Optimization of PID Controller Parameter using the Geometric Mean Optimizer

Osama Abdellatif[1], Mohamed Issa[2], Ibrahim Ziedan[3]
Computer and Systems Engineering Dept., Faculty of Engineering, Zagazig University, Zagazig, Egypt[1, 2, 3]
Faculty of Computer Science and Information Technology, Egypt-Japan University, New Borg El Arab, Egypt[2]

***Abstract*—The PID controller is a crucial element in numerous engineering applications. However, a significant challenge with PID lies in selecting optimal parameter values. Conventional methods need extra tunning and may not yield the best performance. In this study, a recently introduced metaheuristic algorithm, Geometric Mean Optimizer (GMO), is employed to identify the most suitable PID parameter values. In conventional methods, a fixed empirical equations are applied to select parameter values of PID. In GMO, there is a wide search space to select the optimal parameter values of PID based on an objective function. The objective function that the GMO seeks to minimize is the Integral of Absolute Error (IAE). GMO is chosen for its effectiveness in balancing exploration and exploitation of the search space, as well as its robustness and scalability. GMO is tested in the context of optimizing PID parameters for an engineering application: DC motor regulations. The results demonstrated GMO's superiority over comparable algorithms.**

***Keywords*—Metaheuristics; PID controller; GMO; DC motor**

I. Introduction

In manufacturing industries, the PID controller is favored for its effectiveness, resilience, and durability. This controller features standard control parameters, including system stability, settling time, and the deviation between desired and actual responses [1]. Given the shared use of processes in factories, tuning these parameters becomes a crucial task. Proper configuration enables the achievement of efficient transient performance, minimizing settling time, steady-state error, maximum deviation, and rise time as much as possible. The PID controller relies on three essential parameters: proportional gain (K_p), integral gain (K_i), and derivative gain (K_d).

The PID controller finds applications in regulating a variety of industrial processes, including pressure, temperature, flow rate, feed rate, weight, speed, and position [1]. Tuning the PID controller's parameters falls into three categories: analytical methods, rule-based methods, and numerical methods [2]. The Ziegler-Nichols (ZN) method, a classic approach for adjusting PID controller parameters, is the most commonly used and falls into the analytical category [3]. However, it's important to note that ZN does not provide optimal performance.

Stochastic optimization techniques, like heuristic algorithms, are well-suited for tuning PID parameters [4] [5] . These methods treat the problem as a "black box," adjusting the parameters and monitoring fitness to reach the optimal value. A meta-heuristic algorithm, which relies on random motion to expedite the exploration of a problem's search space, aims to find a satisfactory solution within a reasonable timeframe [6].

In this work, Geometric Mean Optimizer (GMO) [7] is used to identify the most suitable PID parameter values. The objective function employed to enhance process performance is the Minimum Integral of Absolute Error (IAE) [8].

The main objective of this work is to enhance IAE for estimating the parameters of PID controller. The performance of GMO is evaluated in comparison with other algorithms such as Arithmetic Optimization Algorithm (AOA) [9], Sine-Cosine Optimization Algorithm (SCA) [10], Particle Swarm Optimization algorithm (PSO) [11], [12] and Genetic Algorithm (GA) [13], [14]. GMO is chosen for estimating the parameters of PID due to the following advantages such as balanced exploration and exploitation, robustness, sensitivity control, scalability, convergence and divergence.

II. Methods

A. PID Tunning

The following three terms are the foundation of PID controller [1]:

- Proportional (P) term: Its purpose is to adjust the actual response $y(t)$ in accordance with the error $e(t)$ that is present between the desired response $h(t)$ and the actual response $y(t)$ at the moment as defined in (1). The magnitude of the desired correction increases with $e(t)$ increase.
- Integral (I) term: Its purpose is to modifiy the actual response $y(t)$ according to the cumulative error $e(t)$ over time. By doing this, steady state error (SSE) - $e(t)$ after a long time - is reduced.
- Derivative (D) term: Its purpose is to modify the actual response $y(t)$ according to the error's rate of change. By doing this, overshoot - which occurs when the actual response $y(t)$ is greater than the desired response $h(t)$ - is suppressed.

