k-Modulus Method for Image Transformation

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***Abstract*—In this paper, we propose a new algorithm to make a novel spatial image transformation. The proposed approach aims to reduce the bit depth used for image storage. The basic technique for the proposed transformation is based of the modulus operator. The goal is to transform the whole image into multiples of predefined integer. The division of the whole image by that integer will guarantee that the new image surely less in size from the original image. The k-Modulus Method could not be used as a stand alone transform for image compression because of its high compression ratio. It could be used as a scheme embedded in other image processing fields especially compression. According to its high PSNR value, it could be amalgamated with other methods to facilitate the redundancy criterion.**

I. Introduction

Image transformations have received a significant consideration due to its importance in computer vision, computer graphics, and medical imaging [1]. The transformed images are information about the dissimilarities between input image and the output image and trying to make these dissimilarities as closely as possible [2]. Image transformations are the extremely significant process in image processing and image analysis. Image transforms are used in image enhancement, restoration, reconstruction, encoding and description. A spatial transformation is an image processing process that re-identifies the spatial relationship between pixels in an image. This procedure will make the manipulation of an image layout is easier concerning image size and shape. The mathematical concept of image transformation is a powerful procedure that is recruited in various fields in image processing disciplines and the most important one is image compression [3]. The pixels pattern in natural images is not random, but has a noticeable statistical regularity. This regularity is clearly may be touched at the single pixel level or the block of pixels [4]. Redundancy in data is exemplary reachable through transforming the original data from one representation to another [5]. The basic idea in image compression is that the new pixels are almost smaller than the original pixels. Alternatively, the decoder side enters the transformed pixels and reconstructs the original image by implementing the inverse transform procedure [3]. The image transformation could be imagined as a mapping from one coordinates to another transformed coordinates. It must be mentioned that, there are two types of image transformations according to the reversibility procedure. The first one is lossless transform and the other one is the lossy transform. In case of lossless transform, the original pixels are completely reversible. While in the lossy transforms, the genuine pixels could not be obtained because lossy transformations are fully irreversible [6]. In principle, the idea behind image transformation is the representation of the pixels in the original image in terms of fewer bit length than the original bit stream. Transformation of an image usually decreases the entropy of the original image by eliminating the redundancies of the image pixel sequence [5].

The remainder of the paper is organized as follows. Section II discusses the spatial transformation for image processing. The proposed k-Modulus Method (k-MM) has been introduced by details in section III. Section IV presents the experimental results when applying the proposed k-Modulus Method to a variety of test images. The conclusion of this paper was presented in section V.

II. Spatial Transformation Preliminaries

Spatial transformations are important in many aspects of functional image analysis [7]. The main concept in spatial transformation is to find a mapping for each pixel lies in the original integer lattice to a new grid used to resample the input pixels [1]. The relationship between the image and its corresponding mapping is depending on the structure of the images themselves [4]. Therefore, the selection of the suitable representation of image is only part of the solution for transforming the original image into a suitable mapping for ulterior processing [6]. According to [1], the geometric transformation is completely determined by the spatial transformation because analytic mapping is bijective, i.e. one-to-one and onto. The principle motivation behind transformation could be summarized in two aspects. The firs aspect is that, transformation yielding more efficient representation of the original samples. On the other hand, the transformed pixels must demand fewer bits [8]. Spatial transformation mappings may take different forms and this is depends on the type of application [1]. The choice of the mapping model is essential to obtain reliable results; this choice must be considered seriously and assessed before any processing. As mentioned above, the transformation functions depend on the nature of the scene and on the acquisition system used. However, under some assumptions, and before applying a complicated model, it may be useful to reach the idealistic mapping solicited by consecutive less and less 'rigid' mapping models [9].

