Diagnosis of Wind Energy System Faults Part I : Modeling of the Squirrel Cage Induction Generator

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Abstract—**Generating electrical power from wind energy is becoming increasingly important throughout the world. This fast development has attracted many researchers and electrical engineers to work on this field. The authors develop a dynamic model of the squirrel cage induction generator exists usually on wind energy systems, for the diagnosis of broken rotor bars defects from an approach of magnetically coupled multiple circuits. The generalized model is established on the base of mathematical recurrences. The winding function theory is used for determining the rotor resistances and the inductances in the case of n- broken bars. Simulation results, in Part. II of this paper, confirm the validity of the proposed model.**

Keywords—Induction Generator; Rotor Broken Bars; Faults Diagnosis; MCSA

I. INTRODUCTION

In a general way, the problem of fault diagnosis consists in the determination of the fault type with as much as possible details as the fault size, location and time of detection. From these important details, various approaches using mathematical models are developed for more than three decades, and it has a rich literature of hundreds of papers including many surveys [1, 2].

For the electrical machines, the faults, and their diagnosis techniques are reported in a great number of papers and categorized in three important sections:

Coupled magnetic circuit theory. This theory uses the finite element method associated the state space models, based on the Maxwell's equations by taking account of the machine geometry and the magnetic properties of materials. This method offers considerable advantages, such as the modeling of the topology of the coupled magnetic circuits and the capacity to model magnetic saturation on a high level of precision. However, the finite element method was not the privilege of all the researchers for the simulation of the rotor defects of the induction machines and remains today, little used because of the high computing time and the absence of on line knowledge of the faults [3, 4].

Multiple coupled circuit approach. The approach of the multiple coupled electric circuits is based on the Kirchhoff's laws. If in this method, magnetic saturation is neglected, it is however possible to observe the defects to the rotor and the stator, thanks to the design of a detailed model of an induction machine having m-stator circuits and n-rotor bars, [5-7]. After having drawn up the equivalent

scheme under operation normal conditions, it is possible then to write the equations of the meshed network and to simulate the faults in the rotor by the breaks of the rotor bars and/or an end ring segment.

Spectral analysis and parameter estimation. The procedure is based on the spectral analysis of the stator current signature and on the parameter estimation respectively, to identify the rotor bar defects. It is commonly called "Motor Current Signature Analysis.". The stator current spectra shows that the broken rotor bars give rise to a sequence of sidebands around the supply frequency f [8]. These sidebands are at $f_m=f(1\pm2ks)$, $k=1,\ 2\ldots$ The lower sideband is specifically due to a broken bar, and the upper sideband is due to consequent speed oscillation. Another signal that has been proposed for motor fault analysis is an air-gap torque. Frequency components are observing at characteristic frequencies $f_m=f(2ks)$. The fault spectra are dependent on the slip, which changes with the load.

The air-gap torque spectrum and the current stator spectrum are a potential signature that can be used by on line fault diagnosis. The analytical knowledge in the process is also used to produce quantifiable, analytical information. The first characteristic values are generated by the process analysis using mathematical process models together with parameter estimation, state estimation and parity equation methods. Characteristic values are parameters, state variables or residuals. As an example: Broken bar detection on an induction machine using state and parameter estimation techniques have been reported in [9, 10].

This paper presents a generalized induction machine model for the diagnosis of rotor defects by an approach of magnetically coupled multiple circuits (using ideas *A.R. Muñoz and T. A. Lip*, [11]). The developed inductances (self and mutual inductances) were given for rotors having an unspecified number of broken rotor bars, by using of mathematical recurrences. Considering the configuration of the rotor, it is more realistic to model the cage as m-identical magnetically coupled circuits. Moreover, one advantage of this approach is that it is applicable for all numbers of broken bars per pole pair. From where the need for determining inductances and resistances to the rotor meshes affected by the defect. Experimental results and simulation in Part. II of this paper confirm the validity of the proposed approach.

II. MODELING OF SCIG BY A MULTIPLE COUPLED CIRCUIT APPROACH

The three-phase model does not make it possible to know the actual values of the currents circulating in the rotor bars and thus does not allow determining the defects with the rotor. This is why the model multi-windings were developing. One will represent then, a model of the rotor as being made up with as many phases as bars; what results in regarding the currents circulating in the end ring segments as rotor phase currents [12, 13].

By taking account of the following assumptions:

- ✓ negligible saturation,
- ✓ sinusoidal distribution of the EMF,
- ✓ uniform air-gap,
- ✓ negligible inter-bar currents

A. Stator equations

The stator equations are written by:

$$v_s = R_s.i_s + \frac{d\phi_s}{dt} \qquad (1)$$

With: $v_s = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix}^T$, $i_s = \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}^T$

The resistance matrix (diagonal (3x3)) contains resistance of each winding:

$$R_s = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}$$

And the total stator flux is:

$$\phi_s = \phi_{ss} + \phi_{sr} \qquad (2)$$

$$\phi_s = L_s i_s + L_{sr}.i_r \qquad (3)$$

Where: ϕ_{ss} is stator flux due to the stator currents and ϕ_{sr} is stator flux due to the rotor currents.

