A Comparative Study Between Linear and Affine Multi-Model in Predictive Control of a Nonlinear Dynamic System

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***Abstract*—Model Predictive Control (MPC) is the most successful control strategy that coped in many areas. However, the success of an MPC scheme lies in the accuracy of the adopted prediction model. This paper treats the problem of MPC when there is a need to a larger domain of set-point values and best tracking performances. It presents a novel modeling structure for representing a nonlinear dynamic system based on its static nonlinear characteristic. Then, the Multiple Affine Model (MAM) structure is compared to Multiple Linear Models (MLM) in a Linear MPC (LMPC) scheme. It is noted that the MAM structure offers more precision for modeling and the much smaller number of models. Therefore, it guarantees the best tracking performances in terms of stability, speed and accuracy.**

Keywords—Affine models; linear models; static characteristic; linear model predictive control; prediction horizon; tracking performances

I. INTRODUCTION

The essential objectives in Model Predictive Control (MPC) are good tracking performances and less energy consumption. MPC coped in many industrial areas such as chemical [1], thermal [2] and robotic [3], [4]. However, there were limitations in tracking performances due to the imprecision of adopted prediction models. Indeed, the success of the MPC scheme lies in the accuracy of the employed model in computing the optimal control solution. Generally, the linear model can't represent most physical systems only in few operating points [5]. Therefore, due to limitations in LMPC based on linear model was developed the Nonlinear MPC (NMPC) NMPC strategy [6], [7]. However, this strategy encountered difficulties in optimization problems and computing time requirements [8], [9], [12]. For that reason, the researchers turned to employ the concept of MLM in order to achieve good tracking performances with larger domain of set-point values [11], [12].

MLM concept was previously exploited in adaptive and robust control schemes [13], [14], [15]. Indeed, the model of the treated system changes for the same control domain. In the last decade, the MLM concept became employed in the control of nonlinear systems [11], [12], [16], [17]. Thus, the whole control domain is divided in several sub-domains. For each sub-domain of control, it is considered a different linear model. By analyzing the static characteristic of the nonlinear system, this concept requires the consideration of a great number of models in order to cover all the static characteristics of the system. Indeed, each model approaches the system only for one operating point. There were determined a variety of switching laws allowing the determination of the adequate model for each desired set-point. Nevertheless, the task of identification all models followed by a heavy burden, in computing the switching law leading to the appropriate model, is very hard and consumes a large time. Moreover, in most results, there exist oscillations proving insufficient tracking performances. These drawbacks can't suit fast dynamic systems such as robots. This is due to the insufficient accuracy of the employed models. In effect, all straight lines of the linear models must go through the origin in the input output curve.

In study [18], it was proposed the concept of MAM to achieve more accurate models. At first, it was considered a Hammerstein model with static nonlinearity. Then, it was considered, in [19], a nonlinear dynamic system based on affine modeling in study [20]. The obtained affine models were few and their characteristics are close to the static system curve. There was carried out a comparison between two MPC strategies. LMPC-based MAM and NMPC based on the original system model. The LMPC scheme achieved much better tracking performances in addition to the low calculation time consumed.

This paper presents a comparative study between MLM and MAM in a LMPC scheme of a nonlinear dynamic system. The comparison concerns static characteristics, transient and steady regimes and tracking performances.

The paper is organized as follows:

In the second section, it is presented the problem of modeling where they are explained the two modeling concepts. In the third one, it is treated the LMPC strategy where are highlighted all its steps. Simulation results are illustrated in the fourth section and the paper ends with a conclusion.

II. MODELING PROBLEM

It is considered the nonlinear delayed SISO system, treated in study [19], described by the Eq. (1). It is noted as a hard modeling system.

$$y(k) = u^3(k-2) + u^4(k-3) + \frac{0.8+y^2(k-1)}{1+y^2(k-1)+y^3(k-2)} \quad (1)$$

where, $y(k)$ and $u(k)$ are respectively the output and input of the system. The main goal, in this work, is to control the system to track a reference trajectory. We will be interested, in this study, not in the whole input-output curve of the system but only to the part which allows the less energy consumption. Fig. 1 represents the static nonlinear characteristic of the system (1) with black continuous curve. Indeed, two optimal control solutions correspond to each desired set-point value whereas the best of them is closest to the origin. Moreover, by considering the right part of the input-output curve, the MPC algorithm allows the least variations of the control signal. Therefore, the modeling based on the MLM concept is applied only on the right part.

