DIAUTIS III: A Fuzzy and Affective Platform for Obtaining Autism Mental Models and Learning Aids

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***Abstract*—Autism spectrum disorders (ASD) are conditions characterized by social interaction and communication difficulties, atypical patterns of activities, and unusual reactions to sensations. Characteristics of autism may be detected in early childhood, but diagnosis is often delayed. The diagnosis of autistic children typically aligns with medical and psychological recommendations, but it does not evaluate all the problems, intensity, or changes in symptoms over time. It also does not identify the affective states associated with these deficiencies, making aid less effective. The mental model of autistic children contains their deficits, tasks, and intensities, beyond diagnostics. That why we enhance DIAUTIS platform for achieve our objectives related to helping children with ASD. DIAUTIS I is a platform that aim to diagnosing autism and identifying its severity using cognitive, fuzzy, and affective computing. It presents tests, evaluates results, and presents a final model. Then, we implemented DIAUTIS II by adding KASP methodology, a new methodology of designing serious games, based on knowledge, affect, sensory, and pedagogy, this tool allows to DIAUTIS II agents to designing over 80 games, considering a child's background. In this paper, we will present a new tool of formalization of the autism mental model based on fuzzy and affective computing. DIAUTIS III is the extension of DIAUTIS II platform aim to represent a cognitive fuzzy mental model with using the metrics of category theory. So far, no mental model has been developed for autism. Our mental model, can be obtained anytime as a fuzzy cognitive map or fuzzy graph and the use of affective computing. In addition, the mathematical theory of categories represent this mental model of an autistic child from a fuzzy graph, and it allows for operations like CONS and TRA to evaluate the difference between two mental models. Fuzzy cognitive mental model can be used to develop new techniques for improving the learning and integration of autistic children into social life, which is the focus of our immediate future.**

Keywords*—*Fuzzy computing; affective computing; mental models; learning aids; autism; category theory

I. Introduction

Autism, also known as Autism Spectrum Disorder (ASD), is a condition characterized by difficulties in social skills, repetitive behaviors, speech, and nonverbal communication [1].

Autism spectrum disorder (ASD) is a neurological and developmental disorder that affects social interactions, communication, learning, and behavior. Symptoms typically appear in the first two years of life, making it a "developmental disorder." The Diagnostic and Statistical Manual of Mental Disorders (DSM-5), used by healthcare providers, identifies ASD symptoms as difficulty with communication, restricted interests, repetitive behaviors, and affecting functioning in school, work, and other areas of life [2].

Autism is a spectrum disorder with varying symptoms across genders, races, ethnicities, and economic backgrounds. It can be lifelong, but treatments can improve symptoms and daily functioning. The American Academy of Pediatrics recommends all children receive autism screening, and caregivers should consult their child's healthcare provider about screening or evaluation [2].

Mental models are an analogical representation of knowledge, involving a direct correspondence between the entities and relations in the representation structure and the entities and relations of the real network. They represent a functional form of prior conceptions in relation to a specific and momentary goal, consisting of elements and their relationships that represent the state of things. Each model is predisposed in a way consistent with its intended use.

A mental model is a cognitive simulation of how things work and are related, built over time through learning and understanding. It helps people make sense of the world, make decisions, solve problems, and predict outcomes, enhancing their decision-making abilities.

So far, according to our knowledge, no mental model of autism has been developed, although we have studied the development of other non -mental models.

Our mental model emphasizes the importance of emotional aspects in children, as affective states can trigger learning problems and improve them. It includes all developed affective relationships and uses affective computing to determine and improve these states. This elaborate model can be classified as fuzzy and affective, as it can be used to treat autistic children and improve their emotional well-being. Affective computing began with the works of Rosalind Pickard, followed by a large number of articles and collaborations of different authors [3-4].

Category theory is a mathematical branch that formalizes mathematical structure and has powerful applications in mathematics and programming languages. Concepts like categorical semantics, monads, functors, and proof assistants are deeply connected to category theory, making them essential in various fields throughout one's career.

Category theory is a generalization of algebra that focuses on understanding mathematical objects through structure-preserving transformations called morphisms. It does not embed a single group into a category, but studies groups through the category of all groups Grp, where objects are groups and morphisms are group homomorphism. Instead, we learn about group's relationships through morphisms rather than the elements that make them up.

In this perspective, the article first presents an overview of Autism Spectrum Disorder (ASD), its symptoms, and diagnosis. Moreover, it describe the main features and functionalities of DIAUTIS I and DIAUTIS II. Next, it introduces DIAUTIS III, which will use the mathematics of category theory for formalization of fuzzy cognitive mental model of autism and compare the differences between two mental models for helping autistic children to enhance their learning and exceed their learning disabilities.

II. Autism Spectrum Disorder

American Psychiatric Association define the Autism Spectrum Disorder (ASD) as a set of permanent neuro-developmental described by troubles with social interactivity and restricting and repeating behaviors [5], usually beginning in early childhood, Children with Autism Spectrum Disorder may learn, move, or pay attention with different styles.

Autistic children often have problems with: Keeping eye contact, not responding to their name (9 months of age), Ignoring other children and avoiding play with them (36 months of age), etc. Moreover, they may behave in ways that may seem unfamiliar. as: Repeat voices, words or phrases, Have obsessive hobbies, Flap hands, Spinning in circles, Locked body, Have unusual reactions to some of sounds, odors, flavors, or feels. In addition, there are other characteristics that autistic children may have which include hyperactive, unusual eating and sleeping patterns, anxiety, stress, Not getting scared or getting scared more than expected [6].

A. *Diagnosis*

Over the past few decades, the symptoms of autism spectrum disorders (ASD) have multiplied, leading to increasing recognition of this developmental disorder [7]. For a diagnosis of ASD to be made, children must exhibit characteristics related to both social communication difficulties and restricted, repetitive, and/or sensory behaviors from an early age. The diagnostic criteria for ASDs have evolved considerably over the years. Currently, diagnosis is mainly made using observational tracking tools that measure the child's social, behavioral and cognitive abilities [8]. There are many diagnostic tools, but the two principle ones used in the diagnosis of ASD are DSM-V and M-CHAT.

American Psychiatric Association is publishing a book titled "The Diagnostic and Statistical Manual of Mental Disorder" for presenting a classification of mental patterns with associated criteria, this association classifies autism, Asperger's syndrome and developmental disabilities as Pervasive Developmental Disorders (PDD). It has been recommended that the term "PDD" should turn into "DSM-V" with "Autism Spectrum Disorder". Autism is described by a range of signs, including the existence of permanent deficiencies in social communication, deficiencies in social interaction in a different situations, and restricted and repetitive patterns of behavior in DSM-IV-TR. It was combined into two categories in DSM-V: 1) fusing troubles in social communication and interaction; 2) restricted and repetitive behaviors [9]. The DSM-V is a tool that can be used by wide clinical professionals to help in the diagnosis of mental illness and developmental disabilities. This manual (DSM-5) is used by psychiatrists, psychologists, social employees, doctors and nurses for people with mental illness.

B. *Mental Models: Autism Mental Model*

The Mental model theory proposes a theory of inference that aims to explain different types of thinking according to how the mental model represents. A mental model is a powerful cognitive entity that organizes people's interpretation of their world and their actions in response to it [10]. We view the concept of mental models, described as the ideas people form about the world and the activities they undertake [11].

A Mental Model (MM) is a simulation of the reality that we are producing, with conscience or unintentionally, in our minds [12]. It plays a crucial role in various fields of our life, which a main aim of a MM is to help the model's user to choose the right action of a target system (Johnson-Laird; Norman, 1983) for making sense of, solving problems, making decisions, making plans, or learning/education…etc. A mental model is created, updated and stored through a three-phase process: observation, learning and experience [12].

The purpose of education is to provide access to knowledge. In order to assess the evolution of a learner's knowledge, we believe it is useful to understand the process of learning which has been a principal subject in the field of education [13]. Hence, mental models are a very important part in this process, which play a crucial role in understanding the development of knowledge and the actions of learners. A learner's knowledge passes by three successive stages; these are the Novice, Learner and Expert stages. One of the important contributions of cognitive psychology to learning is the notion of mental models. This notion will allow us to better differentiate between these three stages of learner development [14, 15].

Presenting a mental model visually and formally requires at least three phases: extraction, analysis and formalizing. Finding ways of modeling a mental model presents a challenge to any discipline using this concept as a tool to better understand individuals' internal representations of their environment.

