Nonlinear Condition Tolerancing Using Monte Carlo Simulation

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Abstract—**To ensure accuracy and performance of the products, designers tend to hug the tolerances. While, manufacturers prefer to increase them in order to reduce costs and ensure competition. The analysis and synthesis of tolerances aim on studying their influence on conformity with functional requirements. This study may be conducted in the case of the most unfavorable configurations with the "worst case" method, or "in all cases" using the statistical approach. However, having a nonlinear condition make it difficult to analyse the influence of parameters on the functional condition.**

In this work, we are interested in the tolerance analysis of a mechanism presenting a nonlinear functional condition (slider crank mechanism). To do this we'll develop an approach of tolerances analysis combining the worst case and the statistical methods.

Keywords—Worst case tolerancing; statistical tolerancing; Monte Carlo simulation; nonlinear condition; slider crank system

I. INTRODUCTION

The industrial challenges lead to develop models and design support tools to meet customer needs by optimizing the triptych: cost quality and time. Several works have been conducted in order to analyze the impact of deviating component surfaces on functions, properties and assemblability of the product [1, 2, 3].

Mechanical products tolerancing consist on specifying limits of dimensional characteristics variations (often unidirectional) with a tolerance interval. There are mainly two tolerancing approaches:

- "worst case", ensures the assembly and functionality of the mechanical system, but leads to a higher production comparing to the second approach;

- "statistic", ensures a low cost of production but accepts mechanisms whose functionality is not respected.

The tolerance analysis aims to study the influence of dimensional characteristics variations on the respect of functional requirements. This analysis may be conducted in the case of the most unfavorable configurations "worst case" or "in all cases" using the statistical approach. The analysis is so difficult to be performed in the case of the nonlinear functional conditions, because we cannot write them as an algebraic combination [4]. Our work aims to propose an approach based on the combining of the worst case and the statistical methods that allows the tolerance analysis and synthesis in the case of a nonlinear functional conditions.

This paper is organized as follows: In section 2 we treat tolerance analysis at worst case and we present its limits on tolerancing nonlinear condition. Section 3 describes the statistical tolerancing method using Monte Carlo Simulation. In section 4, we apply our approach on a slider crank system that presenting a nonlinear functional condition.

II. TOLERANCE ANALYSIS "AT WORST CASE"

The analysis at "Worst Case" (WC) is mainly used to operate the system in the most adverse conditions [5, 6, 7]. It allows finding extreme values (maximum and minimum) that can be reached by the resulting operating condition for any combination of the real initial dimensions.

In the case of mono-domain systems, the relationship between the resulting functional requirement and basic characteristics is often translated into a linear additive relationship. Overall, it resulted in a unidirectional relationship as follows:

$$Y = \sum_{k=1}^{n} \alpha_k X_k = \sum_{k=1}^{n} \alpha_k (X_{kn} \pm \frac{t_k}{2}) \qquad (1)$$

Where:

- Y is the functional condition which must be between y_{min} and y_{max};

- X_{kn} is the nominal value of X_k;

- α_k is the influence coefficient ($a_k = \pm 1$);

- t_k is tolerance.

In this case, the functional condition often reaches its limit values only for a linear combination of limit values (minimum and / or maximum) of partial components dimensions such as:

$$y_{min} \leq \left(\sum_{k=1}^{n} \alpha_k X_k \right)_{min} = \sum_{k=1}^{n} \alpha_k X_{kn} - \frac{1}{2} \sum_{k=1}^{n} |\alpha_k| t_k$$

$$\left(\sum_{k=1}^{n} \alpha_k X_k \right)_{max} = \sum_{k=1}^{n} \alpha_k X_{kn} + \frac{1}{2} \sum_{k=1}^{n} |\alpha_k| t_k$$

$$\leq y_{max} \qquad (2)$$

The functional condition is then satisfied if:

$$y_{max} - y_{min} = t_y \leq \sum_{k=1}^{n} |\alpha_k| t_k \qquad (3)$$

Practically, the relationship between the resulting functional requirement and basic characteristics is more complex than the linear additive relationship.

III. TOLERANCES ANALYSIS WITH MONTE CARLO SIMULATION

Monte Carlo simulation (MCS) is a method for predicting errors manufacturing [8]. Its core idea is to use random samples of parameters or inputs to explore the behavior of a complex system or process. For this, the Monte Carlo simulation uses pseudo-random generators with numbers corresponding to different types of statistical distributions. By using Monte Carlo Simulation, the results obtained are more realistic than those obtained by conventional methods of calculation.