With:

$$L_s = \begin{bmatrix} L_{ls} + L_{ms} & -\dfrac{L_{ms}}{2} & -\dfrac{L_{ms}}{2} \\ -\dfrac{L_{ms}}{2} & L_{ls} + L_{ms} & -\dfrac{L_{ms}}{2} \\ -\dfrac{L_{ms}}{2} & -\dfrac{L_{ms}}{2} & L_{ls} + L_{ms} \end{bmatrix}$$

Where L_{ls} and L_{ms} are respectively leakage and magnetizing inductances of stator windings. Magnetizing inductance L_{ms} for a winding having N_s number of stator phase turns in series, is given by:

$$L_{ms} = \frac{\mu_0 lr}{g} N_s^2 \left(\frac{\pi}{4} \right) \qquad (4)$$

Leakage inductance is computed by [11].

Each rotor mesh is made of two adjacent bars and the end-ring segment, which connect them and is magnetically coupled with all the other rotor meshes and the three-phase stator. Mutual inductance matrix $(3 \times N_r)$ stator-rotor is:

$$L_{sr} = \begin{bmatrix} L_{a1} & L_{a2} & \cdots & L_{aN_r} \\ L_{b1} & L_{b2} & \cdots & L_{bN_r} \\ L_{c1} & L_{c2} & \cdots & L_{cN_r} \end{bmatrix}$$

The inductances are computed by using winding function theory "WFT". This method supposes that there is no symmetry according to the winding function theory [6]. Mutual inductance between two unspecified reels "i" and "j" of the machine can be calculated from Eq.(5), by supposing that iron permeance is infinite [14].

$$L_{ij}(\theta) = \mu_0 lr \int_0^{2\pi} g^{-1}(\varphi,\theta) N_i(\varphi,\theta) N_j(\varphi,\theta) d\varphi \qquad (5)$$

Where θ represents the rotor position compared to a given reference (related to the stator). φ is a particular angular position along the interior surface of the stator. $g^{-1}(\varphi, \theta)$ is the inverse function of the air-gap, if we then suppose that the air-gap is constant and small compared to rotor ray, the function is equal to $1/g$. The term $N_i(\varphi,\theta)$ is known as winding function and represent the spatial distribution of the MMF along the air-gap for a unit current circulating in winding "i". Leakage inductances are calculated by only setting $i=j$. From winding function theory, one can determine the MMF of air-gap produced by a current i_a circulating in winding "a" for that it position of the air-gap, as follows:

$$F_a = N_a(\theta).i_a \qquad (6)$$

The created flux in a reel, having N_b whorls, by the current i_a is given by:

$$\phi = F. \lambda \qquad (7)$$

Where λ and F are respectively the permeance and air-gap magneto-motive force. The differential flux across the air-gap from rotor to stator through the cross section $(r.d\theta.l)$ is:

$$d\phi = F_a (\theta).\mu_0.l.r.d\theta/g \qquad (8)$$

The differential flux created in the reel B is:

$$d\Lambda_{ba} = N_a (\theta).N_b (\theta)i_a (\theta).\mu_0 .l.r.d\theta/g \qquad (9)$$

From where the total fluxes expression:

$$\Lambda_{ba} = \mu_0.l.r.i_a.g^{-1} \int_{\theta_{b1}}^{\theta_{b2}} N_a(\theta).N_b(\theta)d\theta \qquad (10)$$

From total flux, it is then possible to determine inductance between reel a and reel b:

$$L_{ba} = \frac{\mu_0.l.r}{g} \int_{\theta_{b1}}^{\theta_{b2}} N_a(\theta).N_b(\theta).d\theta \qquad (11)$$

By considering the distribution of the stator windings and the rotor bars, the winding function for the stator phases is:

$$N_a = \frac{N_s}{2} cos(\theta)$$

$$N_b = \frac{N_s}{2} cos\left(\theta - \frac{2\pi}{3}\right)$$

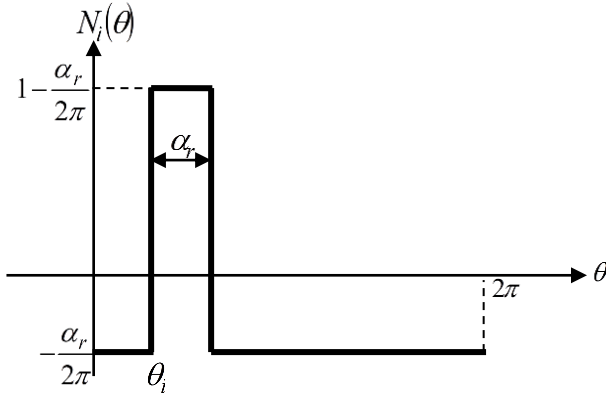$$N_c = \frac{N_s}{2} cos\left(\theta + \frac{2\pi}{3}\right)$$