Our mental models of autistic children contain all the imperfections and activities they do not know how to perform, or which they perform clumsily. This is why their mental models are so important, because they show all their limitations, which can be quite different. These can then serve to identify techniques or strategies for enhancing their learning.

C. *Learning Aids*

Teaching and learning process needs three essential components, identifying the input elements (students, teachers, teaching and learning materials), the procedure, and the output elements (graduation, orientation, skills, diplomats ...). The quality of teaching and learning can be compromised by unfavorable general conditions. It is crucial to provide a sophisticated environment in which to teach and learn effective skills. Using of learning aids help students to hear, see, or perform with efficiency during the learning process that become more enjoyable and less tedious thanks to learning aids [16].

Previous research has repeatedly demonstrated that the use of learning aids can greatly enhance learning from multimedia materials. For example, Renkl (2002) studied learning from examples worked in a computerized learning environment on probability calculations. He found that students learned better not only when they explained themselves, but also when they took advantage of pedagogical explanations offered through an online help system [17].

In this twenty-first century, the technological revolution plays a vital role in the learning process. The development of smart phones and other ICT devices has become increasingly popular in the field of communication, literacy and entertainment. These technologies can contribute to the diverse educational development areas of the autism community [18].

Children with autism cannot learn new things easily and cannot actively receive new information, but they can benefit from various types of learning aids specially designed for autistic children and other children with learning disabilities. There are many different types of beneficial learning aids available today, including sensory and auditory learning aids, as well as motor development aids. These tools enable autistic children to catch on quickly and increase their chances of leading a normal life. [19].

III. DIAUTIS I: A Multi-Agent Platform for the Diagnosis of Autism

M. El Alami, N. Tahiri and F. de Arriaga have proposed an approach based on agents, fuzzy logic and affective computing to build an automatic and autonomous autism diagnosis platform [20]. This platform will offer to clinical teams, doctors, parents, guardians and schools a set of tests that help them to get a model of the kid that allows them to evaluate the gravity of autism and the signs of autistic children. In this section, we will describe with details the features of this platform.

A. Features

This research is the beginning for what could be a long project. Over time, it may lead to developing a tool for helping professionals diagnose autism. Consequently, flexibility was highlighted as an important feature of this work. The aim of this flexibility is to anticipate, as far as possible, unknown elements that could discredit the work carried out to date. In line with these ideas, the method developed comprises the following phases [20]:

- Acquiring knowledge about autism in its various aspects.
- Definition of criteria and tools for diagnosing autism.
- Design of a diagnostic computer model based on a group of tests.
- Selection of a platform to build agents based on its flexibility and conception of the DIAUTIS architecture.
- Determination the DIAUTIS functional proofs assigned to the engineering staff.
- Conduct additional testing with observers, medical and clinical staff.

DIAUTIS proposes a set of tests grouped into seven kinds linked to the troubles identified in DSM-V manual. Each kind contains a set of elements to be evaluated and the evaluating agents that intervene in DIAUTIS (control agent, design agent, cognitive agents for sound, gaze and movement, affective agents, pedagogical agents, rule learning agents, evaluating agents, group evaluating agent, interface agent).

DIAUTIS also uses cameras and sensors to receive information on gaze, face, gestures and movements, as well as software developed in Python to capture emotions and analyze pronunciation and intonation to facilitate analysis of the information provided by the collection of tests proposed by DIAUTIS and approved by medical teams to determine the child's condition. These tests are based on ENT (the test collection generation criterion), fuzzy logic techniques to assess the child's condition and the OOC model introduced in NEOCAMPUS [21] to analyze emotions. Then this platform can facilitate the child's previous diagnostic results and generate a report of each test, indicating the times used, the child's age and the result, which can provide information on the test's difficulty, suitability for a particular age or relationship with the severity of autism.

DIAUTIS agents are independent, intelligent and can work autonomously or with other agents. They have learning capabilities thanks to neural networks and machine learning techniques, as well as specific knowledge to reach affective or cognitive goals. They can collaborate with others when objectives surpass their capabilities, or when another agent needs them. They can be cloned or deleted if necessary, have a natural understanding of language and can receive information from external sensors.

The features of NEOCAMPUS for cooperating and controlling agents [22], [23] have been enhanced to take into account the important set of agents that can try to act simultaneously and the specific roles they perform in DAUTIS.

B. Diagnosis Model

The child's diagnosis prepared by DIAUTIS I is represented by a fuzzy graph consisting of a main node (0) that contains the child's personal details and characteristics, connected to seven other nodes (1-7) representing different test kinds. Each test kind is further divided into subcategories, and the evaluation results of each test are linked to the corresponding category or subcategory node.

These fuzzy sets are then integrated into a single fuzzy set, which is then joined to the main node. The final child's model is obtained by using multi-criteria and incorporated into node 0. The system keeps a record of all diagnosis meetings, allowing analysis and comparison of previous sessions. The integration fuzzy set can be defuzzified into a single value when necessary, making it easy for clinical personnel, tutors, and parents to understand. The system primarily communicates with doctors or clinical teams, who set parameters for the tests and receive immediate results and the final child's model. Communication with parents and tutors is also possible through interviews stored in the system.

DIAUTIS I regularly updates the computer team on system performance and agent operation, including the number and type of agents, cooperation, learning, response time, and diagnosis stages.

DIAUTIS I, in summary, is a software platform that meets the objectives of autism diagnostic aid. However, the use of the results can still be clarified. The information from a diagnostic session provides medical equipment with a clear view of the results obtained and the severity of the disease. This is the essential objective of the platform. Nevertheless, DIAUTIS I can also provide other aids such as:

- Facilitating the comparison of the child's previous diagnostic results.
- Facilitate the history of each test, indicating the periods used, the age of the children who took them and the result they obtained, individually or statistically. In this way, it can provide information on the difficulty of the test, its suitability for a certain age or its relationship with the severity of autism.
- In the light of the last paragraph, DIAUTIS I may in the future begin to develop collections of standardized tests in order to produce test protocols, as indicated by doctors and medical teams, leading to eventual standards.

IV. DIAUTIS II: A Multi-Agent Platform for the Diagnosis of Autism and the Design of Serious Games

A. Features

DIAUTIS II extends the capabilities of DIAUTIS I by adding a methodology for designing serious games based on four dimensions: Knowledge, Affect, Senses, and Pedagogy. The KASP methodology enables DIAUTIS II agents to design games from a set of more than 80 templates, considering the child's background [24].

The results of the tests carried out through DIAUTIS are analyzed in order to deduce the child's basic abilities by developing a result containing five specifications: motor, behavioral, cognitive, socio-emotional and sensory. Using machine-learning techniques, an intelligent system will be set up to classify these specifications, which will indicate the type of disorder the diagnosed child suffers from. The authors propose using SVM, a supervised type algorithm, to classify a child's disorder using an intelligent classifier on selected diagnostic specifications.

DIAUTIS I has been enhanced with KASP (Knowledge, Affect, Sensory, Pedagogy) [25], a new approach to the conception of serious learning games. This is a new solution to aid children with learning disabilities to exceed these barriers by considering their cognitive, emotional and sensory aspects.

B. Serious Game as an Evaluation Tool for the Autistic Children

Headings, A game is a narrative that combines an intention, genre, main concept, and objectives. The balance between the objective and the concepts is crucial for a coherent and interesting story. To assess a child's assimilation of a concept with Autism Spectrum Disorder (ASD), it is necessary to consider the sub-concepts building the main concept. Mathematically, comparing the child's sub-concept weights to the expert's weights helps identify actions that cause significant degradation. The child's actions influence the sub-concepts' weights, generating a new vector with new values. The expert is then encouraged to offer a serious game to assess a concept in children with ASD, consisting of several scenes [26]:

$$\text{Game } S = \{S\alpha, S\beta, S\gamma, \cdots\} \text{ where Si a game scene}$$

The game's main objective is the same as the expert's assessment. Each scene consists of multiple objects representing sub-concepts, which interact with each other based on the child's actions:

$S\gamma = (C\gamma, A\gamma)$ where $\{C\gamma = \{C\gamma1, C\gamma2, C\gamma3, \ldots\}\}$ is the set of sub-concepts of the scene $S\gamma$ and

$A\gamma = \{A\gamma1, A\gamma2, A\gamma3, \ldots\}$ is the set of the actions of the scene $S\gamma$.

Each action is identified by a weight. The Likert scale (Table I) is a reliable tool for measuring perceptions, measures the influence of an action on a concept through a weight, consisting of five levels [27].

$$\forall A\gamma i \Longrightarrow \; W\gamma i \in [1, 5] \tag{1}$$

Likert's scale can also be normalysed by dividing its values by 5. So {0.2, 0.4, 0.6, 0.8, 1}.