The general mapping function can be given in the following form. Let f be the original image and g is the transformed image then there exist an operator T such that:

$$T : f \longrightarrow g, \qquad f, g \in R^{N\times M} \tag{1}$$

and

$$g(x, y) = T\left[f(x, y)\right] \tag{2}$$

where N and M are the image dimensions. It must be mentioned that the operator T is a one-to-one operator, i.e. for each pixel in f there exist a corresponding one and only one pixel is g. Moreover, the one-to-one transformation could be shown in figure 2.

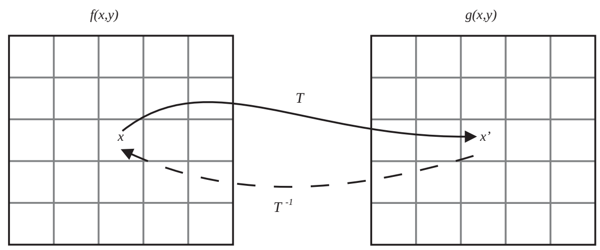

Fig. 1. Spatial one-to-one transformation

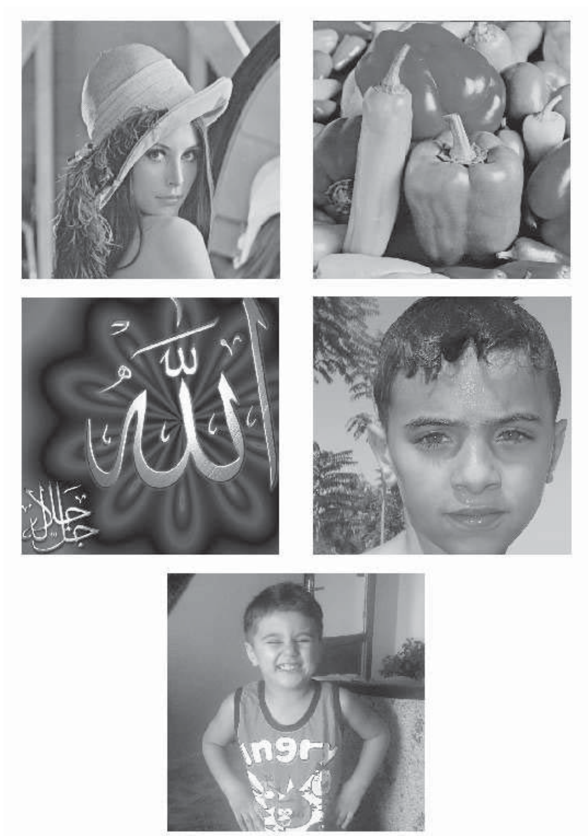

Fig. 2. Test Images (Lena,Peppers, Allah, Boy1, and Boy2)

III. Proposed Transformation

The proposed transform is actually a generalization of the Five Modulus Method (FMM) proposed by [10]. The basic idea behind FMM is to transform the original image into multiples of five only. Hence, it was called Five Modulus Method (FMM). Now, a question may arise and that is why not to generalize FMM into k-Modulus Method (kMM). In another words, instead of using number five only why not to use any integer according to the desired precision?. Therefore, the proposed transform has been established. According to [10], the human eye does not differentiate between the original image and the transformed FMM image. Moreover, for each of the Red, Green, and Blue arrays in the color image are consisting of pixel values varying from 0 to 255. The cardinal impression of k-Modulus Method is to transform the all pixels within the whole image into multiples of k. According to figure 4, it is clear that the k-Modulus Method transformation, up to the 10-Modulus Method, does not affect the Human Visual System (HVS). Now, to simplify the above idea, an illustrative example will be presented. Suppose k=2, then the original range of values 0 to 255 will be transformed as 0, 2, 4, 6,. . . , 254. Similarly, when k=3, the transformed range is 0, 3, 6, 9, 12, . . . , 255. The same previous procedure may be applied for any integer k. The k-Modulus method have been applied to k=2,3,4,. . . 20 have been calculated and presented in figure 3. Obviously, increasing k will lead to high distortion in the output image. Hence, by using try and error, we can see that up to k=10 the human eye could not differentiate between the original and the transformed images. According to figure 3, it can be seen that a selective k values (2, 3, 5, 7, 10, 15, 20) have been used to transform a random 6×6 block from Lena image using k-Modulus Method.

