Modified SFWBP Framework for Vocal Teaching Quality Evaluation Based on the MEREC Technique

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***Abstract*—With the gradual improvement of people's living standards, their pursuit of art is also constantly increasing. Vocal music is not only an important course in the training process of music majors but also an important factor in improving personal qualities and expanding one's own abilities. In the process of vocal teaching, there are many factors that affect the quality of teaching, among which the teacher-student factor is one of the important influencing factors. How to enhance the role of teacher-student factors in improving the quality of vocal teaching has become one of the main directions for the development of vocal teaching. The vocal teaching quality evaluation could be looked as the multiple-attribute group decision-making (MAGDM). Spherical fuzzy sets (SFSs) could portray the uncertainty and fuzziness during the vocal teaching quality evaluation more effectively and deeply. In this paper, based on bidirectional projection, we shall propose the spherical fuzzy bidirectional projection (SFBP) technique and spherical fuzzy weighted bidirectional projection (SFWBP) technique. First of all, the definition of SFSs is introduced. Furthermore, the SFBP technique and SFWBP technique with SFSs are proposed based on the bidirectional projection. Based on the developed SFWBP technique, the MAGDM technique is organized and all computing steps are organized. Finally, a numerical example for vocal teaching quality evaluation is employed to verify the SFWBP technique and some comparisons are employed to verify advantages of SFWBP technique with SFSs.**

Keywords*—Multiple-attribute group decision-making; Spherical fuzzy sets (SFSs); MEREC; bidirectional projection technique; vocal teaching quality evaluation

I. INTRODUCTION

At present, vocal teaching in universities excessively pursues singing techniques while neglecting musical emotions, which goes against the original intention of vocal teaching. In addition, the weak cultural cultivation of students makes it difficult to implement the training plan for vocal education. More importantly, there are frequent problems with the establishment of vocal courses in many universities [1, 2]. Single and one-sided vocal theory subjects emerge one after another, but practical courses are scarce, resulting in vocal courses becoming monotonous singing and imitation classes, hindering students from innovating in music. At present, many universities lack distinctive vocal teaching methods. From singing methods to vocal textbooks, copy the models of other music schools [3, 4]. Vocal teachers did not fully consider the actual situation of students, causing them to imitate a large amount of foreign muic, resulting in students becoming tired of vocal learning, making it difficult to cultivate their musical emotions, and even more difficult to improve their singing skills and emotional expression abilities [5, 6]. At present, most vocal majors in Chinese universities still use traditional teaching methods, which can no longer fully meet the needs of today's society [7, 8]. Teaching methods should be adjusted according to the actual situation of students, stimulate their interest in learning vocal music, improve the quality of vocal teaching, and cultivate qualified vocal talents [9, 10]. As an art discipline, vocal music requires a high level of aesthetic ability from students. However, with the implementation of the policy of expanding enrollment in universities, the number of admissions has gradually increased. Therefore, the examination standards for students' aesthetic ability have also gradually decreased, which will significantly constrain the overall development quality and trend of vocal music majors [11, 12]. The source of students majoring in vocal music in universities is very extensive, and due to the low requirements for cultural standards in the assessment process, the overall quality of students varies. Therefore, it is difficult to effectively control the quality and effectiveness of vocal teaching. Students have a low ability to appreciate and evaluate vocal works, which is also a reflection of their insufficient aesthetic ability[13]. In current vocal teaching, teachers often follow traditional teaching methods, so the lack of diversity in teaching modes has become an important reason for the role of teacher-student factors in controlling the quality of vocal teaching. The traditional teaching mode is relatively rigid, and at the same time, it lacks a more detailed examination of the quality of students' vocal learning, making it difficult to achieve accurate grasp of their learning status. Adopting a relatively fixed teaching mode in the long-term teaching process not only makes it difficult to bring students a fresh experience, but also reduces their interest in vocal learning, affects their initiative in learning vocal music, lacks effective interaction with teachers, and is difficult to promote the improvement of teacher-student relationships [14, 15]. At present, vocal teaching is still difficult to achieve a separate classroom teaching format or a small class teaching format. The conflict between a large number of students and a small number of teachers has become one of the important reasons for the lack of effective communication between teachers and students. In vocal teaching, in order to achieve the expected teaching progress, attention to individual needs of students is often overlooked, leading to a lack of effective communication between teachers and students, which is not conducive to teachers better understanding the learning and ideological status of students, and cannot effectively solve and correct errors made by students in the vocal learning process, thereby affecting the improvement of vocal teaching quality [16-18]. In summary, the development of the times has

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promoted the continuous improvement of vocal teaching level, and has also brought new requirements to vocal teaching activities. Universities need to combine the current problems and shortcomings in vocal teaching, continuously optimize the teaching team, attach importance to the combination of theory and practice teaching, give full play to the long-term effects of teaching, and optimize teaching content, so that vocal teaching can better adapt to the development of the times, keep up with the pace of the times, meet the demand of society for vocal talents, and promote the continuous improvement of vocal teaching quality.