The combination of the three components formulate the PID controller as defind in Eq. (2)

$$e(t) = y(t) - h(t) \tag{1}$$

$$u(t) = K_p\ e(t) + K_i \int_0^t e(t)\ dt + K_d \frac{d}{dt}\ e(t) \tag{2}$$

where $u(t)$ is the output of the PID proccess.

The following time-domain characteristics are essential metrics to keep under careful observation while optimizing IAE:

- The response's rise time (tr): is the amount of time it takes to grow from 10% to 90% of its ultimate value.
- Settling time (ts): Usually expressed as an absolute percentage of the final value, such as 2% or 5%, it is the amount of time needed for the response curve to reach and stay within a given range around the final value.
- Overshooting (Mp): This is the response curve's maximum peak value, as determined by measuring it from unity or a reference point.

IAE is calculated as the summation of disparities between the desired response h(t) and the actual response y(t) during simulation time T_{sim} [1], as defined in (3).

$$IAE = \int_0^{T_{sim}} |y(t) - h(t)| \, dt \tag{3}$$

Fig. 1 shows the entire relationship between the closed loop PID controller and the calculation of its parameters using the GMO based on IAE. At first the parameters are initialized. Then IAE is calculated to decide which best PID parameters should be elected to minimize the IAE and improve the overall response of the system then the parameters are fed to PID function $G_{PID}(s)$ as defined as in Eq. (4).

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d \ s \tag{4}$$

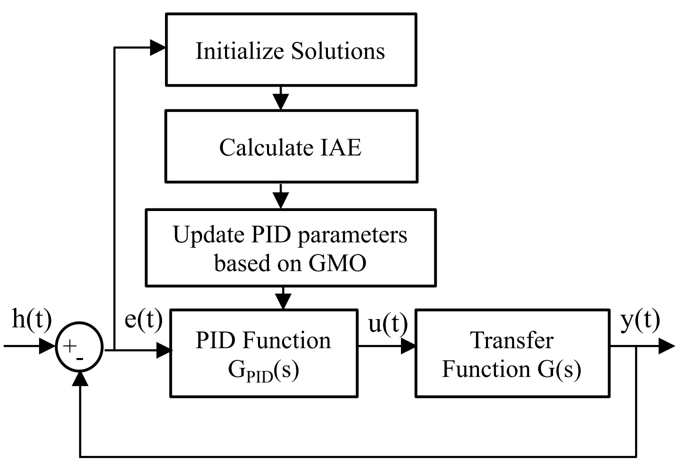

Fig. 1. Calculation of PID parameters based on GMO.

B. GMO Algorithm

GMO is a relatively new metaheuristic optimization algorithm. It is been used to optimize some problems such as [15]–[18]. GMO makes use of the special mathematical characteristics of the geometric mean. This operator allows one to assess search agents' exploration phase and exploitation at the same time. The weight of an agent in GMO is determined by taking the geometric mean of its opposites' scaled objective values (OVs). This means that an agent is appropriately regarded to direct the other agents' search process toward solving an optimization problem by considering the geometric mean of those OVs [7].The flowchart is shown in Fig. 2 and the steps in this strategy are as follows:

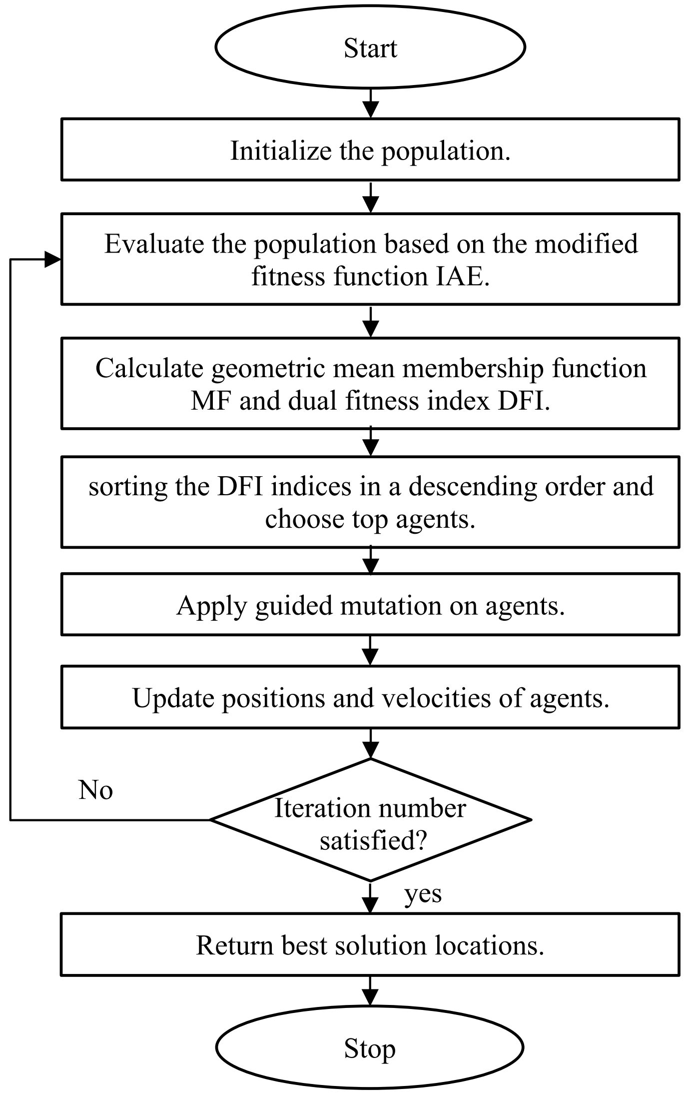

Fig. 2. Flowchart of GMO algorithm.

1) Generate the position and velocity of each searching agent randomly as defined in Eq. (5), Eq. (6).

$$x_i^0 = U\left(x_{min}, x_{max}\right) \tag{5}$$

$$v_i^0 = U(v_{min}, v_{max}) \tag{6}$$

where, $x_{min}, v_{min}, x_{max}, v_{max}$are the lower and upper bounds.

2) Determine each search agent's personal best position by computing their fitness function results as defined in Eq. (7).

$$IAE = \begin{cases} 1000, & \text{if unstable.} \\ \int_0^{T_{sim}} |y(t) - h(t)| \, dt, & \text{otherwise.} \end{cases} \tag{7}$$

a small ,however effective, modification is applied to the fitness function IAE for this problem. A penalty 1000 is applied if the resulting closed loop system is unstable. This ensures that the resulting search space is always stable.

3) Determine geometric mean of the chosen agent related to best agents fuzzy membership function (MF) and dual fitness index (DFI) as defined in Eq. (8).

$$MF_j^t = \frac{1}{1 + exp\left(-\frac{4}{\sigma^t \sqrt{e}}\right)\ \left(z_{best,j}^t - \mu^t\right)} \quad (8)$$

knowing that j loops over all agents starting from 1 to N where N is the total number of agents, $z_{best,j}^t$ is the personal best objective value of the corresponding agent, and μ^t, σ^t are the mean and standard deviation (STD) of all best-so-far agents.

4) Determine the DFI as the geometric mean of all best agents MF except that the corresponding agent as defined in Eq. (9).

$$DFI_i^t = \prod_{j=1,\ j \neq i}^{N} MF_j^t \quad (9)$$

5) Choose the first top agents (Nbest) by sorting the DFI indices in a descending order.

6) Determine the positions of the unique global guide agent calculated for the agent i at the iteration t as defined in (10).

$$Y_i^t = \frac{\sum_{j \in N_{best}, j \neq i} DFI_j^t * X_j^{best}}{\sum_{j \in N_{best}} DFI_j^t +\ \varepsilon} \quad (10)$$

where X_j^{best} is the personal best position at iteration j, and ε is either 0 or a small positive number.

7) Impose guided mutation on agents to make positions of agents more stochastic as defined in Eq. (11).

$$Y_{i,mut}^t = Y_i^t + w\ randn\ (Std_{max}^t - Std^t) \quad (11)$$

$$w = 1 - \frac{t}{T_{max}} \quad (12)$$

where Std^t is the STD calculated for the personal best-so-far agents at the tth iteration, randn is a random vector from normal distribution, and w is the mutation step as defined in Eq. (12), t is the number of the current iteration, and T_{max} is number of iterations.