A. k-Modulus Method Bit Depth

Bit depth refers to the color information stored in an image. More colors could be stored in an image whenever there is high bit depth in the image. In the case of black and white, it contains one bit either 0 or 1. Hence, it can only show two colors which are black and white. Furthermore, an 8 bit image can store 2^8 which is equal to 256 possible colors, etc. It must be mentioned that, the bit depth specify image size. As the bit depth increase, image size also increases because each pixel in the image requires more information [6]. One way to think about bit depth is to consider the difference between having the capability to make measurements with a ruler that is accurate to the nearest millimeter, compared with one that can only measure to the nearest centimeter [11].

In this paper, a general formula to extract the exact bit depth for each of the k-Modulus Methods has been derived as:

$$n = \left\lfloor \frac{256}{k} \right\rfloor + 1 \tag{3}$$

where k is an integer number ($k = 2, 3, \ldots$).

As with resolution, bit depth determines file size. The higher the depth, the greater the file size. It must not be confounded with the amount of actual colors within an image. Therefore, an image with 25 colors may be created with 16 colors. Hence, we may have thousands instead of millions of possibilities. This would obviously lead to increase in file size and that may not be necessary. A versed understanding of bit depth is essential to any graphic or multimedia application [12].

IV. Experimental Results

In this article, five test images have been tested using the proposed k-Modulus Method. In figure 4, the implementation of the k-Modulus Method was applied using k=2,3,,20 for Lena image. It is clear that, the human eye could distinguish

	175	192	203	211	215	220							
	179	195	205	210	214	220							
Original	181	198	207	211	217	221							
6×6	188	201	208	215	218	219							
Block	194	203	212	215	215	218							
	193	204	212	215	217	217							
	197	209	213	216	216	218							
	174	192	202	210	214	220		87	96	101	105	107	110
	178	194	204	210	214	220		89	97	102	105	107	110
	180	198	206	210	216	220		90	99	103	105	108	110
2-MM	188	200	208	214	218	218	→	94	100	104	107	109	109
	194	202	212	214	214	218		97	101	106	107	107	109
	192	204	212	214	216	216		96	102	106	107	108	108
	196	208	212	216	216	218		98	104	106	108	108	109
	174	192	204	210	216	219		58	64	68	70	72	73
	180	195	204	210	213	219		60	65	68	70	71	73
	180	198	207	210	216	222		60	66	69	70	72	74
3-MM	189	201	207	216	219	219	→	63	67	69	72	73	73
	195	204	213	216	216	219		65	68	71	72	72	73
	192	204	213	216	216	216		64	68	71	72	72	72
	198	210	213	216	216	219		66	70	71	72	72	73
	175	190	205	210	215	220		35	38	41	42	43	44
	180	195	205	210	215	220		36	39	41	42	43	44
	180	200	205	210	215	220		36	40	41	42	43	44
5-MM	190	200	210	215	220	220	→	38	40	42	43	44	44
	195	205	210	215	215	220		39	41	42	43	43	44
	195	205	210	215	215	215		39	41	42	43	43	43
	195	210	215	215	215	220		39	42	43	43	43	44
	175	189	203	210	217	217		25	27	29	30	31	31
	182	196	203	210	217	217		26	28	29	30	31	31
	182	196	210	210	217	224		26	28	30	30	31	32
7-MM	189	203	210	217	217	217	→	27	29	30	31	31	31
	196	203	210	217	217	217		28	29	30	31	31	31
	196	203	210	217	217	217		28	29	30	31	31	31
	196	210	210	217	217	217		28	30	30	31	31	31
	170	190	200	210	210	220		17	19	20	21	21	22
	180	190	200	210	210	220		18	19	20	21	21	22
	180	200	210	210	220	220		18	20	21	21	22	22
10-MM	190	200	210	210	220	220	→	19	20	21	21	22	22
	190	200	210	210	210	220		19	20	21	21	21	22
	190	200	210	210	220	220		19	20	21	21	22	22
	200	210	210	220	220	220		20	21	21	22	22	22
	180	195	210	210	210	225		12	13	14	14	14	15
	180	195	210	210	210	225		12	13	14	14	14	15
	180	195	210	210	210	225		12	13	14	14	14	15
15-MM	195	195	210	210	225	225	→	13	13	14	14	15	15
	195	210	210	210	210	225		13	14	14	14	14	15
	195	210	210	210	210	210		13	14	14	14	14	14
	195	210	210	210	210	225		13	14	14	14	14	15
	180	200	200	220	220	220		9	10	10	11	11	11
	180	200	200	220	220	220		9	10	10	11	11	11
	180	200	200	220	220	220		9	10	10	11	11	11
20-MM	180	200	200	220	220	220	→	9	10	10	11	11	11
	200	200	220	220	220	220		10	10	11	11	11	11
	200	200	220	220	220	220		10	10	11	11	11	11
	200	200	220	220	220	220		10	10	11	11	11	11