Multiple-attribute decision-making (MADM) is an important branch of modern management, which solves the core problem of aggregating effective decision-making information and using scientific and reasonable decision-making techniques to rank and select alternative techniques in the context of considering multiple attributes or indicators of solutions [19-24]. Whether it is a country or an individual, they are constantly faced with various choices, and MAGDM has been integrated into all aspects of life, such as location selection, investment evaluation, supplier selection, etc. [25-28]. Due to the complexity of the economy and society, traditional precise numbers have made it difficult to solve real-world decision-making problems [29-33]. Famous scholar Zadeh [34] proposed fuzzy sets (FSs) that use membership degree to represent uncertain information. Because fuzzy sets only use membership degree as single-dimensional information to represent fuzzy information, it is difficult to solve decision-making problems with hesitation [35, 36]. The intuitionistic FSs (IFSs) [37] were employed in MAGDM. Spherical fuzzy sets (SFSs) [38] could comprehensively portray the fuzziness of decision things [39-45]. The vocal teaching quality evaluation could be looked at as the MAGDM. SFSs [46] could portray the uncertainty during the vocal teaching quality evaluation. The projection technique is a useful technique to cope with MAGDM [47, 48]. The projection technique used the length and angle to obtain the projection value on the positive and negative ideal solutions, to rank the decision alternatives. However, if the projection value is equal, the alternatives can't be distinguished from each other. Thus, Ye [49] organized the bidirectional projection (BP) technique to overcome this shortcoming. Unfortunately, few useful works are been found for the BP technique under SFSs in the current MAGDM. Therefore, it is valuable to manage the novel BP technique under SFSs. In this paper, based on bidirectional projection, the spherical fuzzy bidirectional projection (SFBP) technique and spherical fuzzy weighted bidirectional projection (SFWBP) technique are organized. Based on the developed SFWBP technique, the MAGDM technique is organized and all computing steps are organized. Finally, a numerical example for vocal teaching quality evaluation is employed to verify the SFWBP technique and some comparisons are employed to verify the advantages of SFWBP technique with SFSs. Therefore, the research motivation and aim of this study are organized: (1) the method based on the removal effects of criteria (MEREC) technique [50] is utilized to construct the weight values; (2) the BP techniques are extended to SFSs; (3) some BP techniques with SFSs is organized for managing the MAGDM; (4) numerical example for is organized to demonstrate the SFWBP techniques with SFSs and several comparative techniques are employed to verify the SFWBP techniques.

The study framework of this study is outlined. The SFSs are organized in Section II. Some BP techniques with SFSs are built in Section III. Some BP techniques with SFSs are built for MAGDM in Section IV. A numerical example for vocal teaching quality evaluation is employed to show the SFWBP techniques and some comparative techniques are employed in Section V. The conclusion is organized in Section VI.

II. Preliminaries

Gundogdu and Kahraman [46] organized the SFSs.

Definition 1 [46]. The SFSs BB in Θ is organized:

$$QQ = \left\{ \left(\theta, QT(\theta), QI(\theta), QF(\theta) \right) \middle| \theta \in \Theta \right\} \quad (1)$$

where the $QT(\theta), QI(\theta), QF(\theta)$ is truth-membership, indeterminacy-membership and falsity-membership, $QT(\theta), QI(\theta), QF(\theta) \in [0,1]$ and satisfies $0 \le QT^2(\theta) + QI^2(\theta) + QF^2(\theta) \le 1$.

The spherical fuzzy number (SFN) is organized as $QQ = (QT, QI, QF)$, $QT, QI, QF \in [0,1]$, and $0 \le QT^2 + QI^2 + QF^2 \le 1$.

The score value (SV) and accuracy value (AV) of SFNs are conducted to rank two SFNs.

Definition 2 [46]. Let $QA = (QT_A, QI_A, QF_A)$ be the SFN, the SV is organized:

$$SV(QA) = (QT_A - QI_A)^2 - (QF_A - QI_A)^2, \quad SV(QA) \in [0,1]. \quad (2)$$

Clearly, the greater $SV(QA)$, the larger $QA = (QT_A, QI_A, QF_A)$.

Definition 3[46]. Let $QA = (QT_A, QI_A, QF_A)$ be the SFN, an AV is organized:

$$AV(QA) = (QT_A)^2 + (QT_A)^2 + (QF_A)^2, \quad AV(QA) \in [0,1] \quad (3)$$

Two SFNs are compared as follows [46]:

Definition 4 [46]. Let $QA = (QT_A, QI_A, QF_A)$ and $QB = (QT_B, QI_B, QF_B)$ be SFNs, let $SV\left(QA\right)=(QT_A-QI_A)^2-(QF_A-QI_A)^2$ and $SV\left(QB\right)=(QT_B-QI_B)^2-(QF_B-QI_B)^2$, and let $AV\left(QA\right)=\left(QT_A\right)^2+\left(QT_A\right)^2+\left(QF_A\right)^2$ and $AV\left(QB\right)=\left(QT_B\right)^2+\left(QT_B\right)^2+\left(QF_B\right)^2$, then if $SV\left(QA\right)<SV\left(QB\right)$, then $QA<QB$; if $SV\left(QA\right)=SV\left(QB\right)$, then (1) if $AV\left(QA\right)=AV\left(QB\right)$, then $QA=QB$; (2) if $AV\left(QA\right)<AV\left(QB\right)$, then $QA<QB$.