A. Modeling Based on MLM

The dynamic of the linear models is described by an ARIMAX (Auto-Regressive Integrated Moving Average with exogenous inputs). The model is given by the following equation:

$$A(z^{-1})\hat{y}_L(k) = z^{-d}B(z^{-1})u(k) + \frac{e(k)}{\Delta(z^{-1})} \qquad (2)$$

$\hat{y}_L(k)$, $u(k)$, $e(k)$ and d are, respectively, the linear model output and input, the white noise sequence combining the measurement and modeling errors and d is the system delay. The polynomials $A(z^{-1})$, $B(z^{-1})$ and $\Delta(z^{-1})$ are given by:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{nA} z^{-nA} \qquad (3)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nB} z^{-nB} \qquad (4)$$

$$\Delta(z^{-1}) = 1 - z^{-1} \qquad (5)$$

The employment of MLM concept for modeling nonlinear systems has led to sufficient tracking performances for chemical processes [11], [12]. The modeling procedure used in [12] proposed to adopt a great number of linear models whereas each one gives local precision for fixed operating point. Therefore, the estmation of models is a hard task. Indeed, the number of models enlarges with the number of operating points. This number must be great in order to cover the whole chacteristic of the system. Moreover, the switching law, based on an avereged adaptation of the control, remains hard. More details will be illustrated with the simulation results.

As the considered system is delayed, $d = 2$, second order, the estimated linear models are described by (6) whereas the vector of coefficients to be estimated and its measurement vector are given by Eq. (7) and Eq. (8).

$$\hat{y}(k) = -a_1\hat{y}(k-1) - a_2\hat{y}(k-2) + b_1u(k-2) + b_2u(k-2) \qquad (6)$$

$$\theta_L = [a_1 \quad a_2 \quad b_1 \quad b_2]^T \qquad (7)$$

$$\varphi_L = [-y(k-1) \quad -y(k-2) \quad u(k-1) \quad u(k-2)]^T \qquad (8)$$

The linear models are estimated by using the Recursive Least Square (RLS) algorithm. Then, stability of MLM is verified by the Jury criterion. The static characteristics of the obtained linear models are illustrated in Fig. 1 designated by LM. Because all linear model characteristics must go through the origin, each curve of the linear models intercept that of the system (1) only for one Operating Point (OP). Besides, the closest linear model to the system is LM5. For the rest part of the system curve, there is a need to more linear models as detailed in study [17]. Fig. 1 presents, in addition, the characteristics of seven linear models.

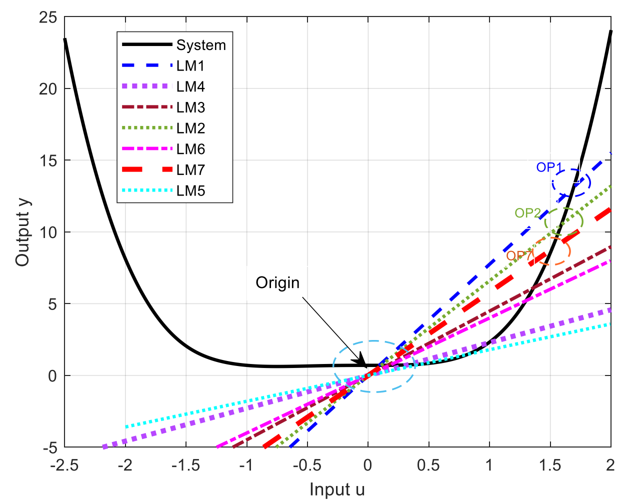

Fig. 1. Static nonlinear characteristic of the system (1) with characteristics of seven linear models.