Fig. 1.   Rotor loop winding function of healthy generator

The winding function for the ith mesh of the rotor is:

$$N_i = \begin{cases} -\alpha_r / 2\pi & 0 < \theta \le \theta_i \\ 1 - \alpha_r / 2\pi & \theta_i < \theta \le \theta_{i+1} \\ -\alpha_r / 2\pi & \theta_{i+1} < \theta \le 2\pi \end{cases}$$

Where α_r is the angle between two adjacent bars and θ_i the angle of the bar i^{th}. By replacing $g^{-1}(\varphi,\theta)$ and $N(\varphi,\theta)$ previously defined in the expression (7), for the phase "a", one will determine following inductances:

$$L_{ai} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta) N_i(\theta) d\theta =$$

$$\frac{\mu_0 lr N_s}{2g} \left(-\int_0^{\theta_i} \frac{\alpha_r}{2\pi} cos\,\theta d\theta + \int_{\theta_i}^{\theta_{i+1}} \left(1 - \frac{\alpha_r}{2\pi}\right) cos\,\theta d\theta - \int_{\theta_{i+1}}^{2\pi} \frac{\alpha_r}{2\pi} cos\,\theta d\theta \right)$$

$$(12)$$

After integration this expression yields

$$L_{ai} = \frac{\mu_0 lr}{g} \frac{N_s}{2} \left(sin\,\theta_{i+1} - sin\,\theta_i\right) \qquad (13)$$

θ_i can be set in the following form: $\theta_i = \theta_r + (i-1)\alpha_r$, with θ_r is an arbitrary angle.

so :

$$L_{ai} = \frac{\mu_0 lr}{g} \frac{N_s}{2} \left[sin(\theta_r + i\alpha_r) - sin(\theta_r + (i-1)\alpha_r)\right] \qquad (14)$$

After transformation by trigonometric relations:

$$L_{ai} = \frac{\mu_0 lr N_s}{g} sin\frac{\alpha_r}{2} cos\left(\theta_r + \left(i - \frac{1}{2}\right)\alpha_r\right) \qquad (15)$$

Let:

$$L_m = \frac{\mu_0 lr N_s}{g} \qquad \text{with} : \ \delta = \frac{\alpha_r}{2}$$

We obtain:

$$L_{ai} = L_m sin\delta cos(\theta_i + \delta) \qquad (16)$$

It is then easy to find mutual inductances for two other phases.

B. Rotor equations

For needs for simple comprehension, the rotor cage is modeled by an equivalent circuit containing $k+1$ magnetically meshes. The rotor bars are numbered from 1 to $k+1$.

It is to be considered that each mesh is defined by two adjacent bars of the rotor and connected between them by end-ring segment. Moreover, each rotor bar and end ring segment are replaced by an equivalent circuit represented by a resistance and an inductance, as the Fig.2 shows it.

From these equivalent circuits of a squirrel cage rotor, one can write the following equations:

$$v_r = R_r.i_r + \frac{d\phi_r}{dt} \qquad (17)$$

With :

$$\phi_r = L_{sr}^T i_s + L_r.i_r \qquad (18)$$

$$v_r = \begin{bmatrix} v_{r1} & v_{r2} & \dots & v_{rk} & \dots & v_{rN_r} & v_e \end{bmatrix}^T = 0$$

$$i_r = \begin{bmatrix} i_{r1} & i_{r2} & \dots & i_{rk} & \dots & i_{rN_r} & i_e \end{bmatrix}^T$$

$$\phi_r = \begin{bmatrix} \phi_{r1} & \phi_{r2} & \dots & \phi_{rk} & \dots & \phi_{rN_r} & \phi_e \end{bmatrix}^T$$

The rotor resistance matrix (N_rxN_r) is:

$$R_r = \begin{bmatrix} R_0 & -R_b & 0 & \dots & 0 & -R_b \\ -R_b & R_0 & -R_b & \dots & . & 0 \\ 0 & -R_b & R_0 & \dots & . & . \\ .. & . & . & \dots & . & . \\ . & . & . & \dots & R_0 & -R_b \\ -R_b & . & . & \dots & -R_b & R_0 \end{bmatrix}$$

With :

$$R_0 = 2(R_b + R_e)$$
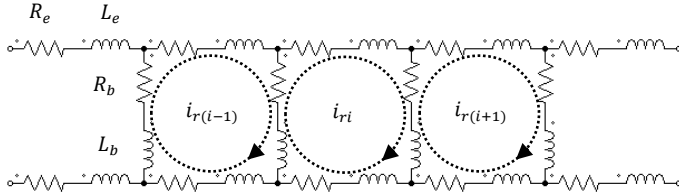
$$L_0 = L_{mr} + 2(L_b + L_e)$$

Fig. 2. Equivalent circuits of a squirrel cage rotor for Healthy induction generator

Self-inductance of the meshes is equal, and they are given by:

$$L_{mr} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_i^2(\theta)d\theta =$$

$$\frac{\mu_0 lr}{g} \left(\int_0^{\theta_i} \left(-\frac{\alpha_r}{2\pi} \right)^2 d\theta + \int_{\theta_i}^{\theta_{i+1}} \left(1 - \frac{\alpha_r}{2\pi} \right)^2 d\theta + \int_{\theta_{i+1}}^{2\pi} \left(-\frac{\alpha_r}{2\pi} \right)^2 d\theta \right)$$

$$= \frac{\mu_0 lr}{g} \alpha_r \left(1 - \frac{\alpha_r}{2\pi} \right) \qquad (19)$$