Definition 5 [46, 51]. Let $QA=\left(QT_A,QI_A,QF_A\right)$ and $QB=\left(QT_B,QI_B,QF_B\right)$ be two SFNs, the basic operations are organized:

$$(1)\ QA\oplus QB=\left(QT_A+QT_B-QT_AQT_B,QI_AQI_B,QF_AQF_B\right);$$
$$(2)\ QA\otimes QB=\left(QT_AQT_B,QI_A+QI_B-QI_AQI_B,QF_A+QF_B-QF_AQF_B\right);$$
$$(3)\ \lambda QA=\left(1-\left(1-QT_A\right)^{\lambda},\left(QI_A\right)^{\lambda},\left(QF_A\right)^{\lambda}\right),\lambda>0;$$
$$(4)\ \left(QA\right)^{\lambda}=\left(\left(QT_A\right)^{\lambda},\left(QI_A\right)^{\lambda},1-\left(1-QF_A\right)^{\lambda}\right),\lambda>0.$$

Definition 6 [52, 53]. Let $QA=\left(QT_A,QI_A,QF_A\right)$ and $QB=\left(QT_B,QI_B,QF_B\right)$, then the Hamming distance between $QA=\left(QT_A,QI_A,QF_A\right)$ and $QB=\left(QT_B,QI_B,QF_B\right)$ is organized:

$$HD\left(QA,QB\right)=\frac{1}{2}\begin{pmatrix}\left|QT_A^2-QT_B^2\right|\\+\left|QI_A^2-QI_B^2\right|+\left|QF_A^2-QF_B^2\right|\end{pmatrix}\quad(4)$$

The SFNWA and SFNWG techniques are organized.

Definition 7 [46]. Let $QA_j=\left(QT_j,QI_j,QF_j\right)$ $\left(j=1,2,3,\cdots,n\right)$ be SFNs, the SFNWA is organized:

$$\text{SFNWA}_{q\omega}\left(QA_1,QA_2,\cdots,QA_n\right)=\overset{n}{\underset{j=1}{\oplus}}\left(q\omega_jQA_j\right)=\begin{pmatrix}\sqrt{1-\prod_{j=1}^{n}(1-QT_j^2)^{q\omega_j}},\\\sqrt{\prod_{i=1}^{n}(1-QT_j^2)^{q\omega_j}-\prod_{i=1}^{n}(1-QT_j^2-QI_j^2)^{q\omega_j}},\\\prod_{j=1}^{n}(QF_j)^{q\omega_j},\end{pmatrix}\quad(5)$$

where $q\omega=\left(q\omega_1,q\omega_2,...,q\omega_n\right)^T$ be the weight of $QA_j\left(j=1,2,3,...,n\right)$ and $q\omega_j>0,\sum_{j=1}^{n}q\omega_j=1.$

Definition 8 [46]. Let $QA_j=\left(QT_j,QI_j,QF_j\right)$ $\left(j=1,2,3,\cdots,n\right)$ be SFNs, the SFNWG is organized:

$$\text{SFNWG}_{q\omega}\left(QA_1,QA_2,\cdots,QA_n\right)=\overset{n}{\underset{j=1}{\otimes}}\left(QA_j\right)^{q\omega_j}=\begin{pmatrix}\prod_{j=1}^{n}(QT_j)^{q\omega_j},\\\sqrt{\prod_{i=1}^{n}(1-QF_j^2)^{q\omega_j}-\prod_{i=1}^{n}(1-QF_j^2-QI_j^2)^{q\omega_j}},\\\sqrt{1-\prod_{j=1}^{n}(1-QF_j^2)^{q\omega_j}}\end{pmatrix}\quad(6)$$

where $q\omega=\left(q\omega_1,q\omega_2,...,q\omega_n\right)^T$ be the weight of $QA_j\left(j=1,2,3,...,n\right)$ and $q\omega_j>0,\sum_{j=1}^{n}q\omega_j=1.$

III. Some BP Techniques Under SFSs

Then, some BP techniques are organized under SFSs based on the traditional BP technique [49]. Then, the spherical fuzzy BP (SFBP) technique and spherical fuzzy weighted BP (SFWBP) technique are built.

Definition 9. Let $QA_i=\left(QT_{ij},QI_{ij},QF_{ij}\right)$ $\left(i=1,2,\cdots,m,j=1,2,\cdots,n\right)$ be a set of SFNs and $QA=\left(QT_j,QI_j,QF_j\right)\left(j=1,2,\cdots,n\right)$ be the ideal solution under SFSs, then the SFBP technique is conducted:

$$SFBP(QA_i,QA)=\frac{1}{1+\left|\frac{QA_i\cdot QA}{\|QA_i\|}-\frac{QA_i\cdot QA}{\|QA\|}\right|} \quad (7)$$
$$=\frac{\|QA_i\|\|QA\|}{\|QA_i\|\|QA\|+\left\|QA_i\|-\|QA\right\|(QA_i\cdot QA)}$$

where

$$\|QA\|=\sqrt{\sum_{j=1}^{n}\left(\left(\left(QT_j\right)^2\right)^2+\left(2\left(QI_j\right)^2\right)^2+\left(\left(QF_j\right)^2\right)^2\right)} \quad (8)$$

$$\|QA_i\|=\sqrt{\sum_{j=1}^{n}\left(\left(\left(QT_{ij}\right)^2\right)^2+\left(2\left(QI_{ij}\right)^2\right)^2+\left(\left(QF_{ij}\right)^2\right)^2\right)} \quad (9)$$

$$(QA_i\cdot QA)=\sum_{j=1}^{n}\left(\begin{array}{l}\left(QT_{ij}\right)^2\times\left(QT_j\right)^2\\ +4\left(QI_{ij}\right)^2\times\left(QI_j\right)^2+\left(QF_{ij}\right)^2\times\left(QF_j\right)^2\end{array}\right) \quad (10)$$

Obviously, the greater $SFBP(QA_i,QA)$, the better alternative QA_i is.