B. Modeling Based on MAM

In order to determine the accurate model for prediction, we propose the concept of multiple affine models to represent the system (1). There are considered delayed second order models which are described by Eq. (9) where c_0 is an added real constant to be estimated. Therefore, the coefficient vector θ_A is that given by Eq. (10) and the measurement vector φ_A is given by Eq. (11) with respect to the system delay.

$$y(k) = -a_1y(k-1) - a_2y(k-2) + b_1u(k-2) + b_2u(k-3) + c_0 \qquad (9)$$

$$\theta_A = [a_1 \quad a_2 \quad b_1 \quad b_2 \quad c_0]^T \qquad (10)$$

$$\varphi_A = [-y(k-1) \quad -y(k-2) \quad u(k-2) \quad u(k-3) \quad 1]^T \qquad (11)$$

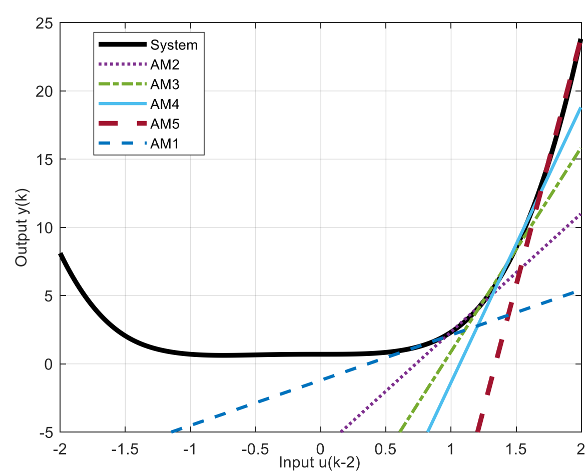

Fig. 2. Static nonlinear characteristic of the system (1) with characteristics of five affine models.

Then, the obtained affine models, corresponding to the least input energy, which is the right part, are estimated by employing the RLS algorithm. The whole control domain is divided into m sub-domains. The input vector is a random sequence in which the magnitude varies differently in each sub-domain allowing a tangency with the nonlinear model. Indeed, the AM changes with the static gain $G_i = \frac{dy(k)}{du(k-2)}$, $i = 1, \dots, m$. Then, stability of MAM is verified by the Jury criterion. The characteristics of the obtained models are traced in Fig. 2, referred by AM1.AM5, with the original nonlinear model (1).

The figure shows that all obtained affine models join the nonlinear characteristic not only for few operation points but also for considered length intervals.

III. Linear Model Predictive Control

The MPC strategy is based on the moving horizon technique. The nonlinear model predicts the output of the system over a specified prediction horizon N_p. The predicted outputs are employed to determine the control signal that minimizes the dynamic criterion given by Eq. (12) as in study [21].

$$J\left(N_p, N_u, k\right) = \sum_{j=1}^{N_p}\left(y_c(k+j) - \hat{y}(k+j)\right)^2 + \sum_{j=1}^{N_u} \lambda \Delta u(k + j - 1)^2 \qquad (12)$$

Subject to:

$$u_{min} \leq u(k) \leq u_{max}$$

$$\Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max}$$

where, $y_c(k+j)$, $\hat{y}(k+j)$, $\Delta u(k)$, u_{min}, u_{max} designate respectively the set-point, the predicted output at instant $k+j$, the control increment and lower and upper bounds of the control signal. The criterion in Eq. (12) is composed of two terms. The first one is the sum of squared prediction errors over the prediction horizon designed by N_p. The second term is formed by the sum of squared control increments over a control horizon N_u, given by Eq. (13), and weighted by the coefficient λ in order to minimize the control energy consumption.

$$\Delta u(k + j - 1) = u(k + j - 1) - u(k + j - 2) \quad (13)$$

In the LMPC strategy, the criterion (5) is quadratic. Therefore, its minimization by annulation of its derivative leads to an equation with degree of the control variable equal to one. Thus, the adaptation law of the control is analytic, and its computing time is minimum. It is detailed in the following as developed in study [21].

The dynamic of the real system is described by an ARIMAX (Auto-Regressive Integrated Moving Average models with exogenous inputs) model which is given by the following Eq. (2). More details of the LMPC algorithm are given in study [19]. With affine models, the control adaptation law remains the same as with the linear models. Indeed, the constant of the model is removed by the calculated difference between two consecutive predicted output measurements.