TABLE I. LIKERT'S SCALE

1	2	3	4	5
Very bad action	Bad action	Acceptable action	Correct Action	Very good action

In addition, the expert attributes a weight to each concept (Table II) to describe its importance in relation to the main concept described on DIAUTIS [21]. The weighting varies from one scene to another:

$$\forall C\gamma i \rightarrow V\gamma i \in [1, 2, 3, 4, 5] \tag{2}$$

TABLE II. WEIGHTS ATTRIBUTED BY THE EXPERT

Concepts	Cγ1	Cγ2	Cγ3	Cγ4
Weight/Concept principal	Vγ1	Vγ2	Vγ3	Vγ4

The expert creates a semantic network of the scene (Fig. 1), representing all concepts and actions, and their weights. A node represents each concept, with an oriented arc connecting two nodes indicating the influence of action on the destination concept. The expert must specify at least five actions for each concept.

After a game session, a child's semantic subnet is created (Fig. 2), inheriting the expert's network and containing only the child's actions and associated concepts.

After that, the semantic network generates two matrices: the expert's matrix (Table III) and the child's matrix (Table IV). The expert's matrix contains all types of actions and concepts, while the child's matrix is a submatrix derived from the expert's matrix. The expert and child's action weights are typically different, as the expert's matrix contains all the varieties of actions and concepts, while the child's matrix is a submatrix derived from the expert's matrix.

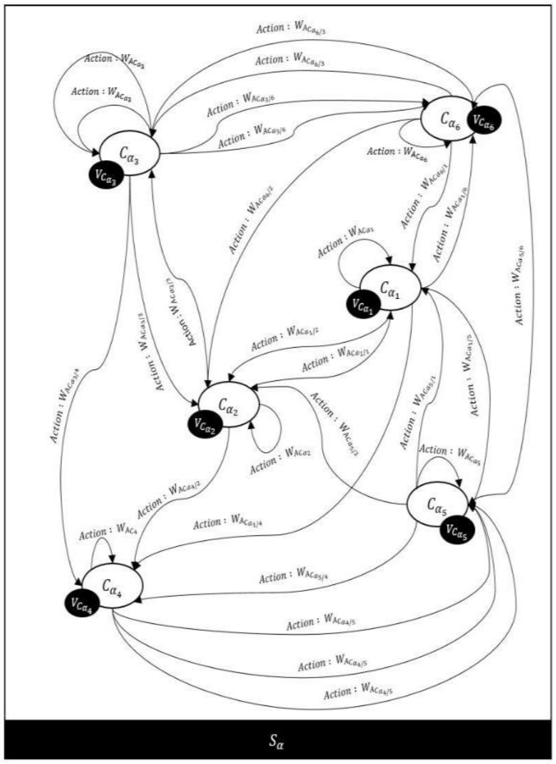

Fig. 1. Expert's semantic network: Scene Sα.

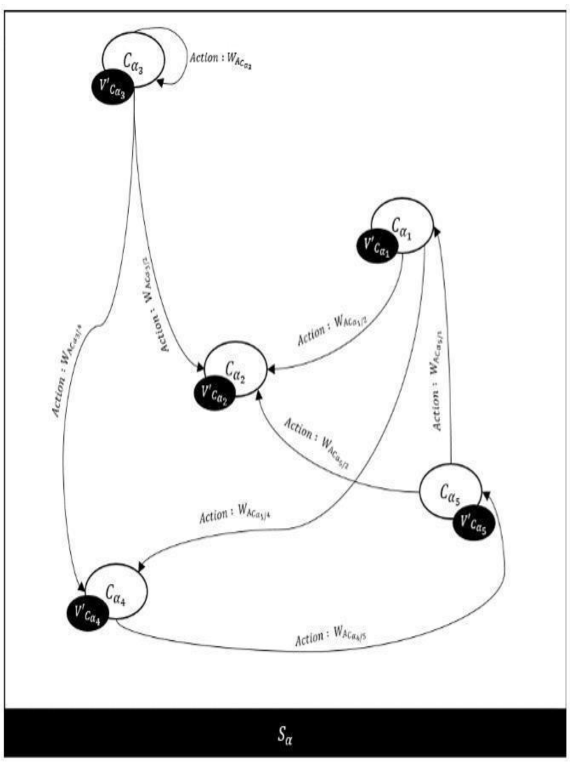

Fig. 2. Child's semantic network: Scene Sα.

TABLE III. EXPERT'S MATRIX: SCENE SA

	Cα1	Cα2	Cα3	Cα4	Cα5	Cα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
…	…	…	…	…	…	…
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6
Action	WACα1	WACα2	WACα3	WACα4	WACα5	WACα6

TABLE IV. CHILD'S MATRIX: SCENE SA

	Cα1	Cα2	Cα3	Cα4	Cα5
Action	WACα1	0	0	0	0
Action	0	WACα2	0	0	0
Action	0	WACα2	0	0	0
Action	0	WACα2	0	0	0
Action	0	0	WACα3	0	0
Action	0	0	0	WACα4	0
Action	0	0	0	WACα4	0
Action	0	0	0	0	WACα5

Then, we have received:

- The expert assigns weights Vαi to scene sub-concepts, which can be represented as an n-dimensional vector, representing the number of scene sub-concepts (Table II).
- The child's sub-concept weights V'αi can now be expressed as: $V'\alpha i = W\alpha j \times W\alpha k \times W\alpha l \times \ldots \times V\alpha i = \Pi\, W\alpha j \times V\alpha i$

The Wαj and Vαi are the normalized child's and expert weights; they are vectors of n dimensions, with some components potentially being 0, based on normalized Likert's scale.

- The cosine of two vectors, a real number between -1 and 1, measures the child's ability to assimilate key concepts.

The study found that an acute angle between two vectors leads to higher understanding in a child, while an obtuse angle makes the main concept unclear. This allows experts to better analyze a child's perception of a concept using ADS measurements, enhancing their understanding of the concept.

- We identify actions causing significant sub-concept weight degradation, selecting actions with a weight $WAC\alpha i \leq 3$ for each scene, ensuring accuracy:
  - o If $WAC\alpha i \leq 3$, a significant degradation is applied to the sub-concept.
  - o If $4 \leq WAC\alpha i < 5$, The sub-concept is subjected to low degradation.
  - o If $WAC\alpha i = 5$, The weighting of the sub-concept remains unchanged.

The expert will identify difficult concepts for each child, allowing tutors to adapt learning programs and highlight gaps in the proposed technique to strengthen understanding for the child with ADS.

V. DIAUTIS III: A Fuzzy and Affective Platform for Obtaining Autism Mental Models and Learning Aids

A. Features

DIAUTIS III extends the capabilities of DIAUTIS II by adding a methodology for formalization of the elaborated mental model of autism (fuzzy graph) based on mathematical theory of categories. That way autism mental models can be automatically obtained. The theory of categories will be used to formalize the graph as well as its operations. We will explain in next sections the steps of this formalization.

The DIAUTIS III tests, based on KASP methodology, analyze the fuzzy mental model's elements. A collection of serious games is designed to cover all deficiencies in the seven families of the model. These games are divided into two kinds: basic discovery of deficiencies and deep exploration of problems, revealing details, intensities, features, and concept assimilation. This allows for a deeper understanding of learning problems and their evolution. The games are displayed on a computer, allowing children to see moving objects, colors, and sounds.

DIAUTIS III has been enhanced with the theory of categories which will be used to formalize the fuzzy and affective mental model of autism, a new approach to the presentation of the abstract concepts. The mathematical theory of categories is a suitable formalism for representing the mental model of an autistic child from a blurred graph, allowing operations like CONS and TRA to evaluate the difference between two mental models or the trajectory of a collection. This mental model allows for the creation of games that require low concentration levels and gradually increase them, thereby facilitating learning and allowing the child's mental model to evolve at any time, reflecting their overall learning experience.

B. Mathematical Theory of Categories

There are various perspectives on the nature and purpose of category theory. Carlos Polanco [28] defines category theory as a mathematical branch that focuses on abstract and unified mathematical structures and their relationships. This Theory is a fundamental aspect of modern mathematics, offering a unified framework for understanding and formalizing mathematical concepts and theories.

Hence, Hoare [29] describes category theory as a broad and abstract branch of pure mathematics, providing little assistance in solving specific problems within its sub-disciplines. As a generalist tool, it offers little benefit to practitioners, making it a valuable tool for generalists.

