T K_{d_i}^T K_{f_i} f_1 + d^T(t) K_{d_i}^T K_{d_i} d_1
\end{aligned} \tag{9}$$

Simplifying Eq. (8) is given as:

$$\psi_i = \int_0^\infty \left\{ \beta^T \varsigma(\varepsilon_1) \ \beta \right\} dt \tag{10}$$

wherein:
$\beta = \begin{bmatrix} x^T & x_{\varepsilon_1}^T & \dot{x}_{\varepsilon_2}^T & u^T(t) & f_1^T & d_1^T \end{bmatrix}^T$ such that ψ_i is defined in the following manner:
$\mu(\varepsilon_1) =$

$$\begin{bmatrix}
\varsigma_{11} & \varsigma_{12} & \varsigma_{13} & \varsigma_{14} & \varsigma_{15} & \varsigma_{16} \\
* & \varsigma_{22} & \varsigma_{23} & \varsigma_{24} & \varsigma_{25} & \varsigma_{26} \\
* & * & \varsigma_{33} & \varsigma_{34} & \varsigma_{35} & \varsigma_{36} \\
* & * & * & \varsigma_{44} & \varsigma_{45} & \varsigma_{46} \\
* & * & * & * & \varsigma_{55} & \varsigma_{56} \\
* & * & * & * & * & \varsigma_{66}
\end{bmatrix} \tag{11}$$

$\varsigma_{11} = P_i(\bar{R}_i + \bar{D}_i) + (\bar{R}_i + \bar{D}_i)^T P_i + Q_i + \varepsilon_1 P_i \bar{D}_i H_i^{-1} \bar{D}_i^T P_i + \omega \bar{R}_i^T H_i \bar{R}_i + C_i^T C$
$\varsigma_{12} = \omega \bar{R}_i^T H_i \bar{D}_i$
$\varsigma_{13} = P_i \bar{J}_i + \omega \bar{R}_i^T H_i \bar{J}_i$
$\varsigma_{14} = P_i \bar{B}_i + \omega \bar{R}_i^T H_i \bar{B}_i$
$\varsigma_{15} = C_i^T K_{d_i} + P_i B_{f_i} + \omega A_i^T H_i E_i$
$\varsigma_{16} = C_i^T K_{d_i} + P_i B_{d_i} + \omega A_i^T H_i F_i$

$\varsigma_{22} = \omega \bar{D}_i^T H_i \bar{D}_i - Q_i$
$\varsigma_{23} = \omega \bar{D}_i^T H_i \bar{J}_i$
$\varsigma_{24} = \omega \bar{D}_i^T H_i \bar{B}_i$
$\varsigma_{25} = \omega \bar{D}_i^T H_i E_i$
$\varsigma_{26} = \omega \bar{D}_i^T H_i F_i$

$\varsigma_{33} = \omega \bar{D}_i^T H_i \bar{J}_i - H_i$
$\varsigma_{34} = \omega \bar{D}_i^T H_i \bar{J}_i \bar{B}_i$
$\varsigma_{35} = \omega \bar{D}_i^T H_i E_i$
$\varsigma_{36} = \omega \bar{D}_i^T H_i F_i$

$\varsigma_{44} = \omega \bar{B}_i^T H_i \bar{B}_i - I_i$
$\varsigma_{45} = \omega \bar{B}_i^T H_i E_i$
$\varsigma_{46} = \omega \bar{B}_i^T H_i F_i$

$\varsigma_{55} = -\gamma_i^2 I_i + K_{d_i}^T K_{d_i} + \omega E_i^T H_i E_i$
$\varsigma_{56} = K_{f_i}^T K_{d_i} + \omega E_i^T H_i F_i$
$\varsigma_{66} = K_{d_i}^T K_{d_i} + \omega F_i^T H_i F_i$

It's clear that inequality (8) $\dot{V}_i < 0$, if $\varsigma(\varepsilon_1) < 0$.
The matrix $\mu(\varepsilon_1) < 0$ is considered monotonic rising according to $\gamma < 0$, thus, keeps towards $0 < \delta \leq \gamma$ if $\Xi(\gamma) < 0$ The inequality(13) can be rewritten by means of the Schur complement.