Rotor-rotor mutual inductance between two rotor meshes is deduced from Eq.(19)

$$L_r = \begin{bmatrix} L_0 & L_{12}-L_b & L_{13} & \dots & L_{1(N_r-1)} & L_{1N_r}-L_b \\ L_{21}-L_b & L_0 & L_{23}-L_b & \dots & . & L_{2N_r} \\ L_{31} & L_{32}-L_b & . & \dots & . & . \\ \vdots & . & . & \dots & . & . \\ \vdots & . & . & \dots & . & L_{(N_r-1)N_r}-L_b \\ L_{N_r1}-L_b & . & . & \dots & L_{N_r(N_r-1)}-L_b & L_0 \end{bmatrix}$$

$$\Gamma_e = \frac{P}{2} L_m \left\{ \left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c \right) \sum_{k=1}^{N_r} i_{rk} \sin(\theta_r + (k-1)\alpha_r) + \frac{\sqrt{3}}{2} (i_c - i_b) \sum_{k=1}^{N_r} i_{rk} \cos(\theta_r + (k-1)\alpha_r) \right\} \qquad (21)$$

C. Broken rotor bar faults

1) Generator with one broken bar

The rupture of a bar decreases the number of equations to the rotor of 1, because the defect imposes the following condition:

If it is the bar traversed by i_{ri} and $i_{r(i+1)}$ which is broken, one has $i_{ri} = i_{r(i+1)}$ which wants to say that current i_{ri} traverses a mesh twice broader and the mesh $i+1$ is eliminated, Fig.(3).

In the inductance matrix the line and the column $i+1$ are eliminated and the terms relating to the column i are thus recomputed by using the expression (5) by taking account of the new function of winding for the rotor mesh i,

$$N_i = \begin{cases} -\alpha_r / 2\pi & 0 < \theta \le \theta_i \\ 1 - \alpha_r / 2\pi & \theta_i < \theta \le \theta_{i+2} \\ -\alpha_r / 2\pi & \theta_{i+2} < \theta \le 2\pi \end{cases}$$

$$L_{ki} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_k(\theta)N_i(\theta)d\theta = -\frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi} \right) \qquad (20)$$

Electromagnetic torque produced by the machine can be expressed from the magnetic co-energy W_{co} as,

$$\Gamma_e = \left[\frac{\partial W_{co}}{\partial \theta} \right]_{(i_s, i_r constant)}$$

It is previously defined that inductances L_s and L_r contain only constant elements and the electromagnetic torque is a scalar quantity. The final expression of the torque is thus reduced to

$$\Gamma_e = \frac{1}{2} P i_s^T \frac{\partial L_{sr}(\theta_r)}{\partial \theta_r} i_r$$

Where P is the number of pole pairs and θ_r is the rotor displacement in electrical radians.
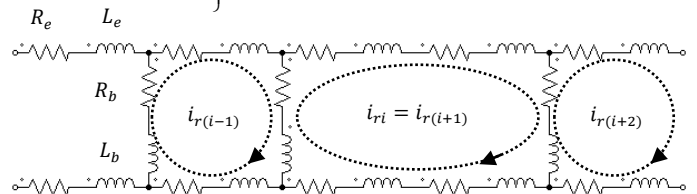

Fig. 3. Equivalent circuits of a squirrel cage rotor with one broken bar

Calculation of new mutual inductance L_{ai} between the mesh i and stator winding a:

$$L_{ai} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta)N_i(\theta)d\theta = L_m \sin(2\delta)\cos(\theta_i + 2\delta) \qquad (22)$$

The mutual inductance matrix stator rotor $(3 \times (N_r-1))$ becomes:

$$[L_{sr}] = \begin{bmatrix} L_{a1} & L_{a2} & \dots & L_{ai} & L_{a(i+2)} & \dots & L_{aN_r} \\ L_{b1} & L_{b2} & \dots & L_{bi} & L_{b(i+2)} & \dots & L_{bN_r} \\ L_{c1} & L_{c2} & \dots & L_{ci} & L_{c(i+2)} & \dots & L_{cN_r} \end{bmatrix}$$