Consider the weight values of SFSs, the SFWBP technique is organized.

Definition 10. Let $QA_i=(QT_{ij},QI_{ij},QF_{ij})$ $(i=1,2,3,\cdots,m,\ j=1,2,3,\cdots,n)$ be a set of SFNs and $QA=(QT_j,QI_j,QF_j)(j=1,2,\cdots,n)$ be the ideal solution under SFSs with weight $qw=(qw_1,qw_2,\ldots,qw_n)$, then the SFWBP technique is organized:

$$SFWBP(QA_i,QA)=\frac{1}{1+\left|\frac{(QA_i\cdot QA)_{qw}}{\|QA_i\|_{qw}}-\frac{(QA_i\cdot QA)_{qw}}{\|QA\|_{qw}}\right|} \quad (11)$$
$$=\frac{\|QA_i\|_{qw}\|QA\|_{qw}}{\|QA_i\|_{qw}\|QA\|_{qw}+\left\|QA\|_{qw}-\|QA_i\|_{qw}\right|(QA_i\cdot QA)_{qw}}$$

$$\|QA_i\|_{qw}=\sqrt{\sum_{j=1}^{n}\left(\begin{array}{l}\left(\left(qw_j\times QT_{ij}\right)^2\right)^2\\ +\left(2\left(qw_j\times QI_{ij}\right)^2\right)^2+\left(\left(qw_j\times QF_{ij}\right)^2\right)^2\end{array}\right)} \quad (12)$$

$$\|QA\|_{qw}=\sqrt{\sum_{j=1}^{n}\left(\begin{array}{l}\left(\left(qw_j\times QT_j\right)^2\right)^2\\ +\left(2\left(qw_j\times QI_j\right)^2\right)^2+\left(\left(qw_j\times QF_j\right)^2\right)^2\end{array}\right)} \quad (13)$$

$$(QA_i\cdot QA)_{qw}=\sum_{j=1}^{n}\left(\begin{array}{l}\left(qw_j\times QT_{ij}\right)^2\times\left(qw_j\times QT_j\right)^2\\ +4\left(qw_j\times QI_{ij}\right)^2\times\left(qw_j\times QI_j\right)^2\\ +\left(qw_j\times QF_{ij}\right)^2\times\left(qw_j\times QF_j\right)^2\end{array}\right) \quad (14)$$

where $qw_j=(qw_1,qw_2,\ldots,qw_n)$ satisfies $0\le qw_j\le 1,\sum_{j=1}^{n}qw_j=1.$ Obviously, the greater $SFWBP(QA_i,QA)$, which indicates the better QA_i.

IV. SFWBP Technique for Managing the MAGDM under SFSs

The SFWBP technique is organized for managing the MAGDM. Let $QA=\{QA_1,QA_2,\ldots,QA_m\}$ be a collection of alternatives. Let $QG=\{QG_1,QG_2,\ldots,QG_n\}$ be attributes, $q\omega=\{q\omega_1,q\omega_2,\ldots,q\omega_n\}$ be weight values of QG_j, where $q\omega_j\in[0,1],\sum_{j=1}^{n}q\omega_j=1$. Assume $QD=\{QD_1,QD_2,\ldots,QD_l\}$ be DMs along with weight values of $qw=\{qw_1,qw_2,\ldots,qw_l\}$, where $qw_k\in[0,1],\sum_{k=1}^{l}qw_k=1$. And $QQ^{(k)}=\left(QQ_{ij}^k\right)_{m\times n}=\left(QT_{ij}^k,QI_{ij}^k,QF_{ij}^k\right)_{m\times n}$ is the overall SFN matrix, $QQ_{ij}^k=\left(QT_{ij}^k,QI_{ij}^k,QF_{ij}^k\right)$ means the SFNs of alternative QA_i regarding the attribute QG_j through QD_k. Subsequently, the designed calculating steps are organized (see Fig. 1).

Step 1. Organize the SFN matrix $QQ^{(k)}=\left(QQ_{ij}^k\right)_{m\times n}=\left(QT_{ij}^k,QI_{ij}^k,QF_{ij}^k\right)_{m\times n}$ and construct the overall values $QQ=\left(QQ_{ij}\right)_{m\times n}$ through SFNWA technique.

$$QQ^{(k)}=\left[QQ_{ij}^k\right]_{m\times n}=\begin{bmatrix}QQ_{11}^k & QQ_{12}^k & \ldots & QQ_{1n}^k\\ QQ_{21}^k & QQ_{22}^k & \ldots & QQ_{2n}^k\\ \vdots & \vdots & \vdots & \vdots\\ QQ_{m1}^k & QQ_{m2}^k & \ldots & QQ_{mn}^k\end{bmatrix} \quad (15)$$

$$QQ=\left[QQ_{ij}\right]_{m\times n}=\begin{bmatrix}QQ_{11} & QQ_{12} & \ldots & QQ_{1n}\\ QQ_{21} & QQ_{22} & \ldots & QQ_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ QQ_{m1} & QQ_{m2} & \ldots & QQ_{mn}\end{bmatrix} \quad (16)$$