IV. Simulation Results

A. Results of LMPC Based on MLM

It was employed the LMPC strategy applied for modeling structures MLM and MAM. There were considered, at first, two set-point values $y_c = 1$ and $y_c = 5$. The control signal is initialized to the sequence $u(k) = 0$, for $k = 1..3$. The retained constraint is $0 \leq u(k) \leq 5$. The models are chosen close to the set-point operating values LM4 and LM6 traced in Fig. 1. The obtained results as temporal responses as depicted by Fig. 3 and Fig. 4 respectively for $N_p = 2$ and different values of N_p. Regarding the figures, it is noted that best tracking performances are achieved with $N_p = 2$ by enlarging the value of the control increment weight λ. Besides, the increasing of N_p attenuates transient overshoot for higher set-point value whereas it causes notable delay for the lower one. Moreover, with $N_p = 4, 6$, there was needed much larger values of λ in order to achieve less overshoot.

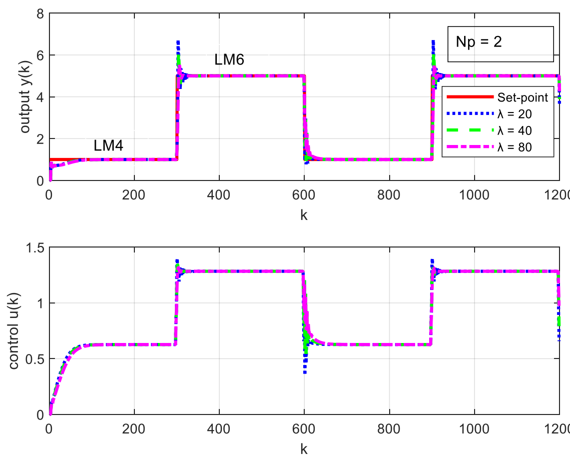

Fig. 3. Results of LMPC based on MLM for $y_c = 1$ and $y_c = 5$ with constraint $0 \leq u(k) \leq 5$ and $N_p = 2$.

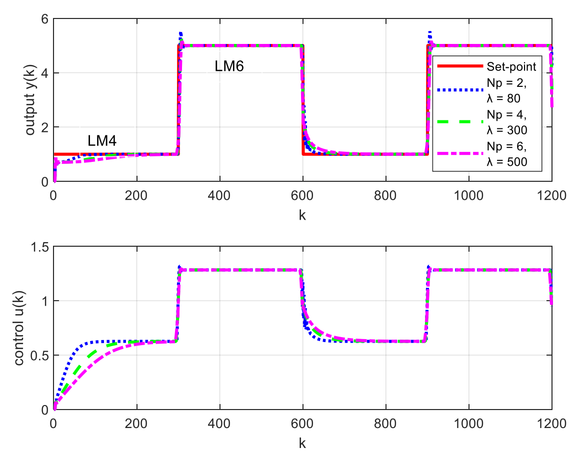

Fig. 4. Results of LMPC based on MLM for $y_c = 1$ and $y_c = 5$ with constraint $0 \leq u(k) \leq 5$ for different values of N_p.

Then, higher set-point values are considered $y_c = 5$ and $y_c = 10$ with the same variations of N_p. The obtained temporal responses are illustrated by Fig. 5 and Fig. 6. Based on figures, it is noted that with $N_p = 2$ best tracking performances can be obtained by tuning the value of λ. As for $N_p = 4, 6$, relatively higher values of the weight λ must be taken in order to reduce the high transient overshoot and oscillations. In effect, this is due to the higher variations of the system output, compared to that of the model, caused by little variations of the control input for higher set-point values.