Category theory is a crucial tool in mathematics, organizing and unifying many fields such as algebraic topology, homological algebra, homotopy theory, representation theory, arithmetic geometry, and algebraic geometry [30]. It is essential for the development of new graphical notations and different levels of abstraction in contemporary mathematics. Category theory and its mathematical disciplines use commutative diagrams, leading to philosophical explorations. However, category theorists have also developed systematic and formal graphical languages to express various forms of argumentations. Mathematics has evolved from being done "up to isomorphism" to "up to equivalence" or "bi-equivalence" or even "n-equivalence." This shift in approach has led to the development of systematic and formal graphical languages to express various forms of argumentations. The level of abstraction in mathematics has evolved from "up to isomorphism" to "up to equivalence" or "bi-equivalence" or "n-equivalence".

Theory of categories focuses on the relationships between structures, offering a highly abstract yet powerful approach to mathematics, focusing on objects and morphisms. The composition operator is a central element, but reducing the theory to a refinement of it is oversimplification and fails for full-featured capture its richness and depth [28]:

1) Abstraction: Category theory is a mathematical field that emphasizes the relations between structures, rather than individual elements. It defines categories by their objects and morphisms, rather than their internal aspects. This abstraction allows for generalization across various mathematical contexts, enabling the transfer of concepts and results across seemingly disparate areas of Mathematics.

2) Unification: Category theory is a powerful tool that unifies various mathematical fields by drawing parallels between seemingly unrelated concepts. It uses factors and natural transformations to connect different categories, demonstrating that different structures and theories are manifestations of underlying ideas, leading to improved understanding and advancements in various mathematical fields.

3) General concepts: Category theory introduces concepts like limits, colimits, and adjunctions, which provide abstract perspectives applicable across various mathematical contexts. Examples include products in abstract algebra and products in topology. This generality enables mathematicians to apply intuitions and results from one area to another, leading to discoveries and advancements in various topics.

4) Applications in other areas: Category theory, a fundamental concept in mathematics, has broad applications in fields like computer science, physics, logic, and philosophy. It is utilized in type theory and programming language denotational semantics, while toposes provide a new framework for intuitionistic logic and set theory in logic.

A category is an abstraction based on objects and morphisms, often studied in groups, rings, and topological spaces. Category theory shifts focus from object elements to morphisms between objects. The axioms of a category do not require objects to be sets, making it unnecessary to speak of an object's elements [31].

A category is a collection of data, denoted by C = (Objects(C), Morphisms(C), ∘, Id) consisting of these items:

1) A set of objects Objects(C), which contains all the items in the category.

2) A set of Morphisms(C), containing the arrows (or Morphisms) between category objects.

3) An operation called composition ∘ that joins Morphisms to form another Morphism, and is associative: $(f \circ g) \circ h = f \circ (g \circ h)$, for any f, g, and h in Morphisms(C).

4) Identity Morphisms Id for each object in Objects(C).

C. Obtainment of the Child's Fuzzy Mental Model

Autism is a group of basic behaviors characterized by difficulties in social reciprocity, communication, and behavioral flexibility. Children with autism spectrum disorder (ASD) struggle to understand others' emotions, feelings, beliefs, and thoughts. Diagnostics for autistic children typically align with existing medical and psychological recommendations, but they do not evaluate all the problems, severities, or individual intensity of the child's symptoms. As they are not repeated, they cannot determine the appearance of new symptoms or deficiencies with age or changes in intensity. Additionally, the diagnosis does not determine the affective states associated with these deficiencies, making aid for autistic children ineffective due to ignorance of these factors. A mental model for autistic children contains all deficits and tasks they cannot perform, but no autism mental model has been developed to date.

In our last article, we propose a fuzzy affective mental model of autism, based on the DSM-V that considers a child's affective states. The model is included their specific deficiencies, intensities, frequencies, and associated affective states. It uses affective computation and fuzzy logic to account for affection and uncertainty. The model can be obtained using the KASP Methodology, and can be obtained in the form of a fuzzy cognitive map or graph. This model can be used to improve learning and social integration for autistic children.

The mental model of an autistic child contains their imperfections, awkward actions, and associated affective states. This model is crucial as it reveals their varied limitations and can help establish techniques or strategies to improve their learning. It is essential for understanding and addressing these difficulties in order to improve their overall development.

1) Autism: New Fuzzy Affective Mental Model: The proposed new conceptual fuzzy and affective model of autism will guide the creation of a specific model for each child, encompassing their specific deficiencies, intensities, frequencies, and associated affective states [32].

The mental model serves as a child's mental radiography, allowing for the identification and treatment of each present deficiency and its intensity. This article does not address this crucial issue, but the model goes beyond diagnosis, as per the DSM-V [33]:

- Persistent deficits in the ability to socialize and interact in a wide variety of contexts.
- Iterative patterns of restricted action, attention, or activity manifesting in at least two contexts.
- These symptoms have to be present in the first stage of the development of the disease.
- These symptoms are the result of a loss of distance in one or more areas of normal functioning.
- An intellectual or general developmental problem does not seem to be the best explanation for these disorders.

The mental model should consider a child's deficiencies and their characteristics to provide appropriate techniques. It also emphasizes the importance of the affective states of the autistic child in learning. Analyzing the link between a deficiency and a specific affective state is crucial for providing emotional and cognitive aids to improve learning deficiencies.

A universal fuzzy model cannot be developed for specific learning deficiencies, as they vary with autistic children's age and present unique characteristics. However, a list of general and concrete deficiencies can guide in expanding, detailing, or specifying them in each case.

The first list of deficiencies has been grouped into seven families:

1) Affective or emotional: This family contains deficiencies related to emotions of the autistic child and her/his affective state when her/his environment changed or meeting people.

2) Verbal and language comprehension: The second family includes all the deficiencies related to the communications and expression that the child with ASD suffers from in her/his daily life and/or in classroom.

3) Monitoring of visual objects: Researchers have identified eight visual processing disorders, each affecting different abilities and presenting unique challenges. These disorders include visual discrimination, optical sequencing, visual figure-ground differentiation, pictorial memory, visual-spatial relation, visual closing, letter and symbol reversal, and visual-motor processing.

4) Attention and response to basic sounds: This family consists of the difficulties related to the identification of sounds, the memory of oral instructions or the sensitivity to the noise…etc.

5) Other social behaviors: The fifth family presents common disabilities and atypical social behaviors of autistic children.

6) Combined tests and deficiencies.

7) Group behaviors.

To better represent the autistic child's specific conditions, each family should include specific deficiencies, rather than general ones, which may belong to multiple families based on the child's characteristics or decompose in multiple families.

Each deficiency has a fuzzy set associated (Table V), with elements as below:

- Severity or intensity: Described by a number that belongs to the whole [0, 1].
- Frequency with which this defect occurs: A number belonging to the range [0, 1] marks it.
- The most commonly associated emotional state. It is defined first by the number of the affective state that belongs to Family 1.
- The intensity of this association is a real number that belongs to [0.1].

2) Obtaining and Formalization the Fuzzy Mental Model: The autistic fuzzy mental model is typically diagnosed through a student progress study to identify learning issues.

To do this, we have implemented the KASP methodology, already tested and used, enables the development of tests based on the theory of serious games. Serious games, which aim beyond entertainment, have significant potential due to the impact of emotions on learning. However, there is a significant academic literature on the nature, composition, and effects of these games.

This methodology evaluates learning deficiencies in autistic children by measuring preschool children's concept assimilation, offering advantages such as identifying continuous learning strategies. It also aids in assessing their learning abilities.

A collection of serious games using KASP methodology has been designed to cover all deficiencies in the seven families of the model. Divided into two categories, the first allows basic discovery of deficiencies, while the second allows deep exploration of problems, revealing details, intensities, features, and concept assimilation. The games are displayed on a computer, allowing children to see moving objects, colors, and sounds.

The process of obtaining the mental model passes via these phases:

- Through the first kind of serious games, or diagnosing, various crucial deficiencies are detected.
- In order to identify new learning issues, serious games of the first kind are used.
- The games of the second kind are used to go deeper into the problems found in phase 1. Getting their intensities, severities and associated emotional states.
- The games of the second kind are used to go deeper into the problems found in phase 2. Getting their intensities, severities and associated emotional states.
- Creation of the fuzzy cognitive map (Fig. 3).

The mental model that is described can be formalized as a fuzzy cognitive map or a fuzzy graph (Fig. 3), which is characterized by its levels:

Level 1: The name of the child with autism, her/his birth date.