$$\begin{bmatrix} S(P_i,Q_i) & 0 & P_i\bar{J}_i & P_i\bar{B}_i & C_i^T K_{f_i} + P\bar{D}_i & C_i^T K_{d_i} + P\bar{B}_i & \bar{R}_i^T & P\bar{D}_i \\ * & -Q_i & 0 & 0 & 0 & 0 & \bar{D}_i^T & 0 \\ * & * & -H_i & 0 & 0 & 0 & \bar{J}_i^T & 0 \\ * & * & * & -I_i & 0 & 0 & \bar{B}_i^T & 0 \\ * & * & * & * & -{\lambda_i}^2 I_i + K_{f_i}^T K_{d_i} & K_{f_i}^T K_{d_i} & E_i^T & 0 \\ * & * & * & * & * & K_{d_i}^T K_{d_i} & F_i^T & 0 \\ * & * & * & * & * & * & -\frac{1}{1+\gamma} {H_i}^{-1} & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\gamma} H_i \end{bmatrix} < 0 \tag{12}$$

where:
$S(P_i,Q_i) = P_i(\bar{R}_i + \bar{d}_i) + (\bar{R}_i + \bar{D}_i)^T P_i + L_i + C_i^T C_i$

From $diag(\Xi_{1_i}, \Xi_{2_i}, \Xi_{3_i}, I_i, \Xi_{3_i})$ which can be multiplied on both sides of the Eq. (12)and after that, using the Schur complement, one gets

$$[\Gamma_{ij}]_{10\times 10} < 0 \tag{13}$$

$\Gamma_{11} = (\bar{R}_i + \bar{D}_i)\Xi_{1_i} + \Xi_{1_i}(\bar{R}_i + \bar{D}_i)^T$
$\Gamma_{13} = \bar{J}_i \Xi_{3_i}$
$\Gamma_{14} = \bar{B}_i$
$\Gamma_{15} = E_i + C_i^T K_{f_i}$
$\Gamma_{16} = F_i + C_i^T K_{d_i}$
$\Gamma_{17} = \Xi_{1_i} \bar{R}_i^T$
$\Gamma_{18} = \Xi_{1_i} C_i^T$
$\Gamma_{19} = \Xi_{1_i}$
$\Gamma_{110} = \bar{D}_i \Xi_{3_i}$

$\Gamma_{22} = -\Xi_{2_i}$
$\Gamma_{27} = \Xi_{2_i} \bar{D}_i^T$
$\Gamma_{33} = -\Xi_{3_i}$
$\Gamma_{37} = \Xi_{3_i} \bar{J}_i^T$
$\Gamma_{44} = I_i$
$\Gamma_{47} = \bar{B}_i^T$
$\Gamma_{55} = K_{d_i}^T K_{d_i}$

$\Gamma_{57} = E_i^T$
$\Gamma_{66} = -\lambda_i^2 I_i + K_{f_i}^T K_{d_i}$
$\Gamma_{67} = F_i^T$
$\Gamma_{77} = -\frac{1}{1+\gamma} Y_i$

$\Gamma_{88} = -I_i$
$\Gamma_{99} = -\Xi_{2_i}$
$\Gamma_{1010} = -\frac{1}{\gamma}\Xi_{3_i}$

$\Gamma_{ij} = 0 \quad$ if not

End demonstration
As a result, from Theorem 1, one holds that $\dot{V}_i < 0$.

IV. Numerical Example

This section presents an illustrated example that was obtained from [22]. The pertinence of the developed theorems is shown and considered in this letter.
Consider a system of uncertain switched neutral (1) that consists of two subsystems. The parameters of these subsystems are as follows:

- Mode 1

  $R_1 = \begin{bmatrix} -5 & 0 \\ 0 & -3 \end{bmatrix}$, $D_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$

  $J_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & -0.1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

  $C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F_1 = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 0.1 \end{bmatrix}$

  $E_1 = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 0.1 \end{bmatrix}$, $K_{d_1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$

  $K_{f_1} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}$ and $\gamma = 0.5$

  $Z_{11} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $Z_{12} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$, $Z_{13} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$,

  $Z_{14} = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$, $W_{11} = \begin{bmatrix} 0.01 & 0.3 \end{bmatrix}$, $W_{21} = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix}$, $W_{31} = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix}$, $W_{41} = 0.02$

- Mode 2

  $R_2 = \begin{bmatrix} -4.5 & 0 \\ 0 & -0.1 \end{bmatrix}$, $D_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$