The user must define the random distribution of input variables. The number of experiments generated must be large enough to reliably determine the statistical parameters of output variables.

The simulation defines a statistical data generally described by the mean dimension:

$$\mu = \frac{1}{N} \sum_{i=1}^{N} Z_i \qquad (4)$$

And standard deviation:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Z_i - \mu)^2} \qquad (5)$$

Where:

- Z_i is the value of the resulting dimension to the 'i_{th}' simulation cycle;

- N is the total number of simulation cycles;

The general approach of applying the MCS method is presented in analyzing functional condition tolerances is presented in Figure 1.

For the evaluation of the realization frequency of the functional condition, the statistical group is converted into a histogram (Figure 2).

The standard deviation 'σ' is a parameter characterizing the dispersion or variation of the values distribution around an average. Higher are the values concentrated around the average, lower is the standard deviation. In a normal distribution, the standard deviation 'σ' is used to establish

confidence intervals for desired confidence levels. The production process is often considered satisfactory at ±3σ. So 99.73% of assemblies are in the interval $[\mu_J - 3\sigma_J \; ; \; \mu_J + 3\sigma_J]$ (Figure 3). For centric distribution, the functional requirement will be respected for 99.73% of assemblies if:
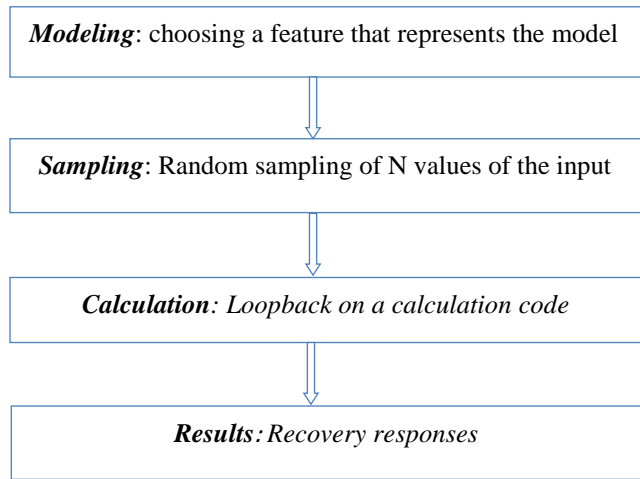
Tolerance on the requirement = 6 σ
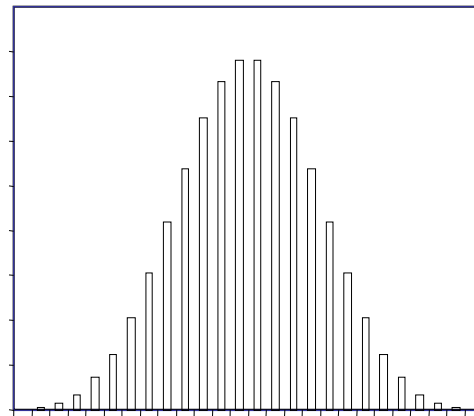

Fig. 1. General approach of applying the MCS
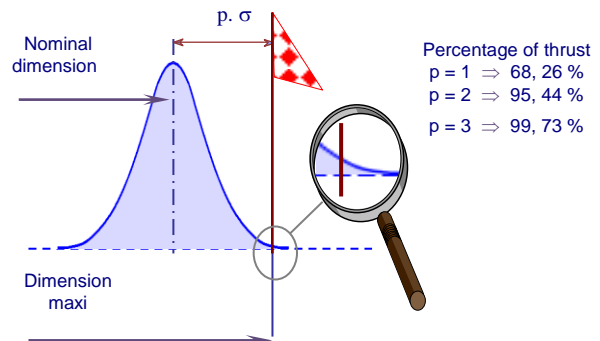

Fig. 2. View as histogram

Fig. 3. Percentage of trust in an interval

IV. APPROACH OF TOLERANCING

To solve the problem of the functional condition non-linearity, the following approach is adopted based on both:

Worst Case tolerancing and Monte Carlo Simulation. This combination will ensure system's functionality with large
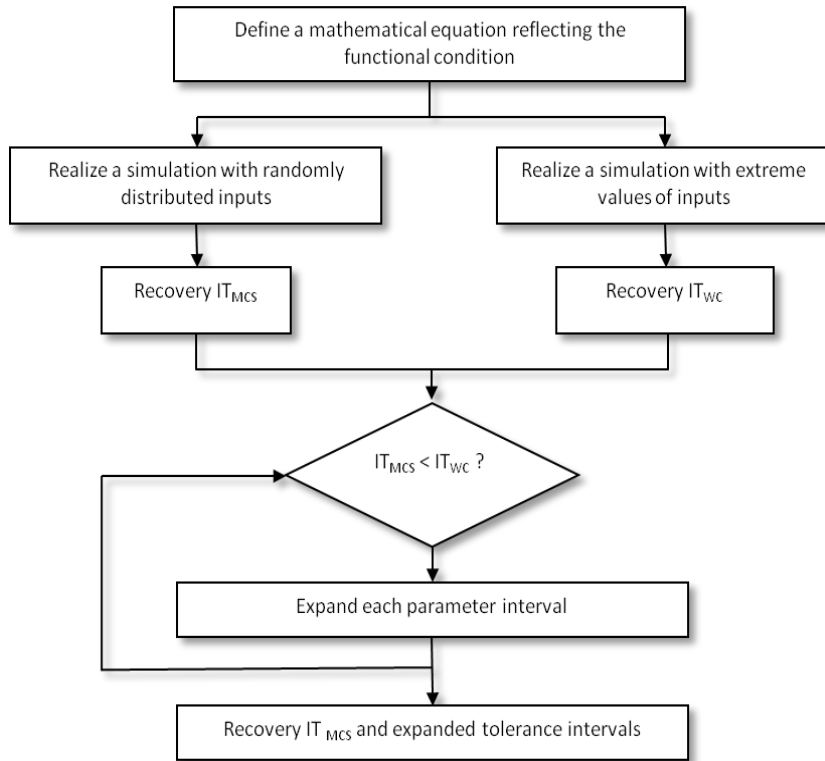
tolerance intervals (figure 4).

Fig. 4.   flowchart of analyzing nonlinear condition tolerance

V.   CASE OF STUDY

A. System Overview

The case of slider crank system (Figure 4), the position X is given by:

$$X = rcos(\alpha) + \sqrt{L^2 - (rsin(\alpha) + A)^2} \qquad (6)$$

The maximum 'X$_{max}$' position of the piston relative to the axis of the crankshaft is illustrated in the simplified diagram below:

$$X_{max} = \sqrt{(r + L)^2 - A^2} \qquad (7)$$

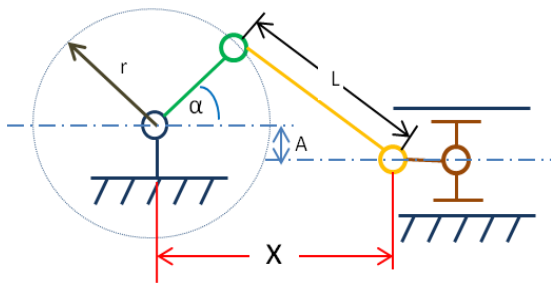Note that the equation reflecting the functional condition ($X_f = X_{max}$) is not linear.

Fig. 5.   slider crank system

B. Tolerances analysis «worst case"

We implement the mathematical equation reflecting the functional condition in the 20-sim software [9]. The program translating the mathematical model is:

parameters

    real L {mm};

    real r {mm};

    real A {mm};

    real omega {rad/s};

    variables

    real X {mm};

    real Cf {mm};

equations

    X=r*cos(omega*time)+sqrt ((L)^2-(r*sin(omega*time)- A)^2);

    Cf = sqrt ((L+r)^2-A^2);

The simulation is launched using the extreme values and then we recover 'L$_i$' limits (lower limit) and 'L$_s$' (upper limit) of the functional condition, Then the tolerance interval at "worst case"(IT$_{wc}$) is deduced.

TABLE I.   PARAMETERS AND NOMINAL DISPERSION

Parameters	Nominal values	Li	Ls	IT	σ
L	178	177,300	178,700	±0,70	0,233
r	39	38,300	39,700	±0,70	0,233
A	13	12,300	13,700	±0,70	0,233

TABLE II.   RESULTS OF DIMENSIONAL TOLERANCING AT WORST CASE

Parameters	Nominal values	L_i	L_s	IT_{wc}
L	178	177,3	178,7	±0,7
r	39	38,3	39,7	±0,7
A	13	12,3	13,7	±0,7
C_f	216,6103	215,1642861	218,053365	±1,**444539**

It is noted from the results of the simulation that:

$$Li(\mathbf{C_f}) = \sqrt{\left(Li(\mathbf{L}) + Li(\mathbf{r})\right)^2 - \left(Ls(\mathbf{A})\right)^2} \qquad (8)$$

$$Ls(\mathbf{C_f}) = \sqrt{\left(Ls(\mathbf{L}) + Ls(\mathbf{r})\right)^2 - \left(Li(\mathbf{A})\right)^2} \qquad (9)$$

The presented method has made it possible to conduct an analysis of the tolerances at "worst case" where the relationship between the resulting functional requirement and basic characteristics is nonlinear. These results guarantee the mechanism operation with parameters within their limits configurations.