Level 2: The seven families of deficiencies presented above.

Level 3: List of deficiencies contained in each family [32].

Level 4: For each deficiency, the associated fuzzy set formed by: intensity or severity of the deficiency, frequency, associated emotional state, and intensity of that association.

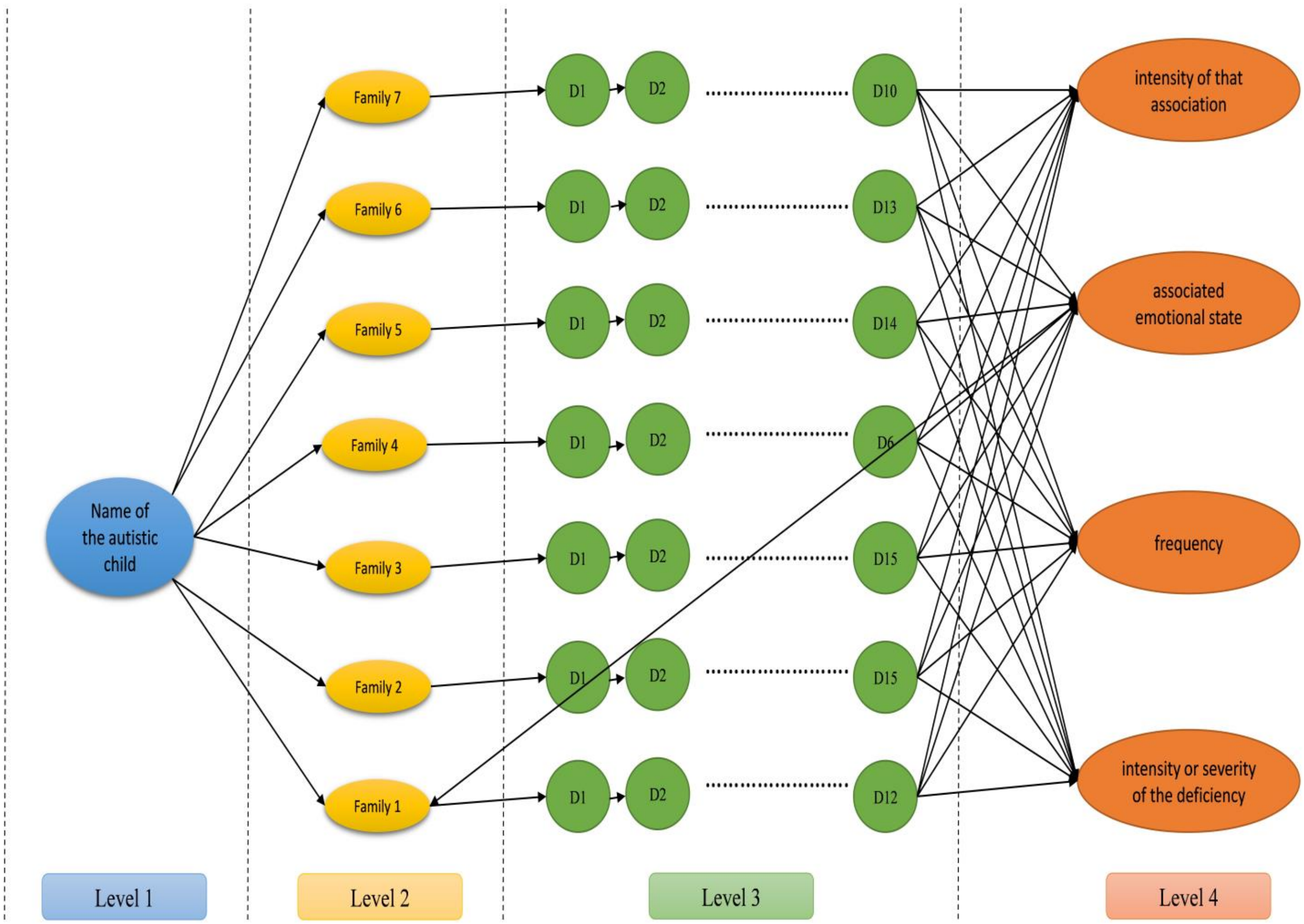

Fig. 3. Detail of the fuzzy cognitive map.

TABLE V. THE ASSOCIATED FUZZY SET OF EACH ELEMENT OF DEFICIENCY

Elements	The associated fuzzy set			
Intensity of the deficiency	[0,0.25] (weak)	[0.25,0.5] (medium)	[0.5, 0.75] (important)	[0.75, 1] (very important)
Frequency of the deficiency	[0, 0.25] (weak)	[0.25,0.5] (medium)	[0.5, 0.75] (important)	[0.75, 1] (very important)
The most frequent emotional state associated	It is determined by the number of the emotional state that appears in Family 1 at the list of deficiencies.			
The intensity of association	[0, 0.25] (light)	[0.25,0.5] (medium intense)	[0.5,0.75] (intense)	[0.75, 1] (very intense)

D. *Metrics for the Evaluation of the Difference Between Fuzzy Mental Models of the Same Child*

So far, we have used the fuzzy graph as a formalization of the elaborated mental model of autism, composed of a three-level graph. The first, of a single node, is constituted by the name of the child and the date on which the mental model is carried out; the second level has as many nodes as families of learning problems we have considered. The third level is formed by the nodes that represent the different learning problems, in total n, that we have found in the child, corresponding to each family (Fig. 3).

This fuzzy graph will be used as the basis of the mathematical concepts we are going to introduce. Next, we will consider first the theory of categories [34], to formalize the graph as well as its operations. We will try to explain in an intuitive manner the steps of this formalization. Subsequently, we will incorporate topological notions, vector spaces and basic algebraic structures.

1) Concept of mental model: The mental model j of the autistic child i, which we represent by Mentij, is composed of a set of three sub-sets, which are:

a) A subset Dij = (a,b,c,..s) of some (or all) of the nodes of level 3 of the fuzzy, which are the difficulties presented by the autistic child. In the particular case adopted in this paper, the total number of learning problems considered in the seven families is:

$$N = 10+13+14+6+15+15+12 = 85 \qquad (3)$$

The cardinal of Dij, i.e. the number of elements containing this subset is s, which will vary in each case of autism. This number s will always be less than or equal to N.

b) A set ₵ij =(Cija, Cijb,..., Cijs) of s fuzzy sets that hang from the nodes of the level 3 of the fuzzy graph, corresponding to the problems of the child. In our case, as we have already said, s<=85.

c) A set of Morphisms and morphisms, Mij = (Morfija, Morfijb,...Morfijs, morfijr,n,... morfijh,t); the Morphisms, called Morfijq apply the fuzzy set Cijq to the node q of level 3. These Morphisms, (with capital letter), are the first-class morphisms; besides, there are other morphisms, (with lower case): morfijr,n,.......,morfijh,t, which swap the fuzzy sets Cijr and Cijn hanging on the nodes r and n.

So, the mental model is composed of:

$Ment^{ij}$ = (Dij, ₵ij, Mij) ; Dij = (a,b,c,..r) ; its cardinal s<=N ;

₵ij =(Cija, Cijb,..., Cijr) ; its cardinal is s<=N; (4)

Mij= (Morfija. Morfijb,...Morfijs, morfijr,n,...,morfijh,t) ;

If we consider that morfijr,n equivale a morfijn,r ,its cardinal is s+(s-1)+...+2+1= (s+1).s/2

d) Moreover, it has two operations, dom and codom, assigned to each Morphism or morphism. For Morphisms, Morfijs, the dom is the fuzzy set to apply, which is Cijs, and the codom is s, the node of the learning problem to which it is applied. For morphisms, morfijh,t the dom is the starting node, h, and the codom is its new position, t, of the node. In short:

dom (Morfijs) = Cijs , codom (Morfijs) = s, s€Dij

dom (morfijh,t) = h, h€Dij , codom (morfih,t) = t, t€Dij (5)

The first Morphisms are those that assign a fuzzy set to a node; without them there is no mental model. The second morphisms exchange the nodes and the fuzzy sets hanging from them. Although dom(morfijh,t) and codom(morfijh,t) are different from dom(morfijt,h) and codom (morfijt,h) we consider both morphisms equivalent, because their effect on the fuzzy sets is the same.

e) It also has an id operation, which assigns to each node b of level 3, or to each fuzzy set h, the morphism idh which is the identity of h, so that its dom and codom coincide with h. In addition, it is verified:

- Identity law: idh.Morfijh= Morfijh, when h is a node; Morfijh.idh= Morfijh when h is a fuzzy set; in this last case the composition idh.Morfijh cannot be done
- Identity law: idh.morfijr,h= morfijr,h, when h is a node; morfijh,t.idh=morfijh,t when h is a node; when h is a fuzzy set the compositions idh.morfijr,h or morfijr,h.idr cannot be done.

f) It also has a composition operation of two morphisms (f.r) such that:

- The dom of (f.r) is the dom of r, and
- The codom of (f.r) is the codom of f.