The same calculation is applied for the ith line, the new matrix inductance of rotor is $(N_r\text{-}1)$ x $(N_r\text{-}1)$ becomes:

$$L_r = \begin{bmatrix} L_0 & L_{12}-L_b & L_{13} & \cdots & L_{1(N_r-1)} & L_{1N_r}-L_b \\ L_{21}-L_b & L_0 & L_{23}-L_b & \cdots & \cdot & L_{2N_r} \\ L_{31} & L_{32}-L_b & \cdot & \cdots & \cdot & \cdot \\ \vdots & \vdots & \vdots & L_{0i} & \vdots & \vdots \\ \vdots & \cdot & \cdot & \cdots & L_0 & L_{(N_r-1)N_r}-L_b \\ L_{N_r1}-L_b & \cdot & \cdot & \cdots & L_{N_r(N_r-1)}-L_b & L_0 \end{bmatrix}$$
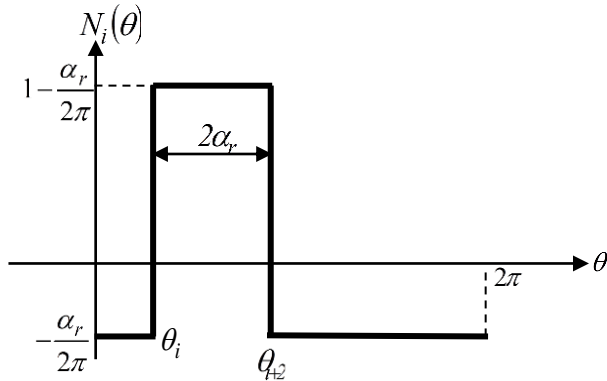

Fig. 4. Rotor loop winding function of faulty generator (1bb)

The rotor resistance matrix is affected by the break of the bar, and is obtained from the new representation $(N_r\text{-}1)$ x$(N_r\text{-}1)$ of the rotor:

$$R_r = \begin{bmatrix} R_0 & -R_b & \cdots & 0 & \cdots & -R_b \\ -R_b & R_0 & \cdots & \cdots & \cdot & 0 \\ \vdots & \vdots & \cdots & \vdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & R_{0i} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \vdots & \vdots & \vdots \\ -R_b & \cdot & \cdot & \cdots & \cdots & R_0 \end{bmatrix}$$

With:

$$R_{0i} = 2(R_b + 2R_e)$$

Determination of the new self-inductance L_{0i} of the rotor:

$$L_{0i} = L_{ii} + 2(L_b + 2L_e)$$

The self-inductance of the ith rotor mesh becomes:

$$L_{ii} = \frac{\mu_0 lr}{g}\int_0^{2\pi} N_i^2(\theta)d\theta = \frac{\mu_0 lr}{g}\alpha_r\left(2 - 3\frac{\alpha_r}{2\pi}\right) \qquad (23)$$

The mutual inductance between the kth mesh and the ith mesh of the rotor is recalculated as follows:

$$L_{ki} = \frac{\mu_0 lr}{g}\int_0^{2\pi} N_k(\theta)N_i(\theta)d\theta = -2\frac{\mu_0 lr}{g}\left(\frac{\alpha_r^2}{2\pi}\right) \qquad (24)$$

2) Generator with two adjacent broken bars

In this case, the number of rotor meshes falls by 2, if the first bar traversed by the currents i_{ri}, $i_{r(i+1)}$ and second bar traversed by the currents $i_{r(i+1)}$ and $i_{r(i+2)}$ are broken, one have

$i_{ri}=i_{r(i+1)}=i_{r(i+2)}$ means that i_{ri} traverses a mesh three times broader as shows it in Fig.(5). At the time of the break of two bars the function of winding becomes:

$$N_i = \begin{cases} -\alpha_r/2\pi & 0 < \theta \le \theta_i \\ 1-\alpha_r/2\pi & \theta_i < \theta \le \theta_{i+3} \\ -\alpha_r/2\pi & \theta_{i+3} < \theta \le 2\pi \end{cases}$$
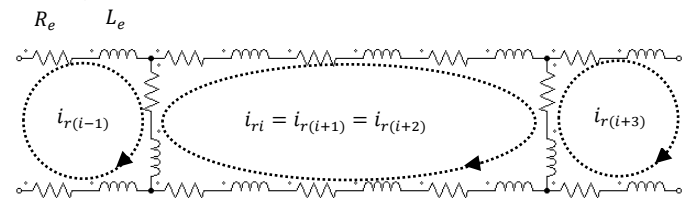

Fig. 5. Equivalent circuits of a squirrel cage rotor with two broken bars

The same type of relation applied in the case of a broken bar is used for the calculation of the new inductance and resistance matrices.

The mutual inductance matrix stator rotor $(3\text{x}(N_r\text{-}2))$ becomes:

$$[L_{sr}] = \begin{bmatrix} L_{a1} & L_{a2} & \cdots & L_{ai} & L_{a(i+3)} & \cdots & L_{aN_r} \\ L_{b1} & L_{b2} & \cdots & L_{bi} & L_{b(i+3)} & \cdots & L_{bN_r} \\ L_{c1} & L_{c2} & \cdots & L_{ci} & L_{c(i+3)} & \cdots & L_{cN_r} \end{bmatrix}$$