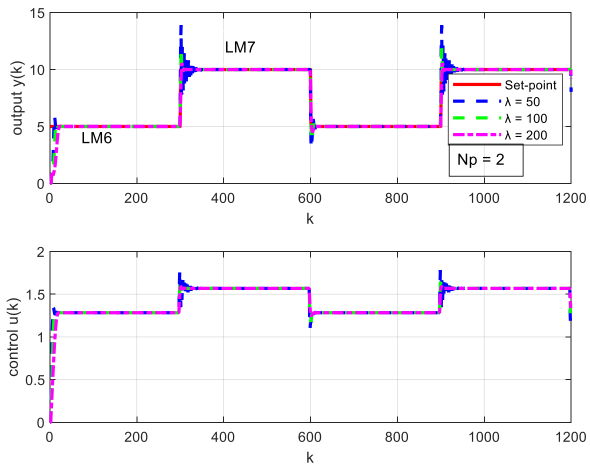

Fig. 5. Results of LMPC based on MLM for $y_c = 5$ and $y_c = 10$ with constraint $0 \leq u(k) \leq 5$ and $N_p = 2$.

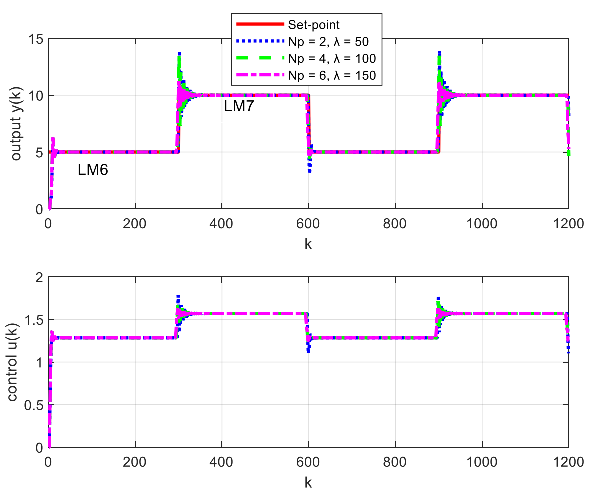

Fig. 6. Results of LMPC based on MLM for $y_c = 5$ and $y_c = 10$ with constraint $0 \leq u(k) \leq 5$ for different values of N_p.

It is well noted, in this part, that errors of modeling with MLM structures have more effects when the prediction horizon is enlarged.

B. Results of LMC Based on MAM

In this case, the affine models traced in Fig. 2 are employed. The switching law of the affine models is described by Table I where M designates the employed model.

TABLE I. AFFINE MODEL SWITCHING LAW

Interval of $y_c(k)$	***Model***
$1 \leq y_c(k) < 2$	M = AM1
$2 \leq y_c(k) < 3.5$	M = AM2
$3.5 \leq y_c(k) < 6$	M = AM3
$6 \leq y_c(k) < 11$	M = AM4
$11 \leq y_c(k) < 24$	M = AM5

The same pairs of set-point values are considered with the same prediction horizon values. The resulting temporal responses, for $y_c = 1$ and $y_c = 5$, are illustrated in Fig. 7 and Fig. 8 respectively for $N_p = 2$ and $N_p = 4$. Regarding the figures, it is well observed the superiority of the MAM according to MLM. Indeed, the transient oscillations take shorter duration. In addition, the overshoot is easily removed with slight increasing of the weight λ. This is noted for the two values of N_p whereas for $N_p = 4$, higher values of λ must be considered. As for the second pair of set-points for $y_c = 5$ and $y_c = 10$, the obtained temporal responses are given by Fig. 9 and Fig. 10. These figures show the best tracking performances achieved with the MAM structure. However, the necessary values of λ for annealing the overshoot are much higher. Indeed, this is due to the higher slope of the affine model which allows large variations of the output for little variations of the input.

With this modeling structure, the variations of the system output is closer to that of the model. Therefore, the increasing of N_p or λ improves the tracking performances without causing any delay.