It is worth remembering that, as in other mathematical formalisms, composite expressions or formulas are read from right to left, that is, in the case of (f.r), first acts r, and then f. In order for this composition to exist, the morphisms f and r must have a particular nature; r must be a Morphism (with capital letter) to be able to act on f, which must be one morphism with lower case. This way:

dom (morfijh,t.Morfijh) = dom (Morfijh) ; codom (morfijh,t.Morfijh) = codom (morf ijh,t) (6) *g) Composition is also complied with:*

Associative law (morfijg,h.morfijb,g).Morfijb = morfijg,h.(morfijb,g.Morfijb)

When, as previously verified

codom(Morfijb)= dom(morfijb,g) and codom(morfijb,g)=codom(morfijg,h) (7)

In this associative law must appear a Morphism (with capital letter), because without it there is no mental model; only one can appear, because if more are included the conditions of equality required of doms and codoms cannot be fulfilled. On the other hand, this Morphism must be the first on the right, for the same reason of equality of doms and codoms.

The associative law is also fulfilled with three morphisms (with lower case), when the appropriate conditions of doms and codoms are verified, since as there is no Morphism, there is not any fuzzy set in any node, therefore, the changes of the material hung in the nodes are inoperative. The mental model does not actually exist because the learning problem nodes do not have any fuzzy set to evaluate them.

We will elementally check that this associative law, which we will describe intuitively in a particular case, is complied with.

Suppose a set of five segments followed by numbers 1 to 6; these numbers represent the positions of the third-level nodes of the fuzzy graph, i.e., the learning problems observed in this autistic case:

1-----2-----3-----4-----5-----6

Let's imagine that, initially, under each node hangs an empty box or contains a fuzzy set in the box. Each box, though empty, contains the number of the node it initially hangs from

1-----2-----3-----4-----5-----6

! ! ! ! ! !

U1 U2 U3 U4 U5 U6

If we consider the associative law (morf4,6.morf2,4).Morf2 = morf4,6.(morf2,4.Morf2) we have:

When performing the right hand member of this equation we find:

- *According to the parenthesis,* Morf2 hangs the fuzzy set C2 in the box U2. And subsequently morf2,4 transfers C2 to the box U4.
- *Subsequently morf4,6* moves the fuzzy set C2 from the box U4 to the boxU6 . That's the final state.

By now carrying out the left hand member of the equation, we have:

- Morf2 hangs the fuzzy set C2 in the box U2.
- *According to the parenthesis,* morf2,4 changes C2 from the box U2 to U4, and morf4,6 also to U6.

Both end states corresponding to the left hand and right hand members of the last equation coincide.

The formal demonstration of this general associative law would follow the same steps commented, only that instead of using concrete numbers for nodes like 2, 4, and 6, we would use generic denominations like r, s, and t. and an arbitrary number of nodes. Accordingly, it can be said:

Theory 1. The Mentij mental model is a finite category.

Theory 2. The number of Morphisms (with capital letter) is s.

Theory 3. The number of morphisms (with lower case) also depends on s and is s+(s-1)+ (N-2)+...1= s(s+1)/2

It can also be demonstrated that:

Theory 4. The set of Morphisms constitutes an isomorphism between the subset of learning problems (nodes), and the subset of fuzzy sets that assess the intensity and other characteristics of these problems.

For if the doms of two Morphisms are different, so are their codoms and reciprocally.

Theory 5. The set of morphisms constitutes an epimorphism between the set of learning problems (nodes), and itself.

Indeed, if we remember that in monomorphisms, if the doms are different, the codoms must also be, we see that in this case it is not fulfilled, because several doms can lead to the same codom. In the case of isomorphism, all of them with different domains, have different codoms, and also different codoms also correspond to different doms. It is an epimorphism. Whose morphisms cover all nodes as doms and codoms.

Let us now present a concrete example to demonstrate this formalism.

2) Example: mental model of the autistic child Juanito: Since we will only consider one mental model for the time being, we can remove the first subindex, i, of Mentij, and place the name of the child in place of the second sub-index, j. So, we will have as the name of this mental model MentJuanito.

Suppose now that the autistic child Juanito has: problems learning grammar (family 2, number 7), he experiences disgust when writing (family 3, problem 6), and he does not allow or maintain social relationships with his peers (problem 13 of family 5). These problems are associated respectively with a tendency to be distracted (problem 2 of family 1), with irritability and physical over activity (problem 9 of family 1) and feelings of anxiety (problem 4 of family 1).

In this case, the fuzzy graph of Juanito's mental model (Fig. 4) has only three nodes at level 3, which are his three learning problems, which correspond with the numbers 19= (12+7), 36= (12+18+6), and 64= (12+ 18+15+6+13). We have replaced the numbering by the family and their respective number, by a total numbering that already includes the family number.

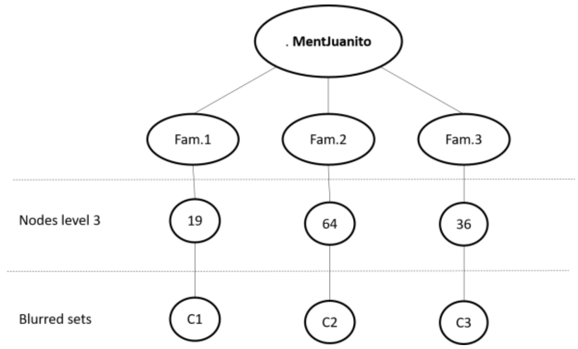

Fig. 4. Juanito's fuzzy graph.

In addition, three fuzzy sets have been obtained that reflect the intensity of the three problems, and are:

a) C1=[intensity 0.4, frequency 0.4, associated affective problem 2, association intensity 0.], for node 19.

b) C2=[intensity 0.5, frequency 0.5, associated affective problem 9, association intensity 0.4], for node 36.

c) C3=[intention 0.6, frequence 0.4, associated affective problem 4, association intensity 0,4], for node 64.

The Morphisms, (with capital letter), available in this case, which we could intuitively consider as arrows, are three: the first, MorfC1-19, takes the first fuzzy set and hangs it on node 19; the second, morfC2-36, takes the second fuzzy set and hangs it on node 36, and the third, morphC3-64, does the same with the third fuzzy set and hangs it on node 64. These Morphisms create a bi-univocal relationship between the set of the three nodes of the mental model and the three fuzzy sets; therefore, they constitute an isomorphism between these two sets.

The morphisms, (with lower case) that we have, are like circles that swap the position and hanging content of two nodes. Therefore, for example, the morf19-36, exchanges node 19 and what hangs from it with node 36. The number of existing morphisms in this example is 3: the morf19-36, the morph19-64, and the morp36-64; as we have already said, we consider morphism morf36-19 equivalent to morf19-36, because their effect on the fuzzy sets is the same (Fig. 5).

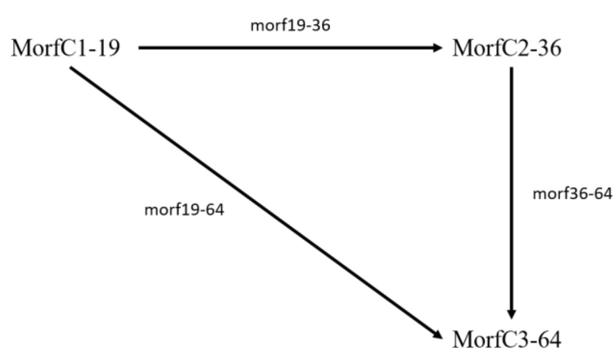

Fig. 5. Existing morphisms.

Furthermore, for each node, the 19 is considered as an example, the morphism id19 that does not change anything of that element, and that satisfies the:

identity law: id19.Morf1-19 = Morf1-19; Morf1-19.id19 = Morf1-19 (8)

and the same would happen with the Morphisms associated with nodes 36 and 64.