8) Finally, update the positions and velocities of agents as defined in Eq. (13) to Eq. (15).

$$V_i^{t+1} = w\ V_i^t +\ \varphi\ (Y_{i,mut}^t - X_i^t) \quad (13)$$

$$\varphi = 1 + (2\ rand - 1)\ \ w \quad (14)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (15)$$

where V_i^t is the velocity vector on ith agent and tth iteration, V_i^{t+1} is the velocity at $(t+1)$th iteration, $Y_{i,mut}^t$ is global guide position for the agent i, X_i^t is a position of the i th agent's,and φ is a scaling parameter, and rand is a random number within (0,1).

III. The Experimental Results and Discussion

Experimental trials are conducted on a DC motor system in order to regulate the speed of it. It is a common subject in numerous related studies [19]–[24].

The experimental findings are compared with related results from AOA, SCA, PSO, and GA algorithms. The fitness function IAE is used to evaluate solutions. A regulated process's step response may be described by the following time-domain characteristics [1]: rising time, settling time, and overshoot.

Table I provides the parameter values for the DC motor utilized as a case study [19]. In the table, Ra denotes the armature resistance, La represents the inductance of the armature winding, J signifies the equivalent moment of inertia of the motor and load referred to the motor shaft, D stands for the equivalent friction coefficient of the motor and load referred to the motor shaft, K indicates the motor torque constant, and Kb represents the back EMF constant.

TABLE I. Parameters of DC Motor

Parameter	Value
Ra	0.4 Ω
La	2.7 H
J	0.0004 kg. m2
D	0.0022 N.m.sec / rad
K	15 e-03 kg. m / A
Kb	0.05 V.s

The transfer function that describes the open-loop speed control system of a DC motor, as expressed in (16). It completes the transfer function in Fig. 1 so that the input of the transfer function is u(t) and the output of it is y(t).

$$G\left(s\right) = \ \frac{15}{1.08\ s^2 + 6.1\ s + 1.63} \quad (16)$$

The speed regulation of an electrical DC motor [20] is managed through a PID controller, with heuristic algorithms employed to determine the most effective parameters for achieving optimal performance. The parameter configurations for PSO, SCA, GA, and GMO can be found in Table II. These settings are determined through experimental estimation to yield the most favorable outcomes.

Fig. 3 shows the open loop response of the DC motor. It has a rise time of 7.8251 sec., settling time of 14.1030 sec., and no overshoot. It also has a peak of velocity of 9.1960 m/sec. After running the PID controller the transient response of the system will improve and the new desired response h(t) can be set to be 9.1960 m/sec.

Table III displays the optimal parameter values for the PID controller that are determined for the purpose of enhancing the speed regulation of the DC motor. These values are achieved through the application of the GMO algorithm and are compared to results obtained from other related algorithms. The GMO algorithm is designed to optimize a single objective, specifically IAE, aiming to find the PID controller parameters that yield the lowest IAE. In addition to IAE, other performance criteria such as settling time, rise time, and overshoot

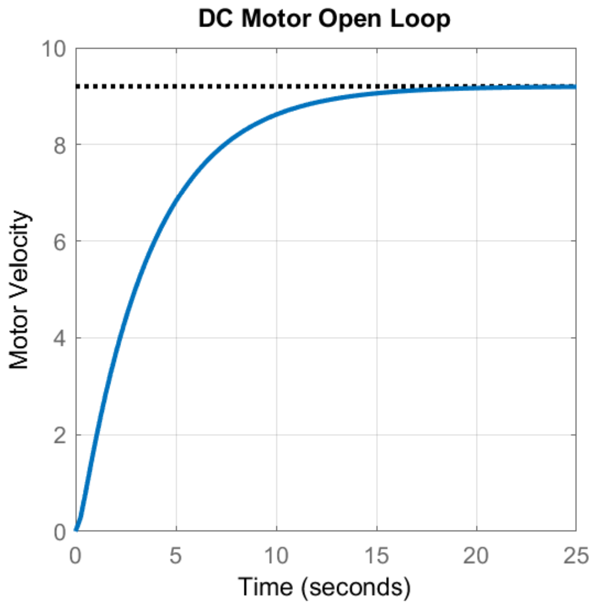

Fig. 3. DC motor open-loop behavior over time, measured in seconds.