Fig. 3. A random 6×6 block from Lena image (Left) k-Modulus Method ($k = 2, 3, 5, 7, 10, 15,$ and 20) (Right) The division of block by the corresponding k

TABLE I. BIT DEPTH OF k-MODULUS METHOD

k-Modulus Method	Range	Binary representation	Length of pixel
2-MM	0..128	10000000	8
3-MM	0..85	1010101	7
4-MM	0..64	1000000	7
5-MM	0..51	110011	6
6-MM	0..42	101010	6
7-MM	0..36	100100	6
8-MM	0..32	100000	6
9-MM	0..28	11100	5
10-MM	0..25	11001	5
11-MM	0..23	10111	5
12-MM	0..21	10101	5
13-MM	0..19	10011	5
14-MM	0..18	10010	5
15-MM	0..17	10001	5
16-MM	0..16	10000	5
17-MM	0..15	1111	4
18-MM	0..14	1110	4
19-MM	0..13	1101	4
20-MM	0..12	1100	4

dissimilarities up to k=10. Higher k may produce noticeable distortion. Moreover, for more illustration, a magnified image of Lena for k=3,5,7,10,15,20 have been presented in figure 6. Also, according to figure 5, the image histogram for the same previous k values k=3,5,7,10,15,20 have been obtained. Here, the same assumption that was stated previously, and that is, up to k=10 the shape of the histogram is approximately similar to the original histogram. Finally, The Peak Signal to Noise Ratio (PSNR) [13] was used to measure the dissimilarities between the transformed and the original images, table (II).

TABLE II. PSNR VALUES OF k-MODULUS METHOD

k-MM	Lena	Peppers	Allah	Boy1	Boy2
2-MM	50.7787	50.4276	51.1058	50.9156	51.1518
3-MM	49.5941	49.1845	49.8984	49.6841	49.8853
4-MM	46.0309	45.7001	46.4233	46.1599	46.3588
5-MM	44.7639	44.4059	45.0469	44.9447	45.1021
6-MM	42.7984	42.4311	43.0906	42.9874	43.1331
7-MM	41.7680	41.4927	41.8433	41.9210	42.0895
8-MM	40.4774	40.0450	40.6093	40.6246	40.7477
9-MM	39.5383	39.2124	39.7726	39.9181	39.9031
10-MM	38.4859	38.1514	38.8397	38.7334	38.8230
11-MM	37.8295	37.4656	38.5854	38.2731	38.1404
12-MM	36.9491	36.7879	37.2798	37.0228	37.2974
13-MM	36.4654	35.8355	36.9229	36.5131	36.6772
14-MM	35.9254	35.3525	35.9268	35.9884	35.9164
15-MM	35.0014	34.8275	35.1099	35.4527	35.4560
16-MM	34.7193	34.4729	35.2276	34.6802	34.8957
17-MM	33.9161	33.7502	34.7368	34.3753	34.2375
18-MM	33.6590	32.9851	33.8404	33.7727	33.7870
19-MM	33.0598	32.7615	33.2398	32.9910	33.4654
20-MM	32.5316	32.3079	33.0107	32.8999	32.8903