Similarly, for existing morphisms it is verified:

identity law: id36.morf19-36= morf19-36 ; morf19-36= morf19-36.id19 (9)

Furthermore, the following properties are met for the morphism composition operation:

Associative law (morf19-36.morf19-36).MorfC1-19 = morf19-36 (morf19-36.MorfC1-19) (10)

Because here it is verified

codom(MorfC1-19) = dom(morf19-36), and codom(morf19-36)=dom (morf19-36) (11)

In this associative law must appear a Morphism of the first class, because without it there is no mental model; only one can appear, because if more are included the conditions of equality required of domains and codominions cannot be fulfilled. On the other hand, this first-class Morphism must be the last of the right, for the same reason of equality of domains and co-dominios.

The association law is also complied with:

(morf64-19.morf36-64).morf19-36 = morf64-19.(morf36-64.morf19-36) (12)

However, it is a trivial law, because since no Morphism intervenes, no node has hanged a fuzzy set of evaluation; therefore, it does not produce the MentJuanito model.

Consequently, Juanito's mental model is a category.

3) *New transactions with categories*: Next, we will define several operations with mental models of the same autistic child. We will try to obtain metrics to evaluate the difference among them.

Value consequence of two mental models

Let us now go to the study of a new operation between two mental models of the same child, which somehow calculates the separation that exists between them. For this, it is necessary to introduce, in addition to the categories, notions of elementary topology, and algebraic structures; for the time being the arithmetic operations of numbers, and their algebraically structures, may suffice. Thus, it is possible to define as a value consequence of two mental models of a child, CONS($Ment_{ij}$, $Ment_{il}$),as a new mental model $Ment_{ijl}$ defined as:

$$CONS(Ment_{ij}, Ment_{il}) = Ment_{ijl} \quad (13)$$

a) $Ment_{ijl}$ contains the same D_{ij} set of Level 3 nodes of $Ment_{ij}$ and $Ment_{il}$, which reflect their learning problems.

b) It has the same morphisms and morfisms as the two previous models.

c) As for their collection of fuzzy sets, each of them is obtained as an arithmetic difference of the numerical values that make up them.

Thus, if the fuzzy set of a certain learning problem in the first model is C_{ija} = [0.5, 0.4, 9, 0.3], and in the second model is C_{ila} =[0.3, 0.3, 9, 0.2], CONS ($Ment_{ij}$, $Ment_{il}$) has as fuzzy set for that node, Cijla = [0,5-0.3, 0.4-0.3, 9-9, 0.3-0.2]= [0.2, 0.1, 0, 0.1]

The positive values of this latter set indicate the benefits or achievements obtained from the first mental model to the second.

However, the following abnormalities may occur:

a) One of the obtained values is negative. That must trigger an alarm that starts a collateral study of the causes that have motivated it, to try to overcome them.

b) The second mental model obtained contains more nodes of level 3 that reflect the difficulties of the autistic child, and therefore more fuzzy sets relative to these new nodes. This happens when new problems arise in his/her evolution. In this case, we have to equalize the number of nodes of the problems, incorporating new rows in the first mental model, as well as increasing the Morphisms and morphisms to include these new nodes.

The values of the fuzzy sets created, relative to these new problems of the first mental model are all null values, according to the operation to be performed (in this first case, arithmetic rest); therefore, they would all be zeros. When performing the rest of the numerical values, the values of the new blurred CONS(Mentij, Mentil) set appear with a negative sign causing the corresponding alarm for indicating the appearance of new learning problems.

This new operation has the following properties:

a) There is no commutative property, or rather, there is the anti-commutative property reflected by:

a.b = -b.a, that is:

$$CONS(Ment_{ij}, Ment_{il}) = -CONS(Ment_{il}, Ment_{ij}) \quad (14)$$

b) There is a neutral element only on the right, when $Ment_{il}$ matches $Ment_{ij}$. which has all its elements equal to 0.

$$CONS(Ment_{ij}, Ment_{ij}) = Mentnulo$$

$$CONS(Ment_{ij}, Mentnulo)= Ment_{ij} \quad (15)$$

This null element only acts on the right, because:

$$CONS(Mentnulo, Ment_{ij})= -Ment_{ij} \quad (16)$$

c) There is an associative property:

$$CONS(Ment_{ik}, CONS(Ment_{ij}, Ment_{il})) = CONS(CONS(Ment_{ik},Ment_{ij}), Ment_{il}), \quad (17)$$

because,

$$CONS(Ment_{ik}, Ment_{ijl})= Ment_{ikjl}; \; CONS(Ment_{ikj}, Ment_{il})=Ment_{ikjl}$$

d) There is the reverse of every mental model, which is himself, as we have seen in (2). Accordingly, it can be said:

Theory 6. The CONS operation between categories constitutes a non-Abelic semigroup.

Estimation of the trajectory of several mental models of the same child

It is also possible to analyze a whole trajectory of mental models of a child,

$$TRA(Ment_{ij}, Ment_{ik}, Ment_{il}, , , , , Ment_{is}) = Ment_{ikl...s}$$

to evaluate the overall performance. To do this we will consider the summary of all your mental models as the following mental model, Mentikl...s such that:

a) Contains the same set of nodes of level 3 as the mental models $Ment_{ij}$, $Ment_{ik}$, $Ment_{il}$,..., $Ment_{is}$

b) It has the same Morphisms and morphisms as the previous models.

With regard to the contained fuzzy sets, there are several ways to get them. One would be to obtain the arithmetic average of the corresponding values of the mental models of the trajectory. However, after studying several practical cases, it seems more appropriate to use the geometric average of these values. In any case, both averages can be used. This new TRA operation has the following properties:

a) It is commutative, because it is the product or sum of the numbers that make up the mental models of the trajectory.

b) It is associative, for the same reason above, the associativity of the sum or product of numbers.

c) It has an identity, formed by the mental model whose numerical components are all equal to d) It does not have an inverse element, because there are no fuzzy sets with numbers greater than 1.

Accordingly:

Theory 7. The TRA operation constitutes an Abelian semigroup with the arithmetic operation + (sum) or x (product).

Example: Juanito's mental model

Suppose now that Juanito has two mental models; in the first one he shows two learning problems corresponding to nodes 19 and 36 of the third level of the fuzzy graph, and their associated fuzzy sets are:

C1= [0.5, 0.4, 9, 0.3]; C2= [0.4, 0.4, 13, 0.2]

In the second mental model the associated fuzzy sets are:

C1= [0.3, 0.3, 9, 0.2]; C2= [0.3, 0.2, 13, 0.1]

The CONS model obtained from these two previous models has as its first fuzzy set:

C1= [0.5-0.3, 0.4-0.3, 9-9, 0.3-0.2]= [0.2, 0.1, 0, 0.1]

and as the second set:

C2= [0.4-0.3, 0.4-0.2, 13-13, 0.2-0.1]= [0.1, 0.2, 0, 0.1]

The positive values of these fuzzy sets indicate the benefits or achievements obtained from the first mental model to the second.

Let us now apply the TRA operation, assuming that Juanito has three models of his autistic trajectory, with two learning problems associated with nodes 19 and 36.

If the fuzzy sets associated with these two problems are:

First model: C1 = [0.6, 0.5, 9, 0.4], C2= [0.5, 0.5, 13, 0.3]

Second model: C1= [0.6, 0.4, 9, 0.3], C2= [0,5, 0.4, 13, 0.2]

Third model: C1= [0.4, 0.4, 9, 0.3], C2= [0.4, 0,3, 13, 0.2]

The final values of the fuzzy sets of TRA will be, if we use the geometric average:

C1=[(0.6x0.6x0.4)1/3, (0.5x0.4x0.4)1/3, (9x9x9)1/3, (0.4x0.4x0.3)1/3]= [0.524, 0.43, 9, 0.363]

C2=[(0.5x0.5x0.4)1/3, (0.5, 0.4, 0.3)1/3, (13x13x13)1/3, (0.3x0.2 x0.2) 1/3]= [0.033, 0.02, 13, 0.04]

a) Vector Formalism: Another formal way of considering the mental model is to use vector spaces. In this case it can be assimilated to a column vector in which the fuzzy sets has been aligned within that column; thus the vector has as number of rows 4xs, being s the number of learning problems corresponding to that model.

Using this formalism it is easy to carry out the operations set out in the previous paragraph, which are CONS and TRA. So the result of CONS($Ment_{ij}$, $Ment_{il}$)= $Ment_{ijl}$ can be obtained as the difference between the numerical values of both vectors.

Given that all numerical values are <=1 except the third number of each fuzzy set that is >=1, it is convenient, though not necessary, to normalize that learning problem number by dividing it by the total number of mental problems of your family, and thus all of them will be <=1. The difference vector shows us in its positive terms the advantages achieved, and in the possible negative ones, the alarms for the emergence of new problems or deterioration of the existing ones.