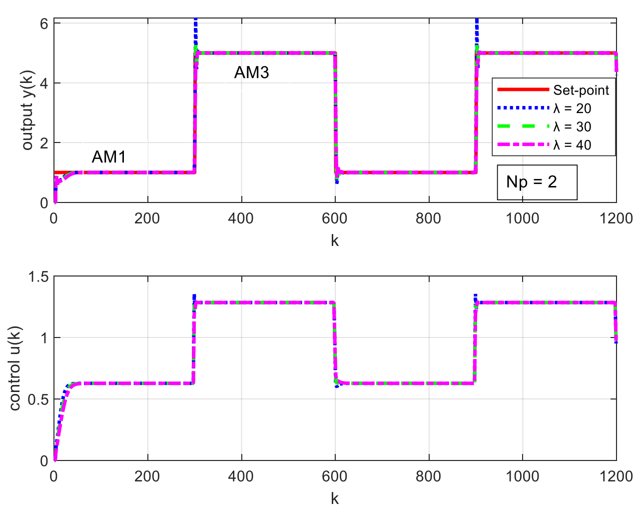

Fig. 7. Results of LMPC based on MAM for $y_c = 1$ and $y_c = 5$ with constraint $0 \leq u(k) \leq 5$ and $N_p = 2$.

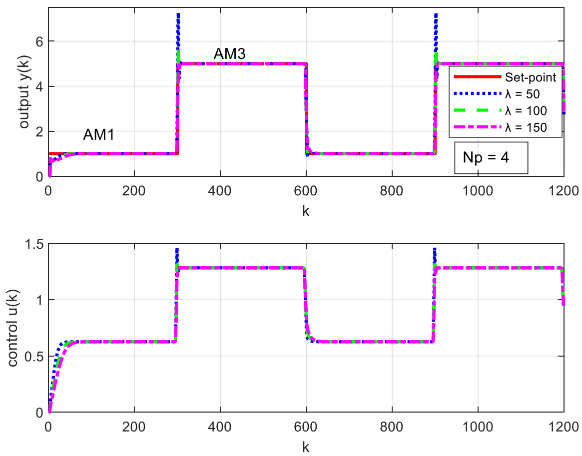

Fig. 8. Results of LMPC based on MAM for $y_c = 1$ and $y_c = 5$ with constraint $0 \leq u(k) \leq 5$ and $N_p = 4$.

C. Results of LMPC Based on MAM and MLM

In this part, results of LMPC based on both MAM and MLM are presented. In addition, higher set-point values and variations are treated $y_c = 8$ and $y_c = 15$. In addition, for the MLM structure, the considered output references don't correspond to operating points. Therefore, for each output reference, the closest linear model is opted. Thus, the linear models present, in this case, important prediction errors. The temporal responses, for different values of N_p and suitable values of the weight λ, are given by the Fig. 11 and Fig. 12 respectively for the MLM and MAM structures. Fig. 11 illustrates hard oscillations for $y_c = 15$ due to the modeling error caused by using LM7. The Fig. 12 shows the superiority of employing the MAM structure in achieving the best tracking performances in terms of stability and speed. The increasing of N_p leads to better results, in terms of least overshoot, with MAM which is due to the accuracy of the models. This result is achieved without increasing the value of λ. This is due to the accuracy of the models even for higher values of set-point. Indeed, little variations of the control have the same effect on both of outputs that of the system and that of the model. Whereas, with the MLM structure, the increasing of N_p produces hard oscillations that can be attenuated by much higher values of λ.

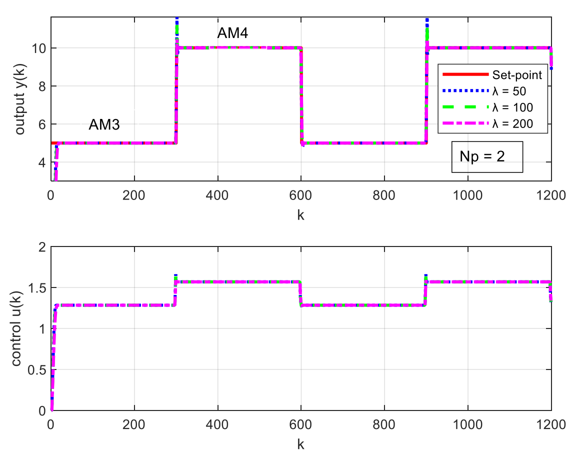

Fig. 9. Results of LMPC based on MAM for $y_c = 5$ and $y_c = 10$ with constraint $0 \leq u(k) \leq 5$ and $N_p = 2$.