The same calculation is applied for the ith line, the new matrix inductance of rotor is $(N_r\text{-}2)$ x$(N_r\text{-}2)$ dimensions, with:
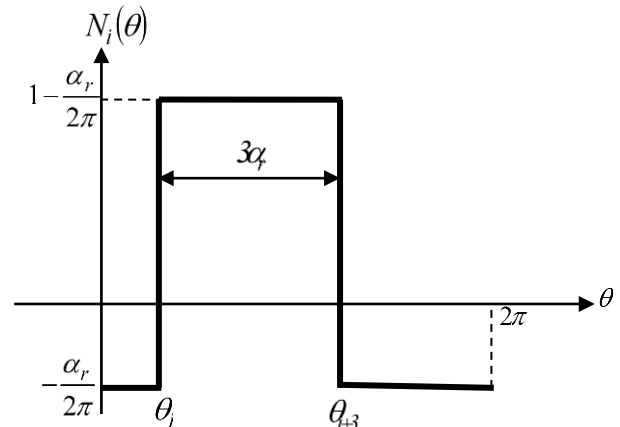
$$L_{0i} = L_{ii} + 2(L_b + 3L_e)$$

Fig. 6. Rotor loop winding function of faulty generator (2 adjacent bb)

And the self-inductance of winding i is

$$L_{ii} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_i^2(\theta) d\theta = \frac{\mu_0 lr}{g} \alpha_r \left(3 - 5\frac{\alpha_r}{2\pi}\right) \quad (25)$$

The mutual inductance between the kth mesh and the ith mesh of the rotor is recalculated as follows:

$$L_{ki} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_k(\theta) N_i(\theta) d\theta = -3\frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi}\right) \quad (26)$$

Determination of mutual inductances

$$L_{ai} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta) N_i(\theta) d\theta = L_m \sin(3\delta) \cos(\theta_i + 3\delta) \quad (27)$$

The rotor resistance matrix is affected by the break of the bar, and is obtained from the new representation $(N_r\text{-}2) \times (N_r\text{-}2)$ of the rotor, with :

$$R_{0i} = 2(R_b + 3R_e)$$

3) Generalized model by mathematical recurrences

The same reasoning, that in the case of a bar and two broken bars, is valid in the case where there are several broken successive adjacent bars. By mathematical recurrences, we determine the generalized model. The number of rotor equations will be fallen according to the number of broken bars, and the meshes concerned with the break are eliminated; the mesh i will be n time broader.

The mutual inductance matrix stator rotor $(3 \times (N_r\text{-}n))$ becomes:

$$[L_{sr}] = \begin{bmatrix} L_{a1} & L_{a2} & \dots & L_{ai} & L_{a(i+(n+1))} & \dots & L_{aN_r} \\ L_{b1} & L_{b2} & \dots & L_{bi} & L_{b(i+(n+1))} & \dots & L_{bN_r} \\ L_{c1} & L_{c2} & \dots & L_{ci} & L_{c(i+(n+1))} & \dots & L_{cN_r} \end{bmatrix}$$

The same calculation is applied for the ith line, the new matrix inductance of rotor is $(N_r\text{-}n) \times (N_r\text{-}n)$ dimensions, with:

$$L_{0i} = L_{ii} + 2(L_b + (n+1)L_e)$$

And the self-inductance of winding I is

$$L_{ii} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_i^2(\theta) d\theta = \frac{\mu_0 lr}{g} \alpha_r \left((n+1) - (2n+1)\frac{\alpha_r}{2\pi}\right) (28)$$

The mutual inductance between the kth mesh and the ith mesh of the rotor is recalculated as follows:

$$L_{ki} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_k(\theta) N_i(\theta) d\theta = -(n+1)\frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi}\right) \quad (29)$$

Determination of mutual inductances

$$L_{ai} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta) N_i(\theta) d\theta = L_m \sin((n+1)\delta) \cos(\theta_i + (n+1)\delta)$$
$$(30)$$

The rotor resistance matrix becomes $(N_r\text{-}n) \times (N_r\text{-}n)$ with:

$$R_{0i} = 2(R_b + (n+1)R_e)$$

4) Generator with two non-adjacent broken bars

Now we suppose that the two broken bars are not adjacent, it is obvious that the matrices resistances and inductances of the previous section are not the same. We will see that there is a new mutual inductance between the two cells and involved two broken non adjacent bars as shown in the following figure:
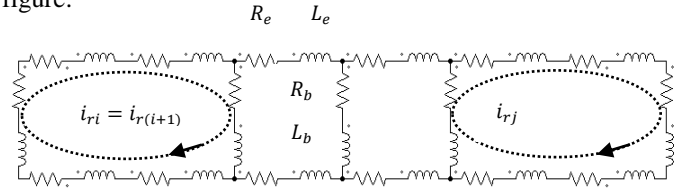

Fig. 7. Equivalent circuits of a squirrel cage rotor with two non-adjacent broken bars