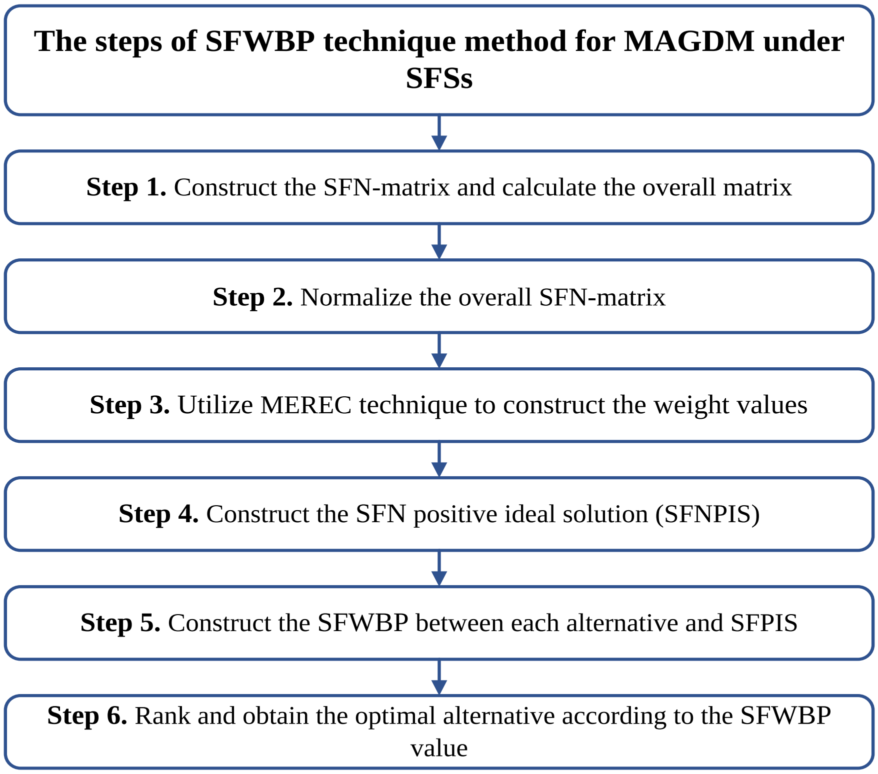

Fig. 1. The framework of SFWBP technique for MAGDM under SFSs

$$QQ_{ij}=\left(QT_{ij},QI_{ij},QF_{ij}\right)=\left(\begin{array}{c}\sqrt{1-\prod_{k=1}^{l}\left(1-\left(QT_{ij}^{k}\right)^{2}\right)^{qw_k}},\\ \sqrt{\prod_{i=1}^{p}\left(1-\left(QT_{ij}^{k}\right)^{2}\right)^{qw_k}-\prod_{i=1}^{p}\left(1-\left(QT_{ij}^{k}\right)^{2}-\left(QI_{ij}^{k}\right)^{2}\right)^{qw_k}},\\ \prod_{j=1}^{n}\left(QF_{ij}\right)^{qw_k},\end{array}\right) \quad (17)$$

where $QQ_{ij}^{k}=\left(QT_{ij}^{k},QI_{ij}^{k},QF_{ij}^{k}\right)$ means the SFNs of QA_i regarding QG_j through QD_k.

Step 2. Normalize the information $QQ=\left(QQ_{ij}\right)_{m\times n}$ to $NQ=\left[NQ_{ij}\right]_{m\times n}$.

$$NQ_{ij}=\left(NQT_{ij},NQI_{ij},NQF_{ij}\right)=\begin{cases}\left(QT_{ij},QI_{ij},QF_{ij}\right), & QG_j \text{ is the benefit attribute}\\ \left(QF_{ij},QI_{ij},QT_{ij}\right), & QG_j \text{ is the cost attribute}\end{cases} \quad (18)$$

Step 3. Employ MEREC model [50] to manage the weight values.

1) Organize the normalized SFN score and accuracy matrix:

$$QSFN\left(NQ_{ij}\right)=\frac{\min_i\left(SV\left(NQ_{ij}\right)+AV\left(NQ_{ij}\right)+1\right)}{\left(SV\left(NQ_{ij}\right)+AV\left(NQ_{ij}\right)+1\right)} \quad (19)$$

2) Organize the overall performance $QSFN\left(NQ_i\right)$.

$$QSFN\left(NQ_i\right)=\ln\left(1+\left(\frac{1}{n}\sum_{j=1}^{n}\left|\ln QSFN\left(NQ_{ij}\right)\right|\right)\right). \quad (20)$$

3) Organize the performance of QA_i by removing each attribute.

$$NSFN\left(NQ_{i(j)}\right)=\ln\left(1+\left(\frac{1}{n}\sum_{k=1,k\neq j}^{n}\left|\ln NSFN\left(NQ_{ik}\right)\right|\right)\right), \quad (21)$$

4) Organize the sum of absolute deviations:

$$SFNSD_j=\sum_{i=1}^{m}\left|\begin{array}{l}NSFN\left(NQ_{i(j)}\right)\\ -NSFN\left(NQ_i\right)\end{array}\right|,\quad k=1,2,\cdots n, \quad (22)$$