TABLE II. THE CONFIGURATION OF PARAMETER SETTINGS FOR DIFFERENT ALGORITHMS APPLIED TO A DC MOTOR

	Parameter	Value
All Algorithms	The population (N)	100
	Iteration Number (T)	50
	Independent run number	20
	Upper bound of (Kp, Ki and Kd)	20
	Simulation Time (Tsim)	5 Sec.
GMO	ε	0
AOA	μ	0.5
	α	4.5
	ε	2
SCA	a	3
	r3	0.5
PSO	C1	0.5
	C2	0.5
	w	0.1

are evaluated based on the estimated parameters for GMO and the other algorithms within the comparative analysis.

For determining the best IAE, GMO outperforms other methods, as Table III and Fig. 4 demonstrate. GA is the closest rival to GMO in terms of IAE. GMO succeeds in decreasing GA's IAE demonstrating how GMO has better exploration and exploitation through the use of DFI, the geometric mean, and guided mutation.

GMO offers no overshoot for overshoot measurement. Other algorithms also succeed to have no overshoot. When overshoot happens, it suggests that the system's responses in certain situations are unfavorable. As previously indicated, the single objective function is chosen to minimize IAE rather than minimizing the settling time. However, GMO does succeed in

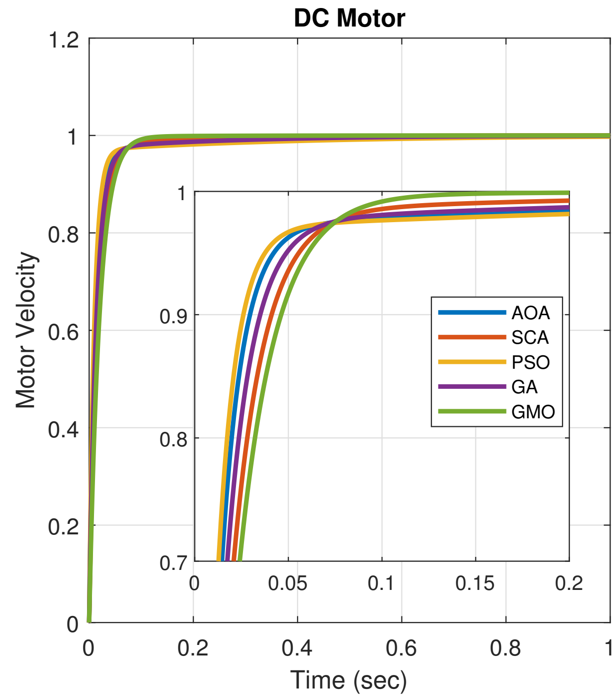

Fig. 4. DC motor behavior over time under the influence of a PID controller, measured in seconds.

having the least settling time which is great for the system stability.

According to Table III, GMO produces the longest rising time and PSO produces the lowest. However, from stability point of view if decreasing rising time affects the settling time or overshoot, it is favorable to make a compromise then. That's assured by Fig. 4, that GMO is the first to reach the target velocity.

The bode diagrams for controlling a DC motor using a PID controller, whose parameters are determined by GMO and the opponent algorithms, are displayed in Fig. 5 GMO has the most bandwidth. This ensures that GMO has a shorter rise time than other algorithms, as seen in Fig. 4 and Table III.

Furthermore, as seen in Fig. 5, the magnitude margin of PSO, for instance, has the highest gain compared to that of GMO, suggesting that PSO reacts more aggressively than GMO and may lead to overshooting (which does not happen here). However, GMO has the most robust response as it has least magnitude and phase margin .

It is evident from Fig. 6, which is concerned with Box Plot diagrams, that the GMO outperforms other optimization techniques when it comes to various parameters. Additionally, it demonstrates that GMO in the systems' component expansion and complexity possesses outstanding performance and convergence ability. As indicated in Table III and Fig. 6, GMO produces favorable results in every experiment carried out in terms of mean, median, and STD. It has the general least IAE, Median, Average, and STD compared to all other algorithms.