This failure reduces the number of rotor equations with two equations, but one will have:

$$i_{ri} = i_{r(i+1)}$$
$$i_{rj} = i_{r(j+1)}$$

The mutual inductances (stator-rotor, rotor-rotor) and the self-inductance of the mesh ir remains the same as we saw in the first part. With the same procedure we can calculate the inductances of the mesh jr, such as:

$$N_j = \begin{cases} -\alpha_r/2\pi & 0 < \theta \le \theta_j \\ 1 - \alpha_r/2\pi & \theta_j < \theta \le \theta_{j+2} \\ -\alpha_r/2\pi & \theta_{j+2} < \theta \le 2\pi \end{cases}$$
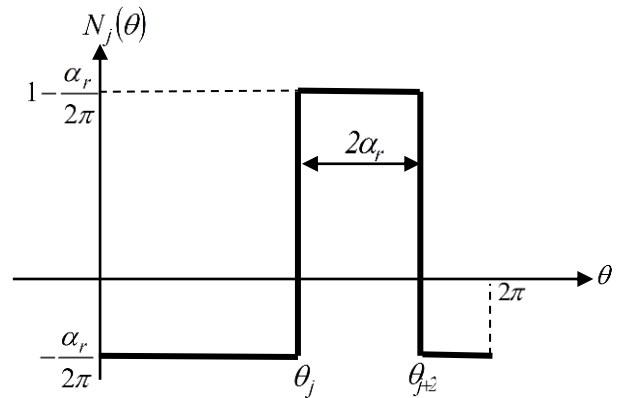

Fig. 8. Rotor loop winding function of faulty generator (2 non-adjacent bb)

Calculation of stator-rotor mutual inductances of the two mesh i_r and j_r :

$$L_{ai} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta) N_i(\theta) d\theta = \frac{\mu_0 lr}{g} N_s \sin(2\delta) \cos(\theta_i + 2\delta)$$

$$L_{aj} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_a(\theta) N_j(\theta) d\theta = \frac{\mu_0 lr}{g} N_s \sin(2\delta) \cos(\theta_j + 2\delta)$$

The matrix of the stator-rotor mutual inductance becomes:

$$[L_{sr}] = \begin{bmatrix} L_{a1} & L_{a2} & \dots & L_{ai} & L_{a(i+2)} & \dots & L_{aj} & L_{a(j+2)} & \dots & L_{aN_r} \\ L_{b1} & L_{b2} & \dots & L_{bi} & L_{b(i+2)} & \dots & L_{bj} & L_{b(j+2)} & \dots & L_{bN_r} \\ L_{c1} & L_{c2} & \dots & L_{ci} & L_{c(i+2)} & \dots & L_{cj} & L_{c(j+2)} & \dots & L_{cN_r} \end{bmatrix}$$

Calculation of self-inductances of ith et j^{th} rotor meshes :

$$L_{ii} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_i^2(\theta) d\theta = \frac{\mu_0 lr}{g} \alpha_r \left(2 - 3 \frac{\alpha_r}{2\pi} \right)$$

$$L_{jj} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_j^2(\theta) d\theta = \frac{\mu_0 lr}{g} \alpha_r \left(2 - 3 \frac{\alpha_r}{2\pi} \right)$$

Calculation of mutual inductances between k^{th} and i^{th} meshes firstly, on the other hand between k^{th} and j^{th} meshes of rotor :

$$L_{ki} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_k(\theta) N_i(\theta) d\theta = -2 \frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi} \right)$$

$$L_{kj} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_k(\theta) N_j(\theta) d\theta = -2 \frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi} \right)$$

Calculation of mutual inductance between i^{th} and j^{th} meshes:

$$L_{ij} = \frac{\mu_0 lr}{g} \int_0^{2\pi} N_i(\theta) N_j(\theta) d\theta = -3 \frac{\mu_0 lr}{g} \left(\frac{\alpha_r^2}{2\pi} \right)$$

The matrix of the rotor inductances becomes:

$$[L_r] = \begin{bmatrix} L_0 & L_{12}-L_b & \dots & L_{1i} & L_{1(i+2)} & \dots & L_{1j} & L_{1(j+2)} & \dots & L_{1N_r}-L_b \\ L_{21}-L_b & L_0 & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ L_{i1} & . & \dots & L_{0i} & L_{i(i+2)}-L_b & \dots & L_{ij} & L_{i(j+2)} & \dots & L_{iN_r} \\ L_{(i+2)1} & . & \dots & L_{(i+2)i}-L_b & L_0 & \dots & L_{(i+2)j} & L_{(i+2)(j+2)} & \dots & L_{(i+2)N_r} \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ L_{j1} & . & \dots & L_{ji} & L_{j(i+2)} & \dots & L_{0j} & L_{j(j+2)}-L_b & \dots & L_{jN_r} \\ L_{(j+2)1} & . & \dots & L_{(j+2)i} & L_{(j+2)(i+2)} & \dots & L_{(j+2)j}-L_b & L_0 & \dots & L_{(j+2)N_r} \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ L_{N_r 1}-L_b & . & \dots & L_{N_r i} & L_{N_r(i+2)} & \dots & L_{N_r j} & L_{N_r(j+2)} & \dots & L_0 \end{bmatrix}$$

The matrix of the rotor resistances is written:

$$[R_r] = \begin{bmatrix} R_0 & -R_b & \dots & 0 & 0 & \dots & 0 & 0 & \dots & -R_b \\ -R_b & R_0 & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ . & . & \dots & R_{0i} & -R_b & \dots & . & . & \dots & . \\ . & . & \dots & -R_b & R_0 & \dots & . & . & \dots & . \\ . & . & \dots & . & . & \dots & . & . & \dots & . \\ 0 & . & \dots & . & . & \dots & R_{0j} & -R_b & \dots & 0 \\ 0 & . & \dots & . & . & \dots & -R_b & R_0 & \dots & 0 \\ . & . & \dots & . & . & \dots & . & . & \dots & -R_b \\ -R_b & 0 & \dots & . & . & \dots & 0 & 0 & \dots & R_0 \end{bmatrix}$$