New metric offered by vector space. An integral or metric evaluation that allows directly to obtain this formalism is the obtaining of the cosine of the angle that the component models of CONS present. For this we have to take into account that, given two vectors A, and B, it is verified that their scalar product A.B is equal to the product of their modules by the cosine of the angle they form; from that expression we can get the cosine of the angle they form.

4) Implementation of these new operations: If we consider each fuzzy set as a column vector, whose components are real numbers belonging to the set [0,1], except for the third which is an integer number, we can construct the matrix formed by these vector columns of learning problems of the autistic child. Thus, each column of the matrix contains all the column vectors of the different learning problems of a mental model.

Considering the previous example of a trajectory consisting of three mental models and their fuzzy sets, which in this case are two, corresponding to two mental problems, the matrix obtained is (Table VI):

TABLE VI. CHILD'S MATRIX

Fuzzy sets	Mental models		
	Mentij	***Mentil***	***Mentis***
Fuzzy set 1	0.6	0.6	0.4
	0.5	0.4	0.4
	8	8	8
	0.4	0.3	0.3
Fuzzy set 2	0.7	0.3	0.3
	0.5	0.4	0.2
	6	6	6
	0.5	0.2	0.2

As you can see, each column vector represents a mental model of the child and contains the two fuzzy sets, related to the two learning problems.

Now we can apply the TRA operation, which using the arithmetic average produces us the vector column: [0.533 0.4333 8 0.333 0.4 333 0.3666 6 0.3]

Similarly, another column vector would be obtained using the geometric average, which would be the cubic root of the product of the terms of each row.

E. DIAUTIS III: Agent Architecture

The various types of agent used in DIAUTIS III are as follows:

1) The control agent is responsible for cloning or eliminating agents, coordinating performance, and resolving conflicts. It grants control to affective or pedagogic agents when requested, centralizes communication with users, and handles initial interviews with parents or instructors. It can communicate with any agent.

2) The design agent is responsible for designing tests based on clinical team indications and criteria. They maintain a database of tests and prepare test records. They communicate with control agents, interface, affective, cognitive, and possibly pedagogic agents, and direct diagnosis achievement. They also design future normalized diagnosis protocols based on qualifications, experience, and medical equipment indications, including the possibility of incorporating affective and pedagogic agents.

3) Cognitive agents (type A) manage voice and sound analysis, collaborating with other pedagogic agents. They receive test information and have a sonorous model of the world. They communicate evaluation inputs from negative elements, design agent, and children's group.

4) Cognitive agents (type B) analyze and answer child's appearance tests, collaborating with other agents and control and design agents. They communicate with evaluation and group evaluation agents, ensuring test evaluation accuracy.

5) Cognitive agents (type C) monitor child movements, analyze movements, and use intelligent toys. They have a spatial model and perform tasks and communications similar to agents A and B.

6) Affective agents analyze a child's emotional state during tests using sensor data, communicate with cognitive agents, and sometimes, under clinical advice, improve their emotional state.

7) Pedagogic agents collaborate on test demonstrations, assuming test voice or recommendations, using simple voices, animals, or friendly objects to gain child confidence and interest.

8) Rule learning agents present various tests using videos, images, or intelligent simulations, similar to cognitive agents, with tasks and communication obligations.

9) The evaluation agent is responsible for obtaining tests, integrating evaluations, and defuzzing them into categories, while maintaining a child's cognitive and affective record and diverse diagnosis possibilities.

10) The group evaluation agent, when present, performs the same tasks as the evaluation agent but differentiates the child suffering from the rest of the group members.

11) The interface agent personalizes the interface based on test situations and the child's state, using elements like screen color, scene background, potential pedagogic agents, messages, and sounds.

F. Validation

Previous experience in assessing Intelligent E-learning Systems [35, 36, 37], has been initially applied and enhanced to take into account the specific features of this platform. According to our methodology already established [39,39], quality assessment of DIAUTIS III has been obtained by working at two different levels: the functional evaluation level and the overall evaluation level. So far, no child has participated in the quality assessment tests. Instead, some errors o anomalous behaviours, randomly chosen, similar to those presented by autistic children, have been introduced as the child's response to the tests, in order to simulate the diagnosis process.

The first level or functional level, with a more reduced scope, follows closely traditional methodologies. The following tests have been carried out:

1) Experimental cross-check of functions (agents) and of auxiliary hardware: design of follow-up exercises with several objects, language questions, social or affective behaviour, by four groups of five observers.

2) Experimental cross-check of the design of simple tests according to doctors' indication and initial test qualification such as: colour, object, words, questions, etc.

3) Experimental cross-check of the design of collection of tests from doctor's indications by using ENT and its built-in criteria.

4) Experimental cross-check of the fuzzy tests evaluations by five observers and members of the clinical equipment.

5) Experimental cross-check of the tests integration into the category fuzzy set and into the child's model by five clinical equipments.

The second level or overall evaluation requires a more creative approach. It includes the evaluation of four different aspects: overall functionality tested by six groups of experts and clinical members, reliability considering that the system learning capability will allow the platform to change parameters according to its experience, evidential validity [40, 41, 42]. In addition, consequential validity [43, 44].

G. Results

The information highlighted in this paper, shows that DIAUTIS III is a software framework that achieves the following objectives:

- Capable of getting for the first time the mental model of an autistic child as a fuzzy graph, by means of affective computing and fuzzy logic.
- Facilitating of the comparison of previous child diagnoses and facilitating their results.
- Providing a comprehensive history of each test, including its usage, age, and results, to understand test difficulty, adequacy at specific ages, and its correlation with autism severity.
- Providing a technique for assessing learning deficits in autistic children that allows the degree of assimilation of a concept by a preschool child to be measured, based on our KASP methodology, that already tested and used, allows the elaboration of the tests, based on the theory of serious games.
- Incorporating the application of the mathematical theory of categories to establish the formalization of the mental model and several other applications.
- Evaluating the difference existing between two mental models, or the trajectory of a collection of mental models of the Autistic child using the metrics of the categories theory.

VI. FUTURE WORKS

The authors suggest using a fuzzy logic approach to establish an efficient evaluation of the KASP based learning system and with the aid of metrics of category theory can extend the functionalities of DIAUTIS III.

For this purpose, we suggest as future works:

- Layout the intensive tests for children, involving doctors, medical equipment, schools, families, and associations. This system can provide crucial aid to all individuals involved with autism, as assessed in previous assessments.
- Design of normalized protocols or tests for autism diagnosis may eventually contribute to the adoption of standards, with the relevance and discriminant power of these tests determined by their history and results.
- Enhance DIAUTIS III with new sensors, software, functionalities, diagnostic tests, and algorithms of artificial intelligence, presenting a new world of possibilities due to rapid technological advancements.
- Use the category theory for a better, deeper, and balanced understanding, of the dimensions and characteristics of autism Among the many ways in which this better understanding could be achieved, it is necessary to identify patterns, relationships, and underlying structures in the behavior and cognition of autistic children.
- Another line of future work is aimed at the analysis of feelings. This is a recent research issue that opens with

great possibilities. So far it has been applied to the analysis of the feelings contained in the written language that is usually sent to social networks to extract opinions and even future actions. It could also be applied to extract deep motivations from the autistic child, analyze them, and even try to modify them where appropriate.

VII. Conclusions

Multi-agent systems with learning, fuzzy, and affective skills have great potential in autism diagnosis and patient assistance, particularly in learning, as they can help with various issues.

DIAUTIS III is a platform designed to assist individuals with autism, particularly in diagnosing this condition.

DIAUTIS III is a diagnostic tool that considers a child's affective and anomalous behavior to accurately diagnose and understand the severity of their illness.

DIAUTIS III is a framework that provides:

1) The mathematical theory of categories constitutes an appropriate formalism to represent the mental model of the autistic child from the blurred graph, and formulate operations such as CONS and TRA that allow evaluating the difference existing between two mental models, or the trajectory of a collection of mental models of the Autistic child.

2) Other formalisms, such as the vector spaces, also allow obtaining the difference between two mental models, as it may be from the cosine of the angle formed by the vectors representing them.

3) The vector formalism also allows a convenient implementation to obtain the CONS and TRA final values.

However, some limitations concerning the results of the platform have to be established:

- DIAUTIS III cannot be applied for the study of other syndromes besides autism.
- It works according to the DSM-V Manual, therefore the platform has to assume all future changes advised by APA.

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