The most notable and useful benefits of GMO are its capacity to assess fitness and diversity simultaneously using

TABLE III. Comparison of GMO and Different Algorithms in Terms of Step Response and IAE Criteria, Rise Time, Settling Time, and Overshoot Percentage

	Kp	Ki	Kd	IAE	Rise time(sec)	Settling time	%Overshoot	Best IAE	Mean IAE	Worst IAE	STD IAE
AOA	19.9888	5.33505	6.13004	0.021144	0.02826	0.12994	0	0.021144	0.022407	0.023667	0.000701
SCA	19.9062	5.65814	4.22508	0.021106	0.03924	0.08215	0	0.021106	0.02187	0.022780	0.000566
PSO	19.5839	5.18908	7.00110	0.021833	**0.02489**	0.16404	0	0.021833	0.022668	0.023589	0.000559
GA	20	5.33333	5.09804	0.020840	0.03348	0.09311	0	0.020840	0.021615	0.023447	0.000703
GMO	19.9914	5.33635	3.61397	**0.020378**	0.04405	**0.07966**	0	**0.020378**	**0.021304**	**0.021890**	**0.000437**

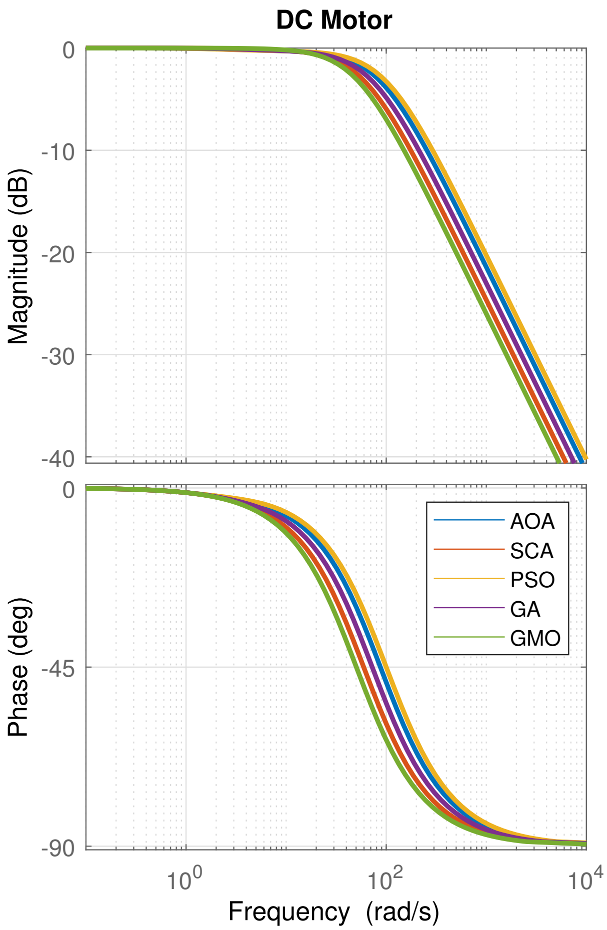

Fig. 5. Bode diagrams for a DC motor under the influence of a PID controller.

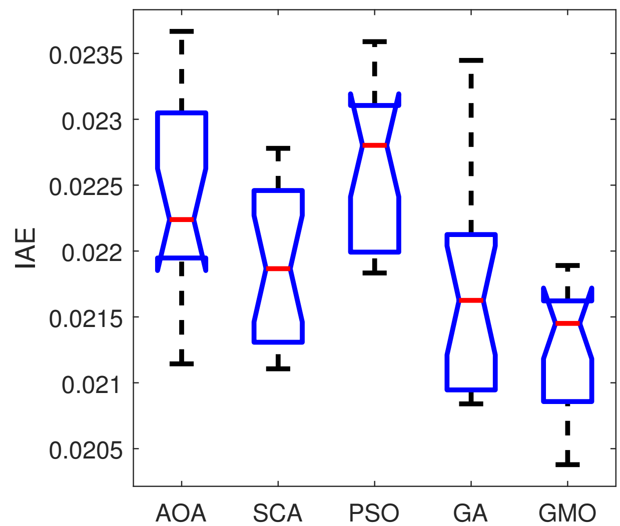

Fig. 6. Box plots of the obtained results from GMO and other optimization algorithms.