Such as:

$$R_{0i} = 2(R_b + 2R_e)$$
$$L_{0i} = L_{ii} + 2(L_b + 2L_e)$$
$$R_{0j} = 2(R_b + 2R_e)$$
$$L_{0j} = L_{jj} + 2(L_b + 2L_e)$$

There is a difference between the two models of equations of a cage rotor with two broken bars, in the first model the two bars are adjacent and in the second are not adjacent. This difference appears particularly at resistance, self and mutual inductances of the relevant mesh breaking bars and the mutual inductance between them and the stator windings.

III. PARAMETER DETERMINATION FOR ROTOR WITH SEVERAL BROKEN BARS

The suggested approach was tested on a 4 kW, 2 pole pairs, 28 bars squirrel cage induction generator which detailed parameters are given in Appendix.

The defects are introduced into the program by removing the blocks defining the bars concerned with the break, and by introducing new inductances and resistances calculated in the previous section. The variations of the equivalent resistance and the inductances of the rotor are given in table1.

TABLE I. PARAMETER VALUES FOR VARIOUS BROKEN ROTOR BARS

Number of BRB	$L_{ii}(\mu H)$	$L_{0i}(\mu H)$	$L_{ki}(\mu H)$	$R_{0i}(\mu\Omega)$
0	8.15	8.78	-0.3	203.88
1	16	16.7	-0.6	213.88
2	23.84	24.61	-0.9	223.88
3	31.68	32.52	-1.2	233.88
4	39.53	40.45	-1.5	243.88
5	47.38	48.37	-1.8	253.88

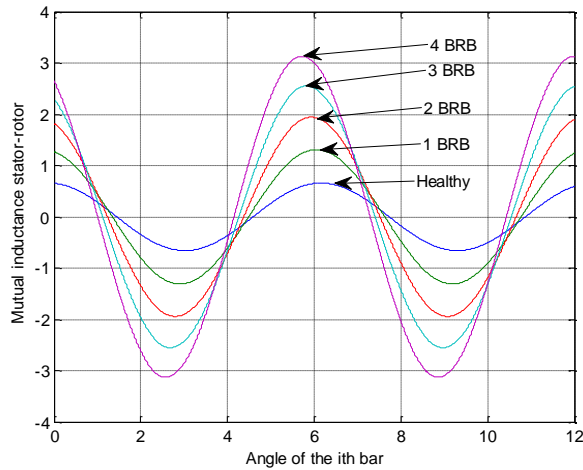| Number of BRB	$L_{ii}(\mu H)$	$L_{0i}(\mu H)$	$L_{ki}(\mu H)$	$R_{0i}(\mu \Omega)$
6	55.22	56.28	-2.1	263.88
7	63.07	64.20	-2.4	273.88
8	70.91	72.11	-2.7	283.88
9	78.76	80.04	-3	293.88
10	86.61	87.96	-3.3	303.88

Fig. 9.   Mutual inductance between *a* stator phase and i^{th} rotor loop

IV.   CONCLUSION

A generalized model of the squirrel-cage rotor in induction generator has been developed at the base of mathematical recurrences. The determination of leakage and the mutual inductances of the rotor meshes is carried out by using the winding function theory.

This model allows for the introduction of the defects and knowledge of the real currents in the rotor bars like in the end ring; what is difficult to realize by the 3-phase model.

One can determine analytical knowledge of the machine to produce quantifiable and analytical information.

The advantage of this model is that it is applicable to cage rotors with non-integral number of rotor bars per pole pair. The proposed approach can be used also for the simulation of variable speed induction machine drives under rotor faults.

APPENDIX

The parameters of our machines are illustrated in the following table:

TABLE II.   INDUCTION GENERATOR PARAMETERS

Parameters	Symbols	Values
Rated power	P	4 kW
Rated voltage	V	220/380 V
Rated current	I	15.2/8.8 A
Rated speed	Ω	1465 rpm
Pole-pair number	p	2
Supply frequency	f_s	50 Hz
Power factor	$cos\varphi$	0.83

Parameters	Symbols	Values
Rotor inertia	J	0.011 kg.m^2
Friction coefficient	f	8.73e^{-4} Nm/rad/s
Stator phase turns in series	N_s	156
Stator phase resistance	R_s	1.5 Ω
Stator phase leakage inductance	L_{ls}	7 mH
Stator magnetizing inductance	L_{ms}	0.55 H
Rotor bar resistance	R_b	96.94 $\mu\Omega$
Rotor bar inductance	L_b	0.28 μH
End-ring segment resistance	R_e	5 $\mu\Omega$
End-ring segment inductance	L_e	0.036 μH
Number of rotor bars	N_r	28
Mean radius air-gap	r	54 mm
Effective air-gap	g	0.28 mm
Length of air-gap	l	120 mm

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