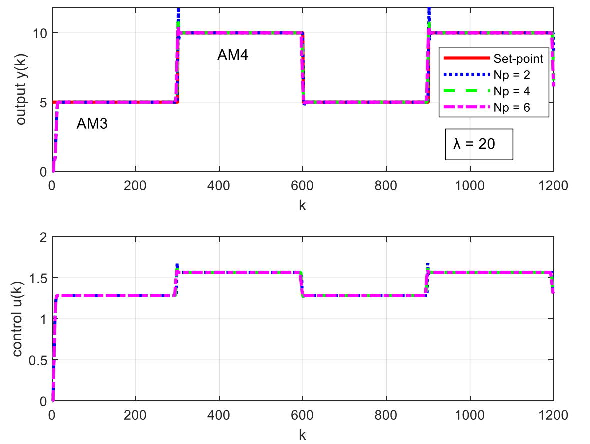

Fig. 10. Results of LMPC based on MAM for $y_c = 5$ and $y_c = 10$ with constraint $0 \leq u(k) \leq 5$ for different values of N_p.

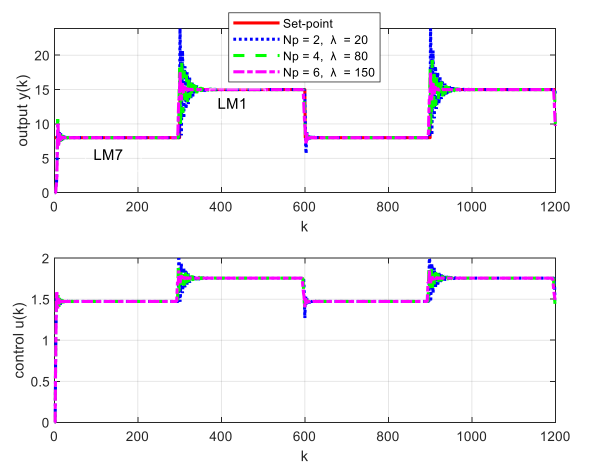

Fig. 11. Results of LMPC based on MLM for $y_c = 8$ and $y_c = 15$ with constraint $0 \leq u(k) \leq 5$.

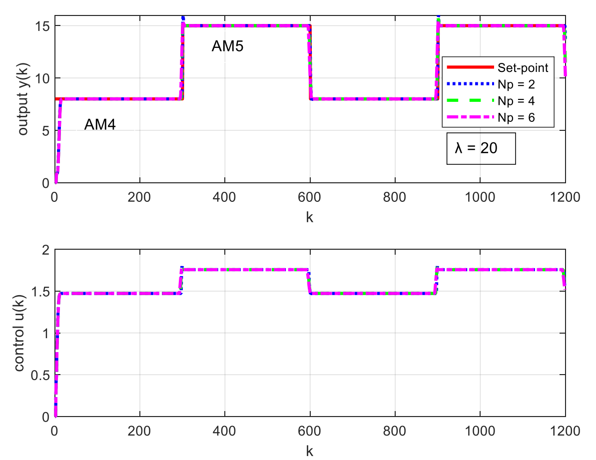

Fig. 12. Results of LMPC based on MAM for $y_c = 8$ and $y_c = 15$ with constraint $0 \leq u(k) \leq 5$ for different values of N_p.

Besides, both of structures attain the null tracking error due to the integrating term considering the prediction error in Eq. (2).

Finally, the effectiveness of the modeling structure should be better validated by employing the static characteristic curve. Thus, if the model gives a curve covering more that of the system it attains better tracking performances in the LMPC scheme.

V. Conclusion and Future Work

In this work, a novel modeling structure MAM is proposed. Then, a comparison between this latter and MLM, employing LMPC strategy has been carried out. The LMPC based on MAM structure with a simple switching law, based on the set-point intervals, has proven its superiority in achieving best tracking performances in terms of stability, speed and accuracy. Indeed, this is due to the higher precision of the modeling structure. Moreover, the switching law is simple and guarantees continuity with all set-point values. In addition, the number of models is much less.

In future works, extension to MIMO systems and real-time application will be addressed.

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