  $J_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

  $C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F_1 = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 0.1 \end{bmatrix}$

  $E_2 = \begin{bmatrix} 0.01 & 0.01 \\ 0.02 & 0.1 \end{bmatrix}$, $K_{d_2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$

  $K_{f_2} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}$ and $\gamma = 0.5$

  $Z_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $Z_{22} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$, $Z_{23} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$,

$Z_{24} = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$, $W_{12} = \begin{bmatrix} 0.01 & 0.3 \end{bmatrix}$, $W_{22} = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix}$, $W_{32} = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix}$, $W_{42} = 0.02$

Theorem 1 proves that LMI (3) is feasible. After solving the LMI, the stability of (1) is dictated. The matrices that correspond are determined as follows:

- Mode 1

$$\Xi_{11} = \begin{bmatrix} 0.4500 & -0.0003 \\ -0.0003 & 0.7218 \end{bmatrix} > 0$$

$$\Xi_{21} = \begin{bmatrix} 12.7884 & -0.0031 \\ -0.0031 & 12.9149 \end{bmatrix} > 0$$

$$\Xi_{31} = \begin{bmatrix} 0.3101 & -0.0004 \\ -0.0004 & 0.3199 \end{bmatrix} > 0$$

- Mode 2

$$\Xi_{12} = \begin{bmatrix} 0.6659 & -0.0005 \\ -0.0005 & 2.2668 \end{bmatrix} > 0$$

$$\Xi_{22} = \begin{bmatrix} 17.6291 & 0.0269 \\ 0.0269 & 15.1129 \end{bmatrix} > 0$$

$$\Xi_{32} = \begin{bmatrix} 2.8709 & 0.0159 \\ 0.0159 & 2.2215 \end{bmatrix} > 0$$

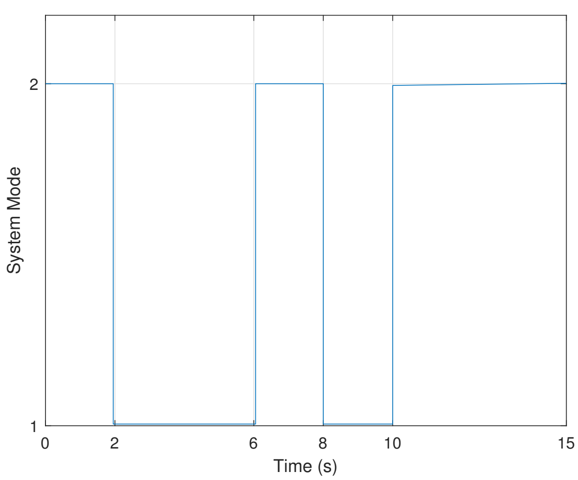

Fig. 1. The switching signal.

The switching signal and output responses are shown in Fig. 1 and 2, respectively.

V. Conclusion

The stability issue related to switched neutral time-delay systems with uncertainties which are norm-bounded has been addressed throughout the present research. It has been illustrated and computed that a new set of criteria can be generated from (MQLF) through resolving a set of LMIs.

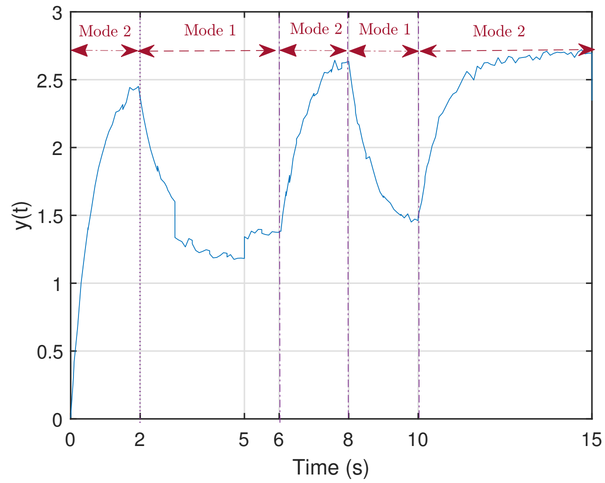

Fig. 2. Output response of uncertain switched neutral system.

Ultimately, the forcefulness and effectiveness of sufficient stability conditions have been illustrated from simulation results.

In forthcoming studies, the proposed methodologies will be expanded to encompass broader, uncertain stochastic switched neural using intervals and time-varying delays.

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