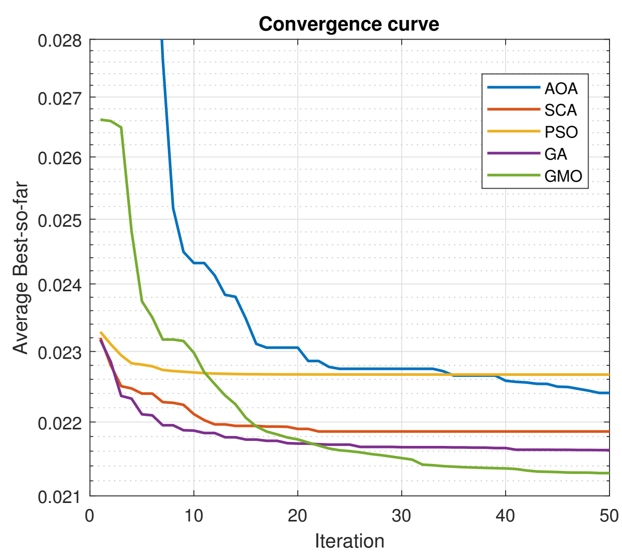

Fig. 7. Convergence curves of GMO other optimization algorithms.

the DFI, computational efficiency, not having a parameter to adjust, and assignment of multiple unique guides for each solution in order to prevent the algorithm from sticking into a local minimum.

As shown in Fig. 7 the convergence curve for GMO gradually decreases until it finds the global minimum for the fitness function. GMO has the second highest average IAE at the beginning, AOA has the first, which explains the exploration capability of GMO algorithm as compared to all other algorithms except AOA. GMO also has the lowest average IAE at the end which means that GMO has the highest exploitation capability among all other algorithms. So, GMO has the greatest balance between exploration and exploitation which makes it the superior algorithm compared to all other algorithms.

IV. CONCLUSION

In this study, the Geometric Mean Optimizer (GMO) algorithm is utilized to estimate PID controller parameters for regulating a DC motor. The primary objective function is the Integral of Absolute Error (IAE). GMO demonstrated superior performance compared to other algorithms in terms of IAE and response characteristics, including overshoot, and settling time, for DC motor control. Furthermore, the frequency response analysis of GMO indicates that it achieves a more favorable bandwidth and gain magnitude margin compared to the other algorithms under comparison. Through the experimental investigation, GMO exhibits its superiority in efficiently estimating PID controller parameters, resulting in improved IAE values in the context of DC motor control. In future work, a multi-objective function may be used to enhance more than IAE objective such as rising time and overshoot. It is recommended to use this multi-objective function to enhance the performance of DC motor or any other closed loop sysytem such as three-tank system.

REFERENCES

[1] K. Ogata, *Modern control engineering fifth edition*, 2010.

[2] T. Mansour, *PID Control: Implementation and Tuning*. BoD–Books on Demand, 2011.

[3] J. G. Ziegler and N. B. Nichols, "Optimum settings for automatic controllers," 1993.

[4] O. Aydogdu and M. Korkmaz, "Optimal design of a variable coefficient fractional order pid controller by using heuristic optimization algorithms," *International Journal of Advanced Computer Science and Applications*, vol. 10, no. 3, 2019. [Online]. Available: http://dx.doi.org/10.14569/IJACSA.2019.0100341

[5] M. R. P. Pillai, S. P. Jadhav, and M. D. Patil, "Tuning of pid controllers using advanced genetic algorithm," in *IJACSA Special Issue on Selected Papers from International Conference & Workshop On Advance Computing*. Citeseer, 2013, pp. 1–6.

[6] E.-G. Talbi, *Metaheuristics: from design to implementation*. John Wiley & Sons, 2009.

[7] F. Rezaei, H. R. Safavi, M. Abd Elaziz, and S. Mirjalili, "Gmo: geometric mean optimizer for solving engineering problems," *Soft Computing*, vol. 27, no. 15, pp. 10 571–10 606, 2023.

[8] S. B. Joseph, E. G. Dada, A. Abidemi, D. O. Oyewola, and B. M. Khammas, "Metaheuristic algorithms for pid controller parameters tuning: Review, approaches and open problems," *Heliyon*, vol. 8, no. 5, 2022.