5) Organize the weight values:

$$q\omega_j = \frac{SFNSD_j}{\sum_{j=1}^{n} SFNSD_j}. \quad (23)$$

Step 4. Organize the SFN positive ideal solution (SFNPIS):

$$SFNPIS_j = \left(NQT_j^+, NQI_j^+, NQF_j^+\right) \quad (24)$$

$$SV\left(SFNNIS_j\right) = \max_i SV\left(NQT_{ij}, NQI_{ij}, NQF_{ij}\right) \quad (25)$$

Step 5. Organize the SFWBP between QA_i and SFNPIS.

$$\begin{aligned} &SFWBP\left(QA_i, SFNPIS\right) \\ &= \frac{1}{1 + \left| \frac{\left(QA_i \cdot SFNPIS\right)_{qw}}{\left\|QA_i\right\|_{qw}} - \frac{\left(QA_i \cdot SFNPIS\right)_{qw}}{\left\|SFNPIS\right\|_{qw}} \right|} \\ &= \frac{\left\|QA_i\right\|_{qw} \left\|SFNPIS\right\|_{qw}}{\left(\begin{array}{l} \left\|QA_i\right\|_{qw} \left\|SFNPIS\right\|_{qw} \\ + \left| \left\|QA_i\right\|_{qw} - \left\|SFNPIS\right\|_{qw} \right| \left(QA_i \cdot SFNPIS\right)_{qw} \end{array} \right)} \end{aligned} \quad (26)$$

$$\left\|QA_i\right\|_{qw} = \sqrt{\sum_{j=1}^{n} \left(\begin{array}{l} \left(\left(qw_j \times QT_{ij}\right)^2\right)^2 \\ + \left(2\left(qw_j \times QI_{ij}\right)^2\right)^2 + \left(\left(qw_j \times QF_{ij}\right)^2\right)^2 \end{array} \right)} \quad (27)$$

$$\left\|SFNPIS\right\|_{qw} = \sqrt{\sum_{j=1}^{n} \left(\begin{array}{l} \left(\left(qw_j \times QT_j^+\right)^2\right)^2 \\ + \left(2\left(qw_j \times QI_j^+\right)^2\right)^2 + \left(\left(qw_j \times QF_j^+\right)^2\right)^2 \end{array} \right)} \quad (28)$$

$$\left(QA_i \cdot QA\right)_{qw} = \sum_{j=1}^{n} \left(\begin{array}{l} \left(qw_j \times QT_{ij}\right)^2 \times \left(qw_j \times QT_j^+\right)^2 \\ +4\left(qw_j \times QI_{ij}\right)^2 \times \left(qw_j \times QI_j^+\right)^2 \\ +\left(qw_j \times QF_{ij}\right)^2 \times \left(qw_j \times QF_j^+\right)^2 \end{array} \right) \quad (29)$$

Step 6. Rank and select the optimal alternative with largest $SFWBP\left(QA_i, SFNPIS\right)$.

V. Numerical Example and Comparative Analysis

A. Numerical Example

In vocal teaching, the teacher-student factor is one of the important aspects of teaching quality control. Through good cooperation between teachers and students, the goal of effectively improving the quality of vocal teaching can be achieved. The teacher-student factor has become one of the important factors affecting the quality of vocal teaching. In the traditional vocal teaching process, it is mainly through the guidance of teachers to help students master the correct vocal techniques. However, this process also limits the initiative of students to explore independently, so the selection of sound specifications is relatively limited. The sound specifications that students can come into contact with are mainly actively taught by teachers, and there is a clear relationship between the knowledge level and reserved sound specifications possessed by teachers. However, there are obvious differences between different musical components, and the sound specifications are usually formed in a gradual development process. In different performance venues, it is also necessary to coordinate with the atmosphere and scenery of the stage to better convey the performance theme. Therefore, by enhancing the participation of teacher-student factors in the vocal teaching process, improving the scope of student expansion, and avoiding students being too limited by the teacher's ability level in the vocal learning process, the goal of better expanding sound specifications can be achieved. Different eras have different aesthetic standards, so in vocal learning, it is necessary to combine the mainstream of music at the current stage of development, and constantly expand new educational models and directions. Therefore, good cooperation between teachers and students can better provide richer choices for the content of vocal teaching. Teachers are limited by traditional thinking patterns in the teaching process, and their acceptance of new perspectives and skills is limited. At the same time, due to some teachers' overly conservative teaching views in the teaching process, it is difficult to promote the optimization of vocal teaching. At the same time, it will also create significant obstacles to the development process of diversified teaching quality evaluation. By improving the interaction between teachers and students in the process of vocal teaching, we can introduce more youthful and diverse performance styles and development directions into the vocal teaching process, transform the teacher-student relationship in traditional courses, and make important contributions to improving the quality of vocal teaching. Teaching philosophy is one of the important reasons that affect the further optimization and development of vocal teaching quality. In traditional classrooms, a teacher-centered teaching philosophy can have a significant negative impact on the expansion of student thinking. Under the traditional teaching philosophy, although teaching activities can also solve some problems in the vocal teaching process, their negative effects are also very obvious, which will create a clear gap between teachers and students, making it difficult to better integrate fashion factors into the process of improving the quality of vocal teaching. Therefore, by enhancing the role of teacher-student factors in the evaluation process of vocal teaching quality, enhancing the position of students as the main body of the classroom, promoting the diversified development of vocal teaching, transforming the rigid and boring atmosphere in traditional vocal teaching classrooms, and integrating more young, fresh, and cutting-edge elements of vocal development into the teaching process. The vocal teaching quality evaluation may be looked as MAGDM. In this section, numerical example for vocal teaching quality evaluation is provided through SFWBP technique. In order to select the optimal music college, some research department sincerely invite three experts $QD = (QD_1, QD_2, QD_3)$ to evaluate the five music college $QA_i\left(i = 1,2,3,4,5\right)$ through four attributes: 1) QG_1 is teaching techniques of music college. 2) QG_2 is teaching contents of music college. 3) QG_3 is teaching satisfaction degree of music college. 4) QG_4 is teaching achievements of music college. Let $qw = (1/3, 1/3, 1/3)^T$ be experts' weight values. The decision evaluation from $QD = (QD_1, QD_2, QD_3)$ are obtained by employing the linguistic scales (see Table I [46]) which are organized in Tables II to IV. Then, the SFWBP technique is employed to help the research department select the optimal music college.