According to [14][15], the typical values for the PSNR in lossy image and video compression are between 30 and 50 dB, where higher is better . Hence, it is clear that the PSNR values in table II are quite acceptable.

V. CONCLUSION

In this paper, a novel spatial image transform has been presented. The proposed transform use modulus operator in a way that transform the whole image array into multiples of one and only one integer k. As a main conclusion from this article is that, the k-Modulus Method could helps in image compression as a scheme but not as stand alone image compression technique. The graphical examples demonstrated that the k-Modulus Method produce better results when k is up to 10. Depending on the application used, the designer may control k. Therefore, higher k (more than 10) could be used whenever there is a need to a low resolution images.

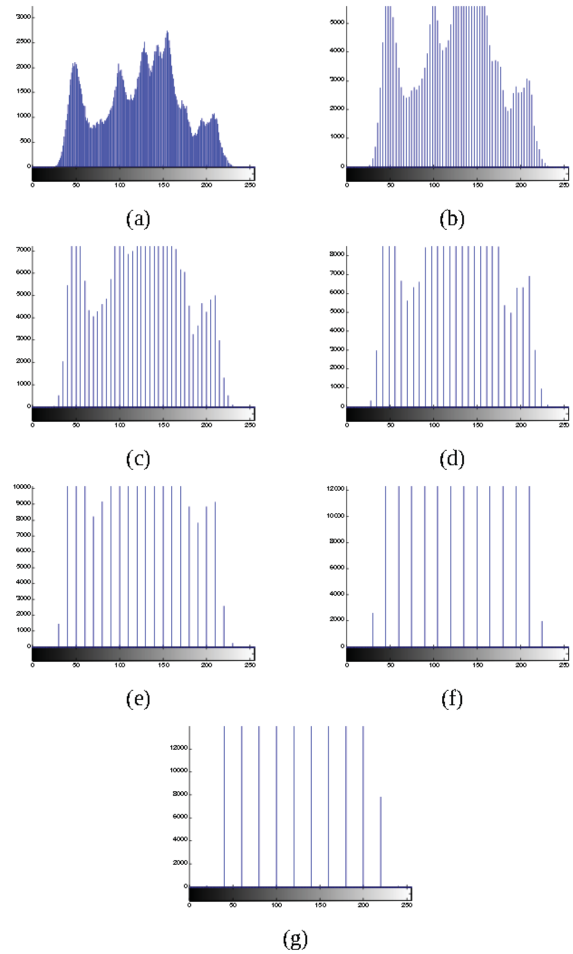

Fig. 5. Image Histogram (a) Original (b) 3-MM (c) 5-MM (d) 7-MM (e) 10-MM (f) 15-MM (g) 20-MM

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Fig. 4. k-Modulus Method (First row) Original, 2-MM, 3-MM, 4-MM, 5-MM (Second row) 6-MM, 7-MM, 8-MM, 9-MM, 10-MM (Third row) 11-MM, 12-MM, 13-MM, 14-MM, 15-MM (Fourth row) 16-MM, 17-MM, 18-MM, 19-MM, 20-MM

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Fig. 6. Top left (3-MM) Top right (5-MM) Middle left (7-MM) Middle right (10-MM) Bottom left (15-MM) Bottom right (20-MM)