[9] L. Abualigah, A. Diabat, S. Mirjalili, M. Abd Elaziz, and A. H. Gandomi, "The arithmetic optimization algorithm," *Computer methods in applied mechanics and engineering*, vol. 376, p. 113609, 2021.

[10] S. Mirjalili, "Sca: a sine cosine algorithm for solving optimization problems," *Knowledge-based systems*, vol. 96, pp. 120–133, 2016.

[11] E. S. Rahayu, A. Ma'arif, and A. Çakan, "Particle swarm optimization (pso) tuning of pid control on dc motor," *International Journal of Robotics and Control Systems*, vol. 2, no. 2, pp. 435–447, 2022. [Online]. Available: https://pubs2.ascee.org/index.php/IJRCS/article/view/476

[12] J. Kennedy and R. Eberhart, "Particle swarm optimization (pso)," in *Proc. IEEE international conference on neural networks, Perth, Australia*, vol. 4, no. 1, 1995, pp. 1942–1948.

[13] S. Tiwari, A. Bhatt, A. C. Unni, J. G. Singh, and W. Ongsakul, "Control of dc motor using genetic algorithm based pid controller," in *2018 International Conference and Utility Exhibition on Green Energy for Sustainable Development (ICUE)*, 2018, pp. 1–6.

[14] J. H. Holland, "Genetic algorithms," *Scientific american*, vol. 267, no. 1, pp. 66–73, 1992.

[15] S. B. Pandya, K. Kalita, P. Jangir, R. K. Ghadai, and L. Abualigah, "Multi-objective geometric mean optimizer (mogmo): A novel metaphor-free population-based math-inspired multi-objective algorithm," *International Journal of Computational Intelligence Systems*, vol. 17, no. 1, pp. 1–29, 2024.

[16] S. Kamel, M. Khasanov, F. Jurado, A. Kurbanov, H. M. Zawbaa, and M. A. Alathbah, "Simultaneously distributed generation allocation and network reconfiguration in distribution network considering different loading levels," *IEEE Access*, 2023.

[17] M. Khasanov, S. Kamel, F. Jurado, A. Kurbanov, and U. Jalilov, "Optimal sizing and sitting of distributed generation in distribution network considering power generation uncertainty," in *E3S Web of Conferences*, vol. 434. EDP Sciences, 2023, p. 01016.

[18] A. E. Khalil, M. Alham, T. A. Boghdady, and D. K. Ibrahim, "Novel single loop load frequency controller for isolated microgrid via geometric mean optimization," in *2023 24th International Middle East Power System Conference (MEPCON)*. IEEE, 2023, pp. 1–8.

[19] J. Agarwal, G. Parmar, R. Gupta, and A. Sikander, "Analysis of grey wolf optimizer based fractional order pid controller in speed control of dc motor," *Microsystem Technologies*, vol. 24, pp. 4997–5006, 2018.

[20] S. Ekinci, B. Hekimoğlu, A. Demirören, and E. Eker, "Speed control of dc motor using improved sine cosine algorithm based pid controller," in *2019 3rd International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT)*. IEEE, 2019, pp. 1–7.

[21] S. Ekinci, B. Hekimoğlu, and D. Izci, "Opposition based henry gas solubility optimization as a novel algorithm for pid control of dc motor," *Engineering Science and Technology, an International Journal*, vol. 24, no. 2, pp. 331–342, 2021.

[22] M. Khalilpour, N. Razmjooy, H. Hosseini, and P. Moallem, "Optimal control of dc motor using invasive weed optimization (iwo) algorithm," in *Majlesi Conference on Electrical Engineering, Majlesi New Town, Isfahan, Iran*, 2011.

[23] N. Razmjooy, Z. Vahedi, V. V. Estrela, R. Padilha, and A. C. B. Monteiro, "Speed control of a dc motor using pid controller based on improved whale optimization algorithm," in *Metaheuristics and Optimization in Computer and Electrical Engineering*. Springer, 2020, pp. 153–167.

[24] T. C. Bora, L. d. S. Coelho, and L. Lebensztajn, "Bat-inspired optimization approach for the brushless dc wheel motor problem," *IEEE Transactions on magnetics*, vol. 48, no. 2, pp. 947–950, 2012.