TABLE I. LINGUISTIC SCALES AND SFNS[46]

Linguistic Terms	SFNs
Exceedingly Terrible-QET	(0.90,0.10,0.10)
Very Terrible-QVT	(0.70,0.30,0.30)
Terrible-QT	(0.60,0.40,0.40)
Medium-QM	(0.50,0.50,0.50)
Well-QW	(0.40,0.40,0.60)
Very Well-QVW	(0.30,0.30,0.70)
Exceedingly Well-QEW	(0.10,0.10,0.90)

Step 1. Organize the SFN-matrix $QQ^{(k)} = \left(QQ_{ij}^{k}\right)_{5\times4}\left(k = 1,2,3\right)$ as in Tables II to IV and construct the overall matrix based on SFNWA technique. The results are organized in Table V.

TABLE II. EVALUATION INFORMATION BY QD_1

	QG_1	**QG_2**	**QG_3**	**QG_4**
QA_1	QVW	QW	QM	QT
QA_2	QVW	VT	QVT	QM
QA_3	QM	QVW	QT	QW
QA_4	QVT	QM	QVW	QW
QA_5	QT	QW	QM	QVW

TABLE III. EVALUATION INFORMATION BY QD_2

	QG_1	**QG_2**	**QG_3**	**QG_4**
QA_1	QW	QM	QT	QM
QA_2	QM	QM	QW	QVT
QA_3	QT	QM	QM	QW
QA_4	QT	QM	QW	QVW
QA_5	QM	QW	QVT	QVT

TABLE IV. EVALUATION INFORMATION BY QD_3

	QG_1	**QG_2**	**QG_3**	**QG_4**
QA_1	QVT	QVW	QM	QVT
QA_2	QT	QVW	QW	QVW
QA_3	QVT	QM	QW	QVW
QA_4	QVW	QT	QM	QW
QA_5	QW	QVW	QVT	QM

TABLE V. THE OVERALL SFNS INFORMATION

	QG_1	QG_2
QA_1	(0.3162, 0.2434, 0.4923)	(0.4536, 0.2152, 0.3743)
QA_2	(0.2135, 0.2654, 0.5726)	(0.5354, 0.3718, 0.3458)
QA_3	(0.3796, 0.1463, 0.4452)	(0.4394, 0.1723, 0.4546)
QA_4	(0.4736, 0.1687, 0.3856)	(0.4657, 0.1812, 0.3583)
QA_5	(0.4324, 0.1895, 0.2263)	(0.4495, 0.2432, 0.2569)
	UG_3	**UG_4**
QA_1	(0.5572, 0.3934, 0.3675)	(0.2247, 0.2765, 0.5928)
QA_2	(0.4823, 0.3564, 0.4186)	(0.3715, 0.1154, 0.4128)
QA_3	(0.2576, 0.1233, 0.5534)	(0.3354, 0.2983, 0.5537)
QA_4	(0.6426, 0.3398, 0.3872)	(0.5347, 0.3768, 0.3645)
QA_5	(0.3156, 0.4376, 0.5126)	(0.3724, 0.4214, 0.5654)

Step 2. Normalize the SFN-matrix $QQ = \left[QQ_{ij} \right]_{5\times 4}$ to $NQ = \left[NQ_{ij} \right]_{5\times 4}$ (see Table VI).

TABLE VI. THE NORMALIZED SFNS

	QG_1	QG_2
QA_1	(0.3162, 0.2434, 0.4923)	(0.4536, 0.2152, 0.3743)
QA_2	(0.2135, 0.2654, 0.5726)	(0.5354, 0.3718, 0.3458)
QA_3	(0.3796, 0.1463, 0.4452)	(0.4394, 0.1723, 0.4546)
QA_4	(0.4736, 0.1687, 0.3856)	(0.4657, 0.1812, 0.3583)
QA_5	(0.4324, 0.1895, 0.2263)	(0.4495, 0.2432, 0.2569)
	UG_3	**UG_4**
QA_1	(0.5572, 0.3934, 0.3675)	(0.2247, 0.2765, 0.5928)
QA_2	(0.4823, 0.3564, 0.4186)	(0.3715, 0.1154, 0.4128)
QA_3	(0.2576, 0.1233, 0.5534)	(0.3354, 0.2983, 0.5537)
QA_4	(0.6426, 0.3398, 0.3872)	(0.5347, 0.3768, 0.3645)
QA_5	(0.3156, 0.4376, 0.5126)	(0.3724, 0.4214, 0.5654)

Step 3. Put forward the attribute weights through MEREC (Table VII).

TABLE VII. THE ATTRIBUTES WEIGHT

	UG_1	UG_2	UG_3	UG_4
weight	0.1684	0.3261	0.2927	0.2228

Step 4. Put forward the SFNPIS (Table VIII).

TABLE VIII. THE SFNPIS

	SFNPIS
QG_1	(0.4736, 0.1687, 0.3856)
QG_2	(0.5354, 0.3718, 0.3458)
QG_3	(0.6426, 0.3398, 0.3872)
QG_4	(0.5347, 0.3768, 0.3645)

Step 5. Put forward the SFWBP between QA_i and SFNPIS (Table IX).

TABLE IX. THE SFWBP VALUES

	SFWBP Values
$SFWBP(QA_1, SFNPIS)$	0.3211
$SFWBP(QA_2, SFNPIS)$	0.6534
$SFWBP(QA_3, SFNPIS)$	0.4340
$SFWBP(QA_4, SFNPIS)$	0.5745
$SFWBP(QA_5, SFNPIS)$	0.9149

Step 6. Relying on SFWBP values, the order of these music college is: $QA_5 > QA_2 > QA_4 > QA_3 > QA_1$ and QA_5 is the best music college.

B. Comparative Analysis

In this section, the SFWBP technique is fully compared with SFNWA technique [46], SFNWG technique [46], spherical fuzzy Hamacher power weighted average (SFHPWA) technique [54], spherical fuzzy Hamacher power weighted geometric (SFHPWG) technique [54], SF-CPT-CoCoSo technique [44], SFN-VIKOR technique [55], SFN-TODIM technique [43], SF-SWARA-CODAS technique [56], SF-CRITIC-EDAS technique [57] and SFN-ITARA-ELECTRE III technique [58]. Then, the results of different techniques are addressed in Table X.

TABLE X. ORDER FOR DIFFERENT TECHNIQUES

Techniques	Order
SFNWA technique [46]	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$
SFNWG technique[46]	$QA_5 > QA_2 > QA_3 > QA_4 > QA_1$
SF-CPT-CoCoSo technique[44]	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$
SFN-VIKOR technique [55]	$QA_5 > QA_2 > QA_3 > QA_4 > QA_1$
SFN-TODIM technique [43]	$QA_5 > QA_2 > QA_3 > QA_4 > QA_1$
SF-SWARA-CODAS technique [56]	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$
SF-CRITIC-EDAS technique [57]	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$
SFN-ITARA-ELECTRE III technique [58]	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$
The proposed SFWBP technique	$QA_5 > QA_2 > QA_4 > QA_3 > QA_1$

From Table X, it is obvious that the order of these techniques is slightly different, however, these techniques have the same optimal music college QA_5 and the same worst music college VA_1. This verifies that the proposed technique is effective. Furthermore, the main advantages of the SFWBP technique not only emphasize the distance values and angle values of the decision alternatives but also emphasize bidirectional projection information. So, the SFWBP technique is more scientific.

VI. CONCLUSION

Vocal music is not only a subject course, but also a way of expressing emotions. Therefore, in the process of vocal teaching, in addition to focusing on professional skills training, emotional education should also be emphasized. The teacher-student factor is one of the main ways to cultivate students' emotions. In the process of teaching professional content, teachers should not only help students master professional vocal knowledge, but also use their language and some auxiliary teaching tools to create a better learning atmosphere, allowing students to have a more intuitive vocal experience environment, and laying an important foundation for students to better understand the emotions conveyed by vocal music. In summary, vocal music is an art that is not only a process of knowledge learning but also a process of emotional integration. Therefore, emphasis is placed on the important influence of teacher-student factors on the quality of vocal teaching. By building a better teacher-student relationship in the teaching process and utilizing the professional competence of teachers, students can better understand the emotions conveyed by vocal works, thereby stimulating their enthusiasm for active learning and making important contributions to the further improvement and optimization of vocal teaching quality. The vocal teaching quality evaluation could be looked as the MAGDM. In this study, based on bidirectional projection, the SFBP technique and SFWBP technique are addressed. First of all, the definition of SFSs is introduced. Furthermore, SFBP technique and SFWBP technique with SFSs are addressed in line with bidirectional projection. Based on developed SFWBP technique, the MAGDM technique is organized and all computing steps are organized. Finally, a numerical example for vocal teaching quality evaluation is employed to verify the SFWBP technique and several comparisons are done to verify the SFWBP technique with SFSs. Therefore, the main contributions of this work are organized: (1) the MEREC technique is employed to construct the weight values; (2) the BP techniques are extended to SFSs; (3) some BP techniques with SFSs are organized for managing the MAGDM; (4) the numerical example for vocal teaching quality evaluation is organized to conduct the SFWBP techniques with SFSs and several comparative techniques are employed to verify the SFWBP techniques. In the future, we shall continue to investigate the vocal teaching quality evaluation with SFSs and extend our organized projection techniques based on the consensus-reaching processes and regret theory [59-63].

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