C. Tolerance Analysis by Monte Carlo: statistical approach

The result of the analysis with Monte Carlo simulation depends on the sample size 'N'. The number of draws 'N' has to be high in order to get accurate results. Indeed, the accuracy of this statistical analysis increases as a proportion to '√N' [10].

If the value of N is sufficiently large, the result of the method reaches a stable value, substantially independent of N. In fact, if the difference between two results of simulation in different draws is greater than 5%, then the number of draws N is not sufficient to reach stability [11].

These tests determine the extent of the result variations and calculate its standard deviation (Figure 6). We will use a sample size of N = 10000 to predict statistically the form of 'X_max'.

The implementation of the MCS method is to first generate the histograms of Figure 8 based on the values of the table II. We chose tolerance intervals (IT) of ± 0.70 and homogeneous distribution of ± 3σ for each parameter.

The histograms in figure 8 show the variation of each parameter and that of 'C_f' in their tolerance range. They essentially provide information on the behavior of each parameter within its variation area.
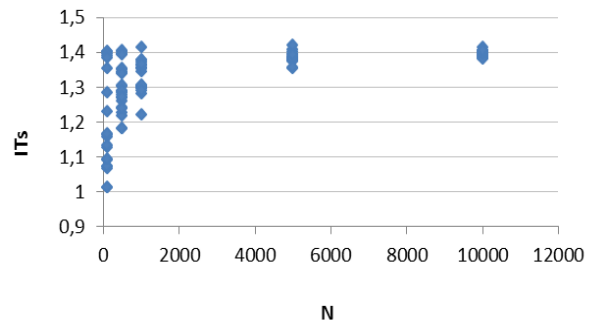

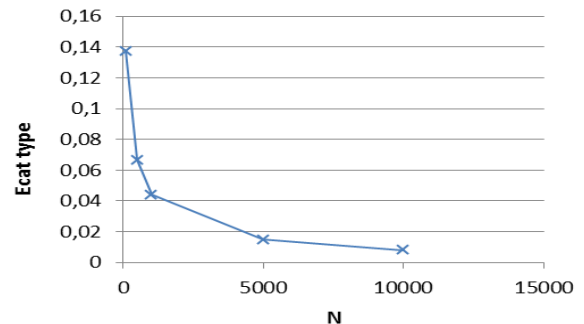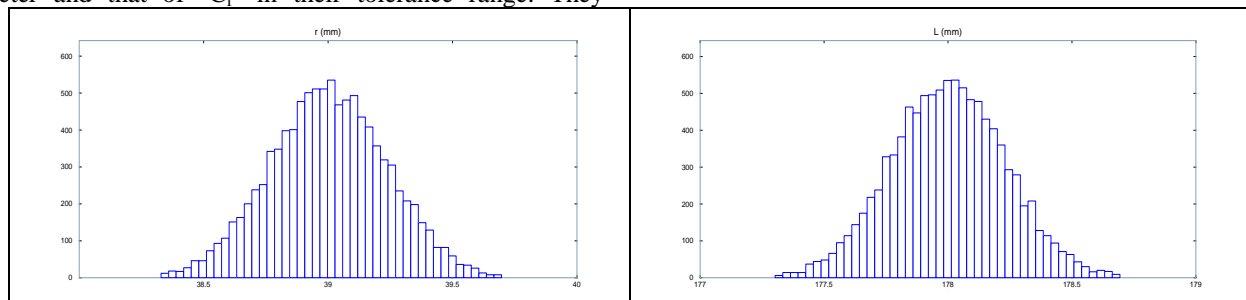Fig. 6.   Evolution of the estimated values of tolerance intervals versus "N"

Fig. 7.   Evolution of the standard deviation of the estimated tolerance intervals versus "N"

The result of the MCS method is also a random variable. To characterize the random variable, twenty Monte Carlo simulations are launched for each value of "N" (figure8).

The effects of changes in basic parameters on the kinematic behavior of the system are presented in Figure 9.
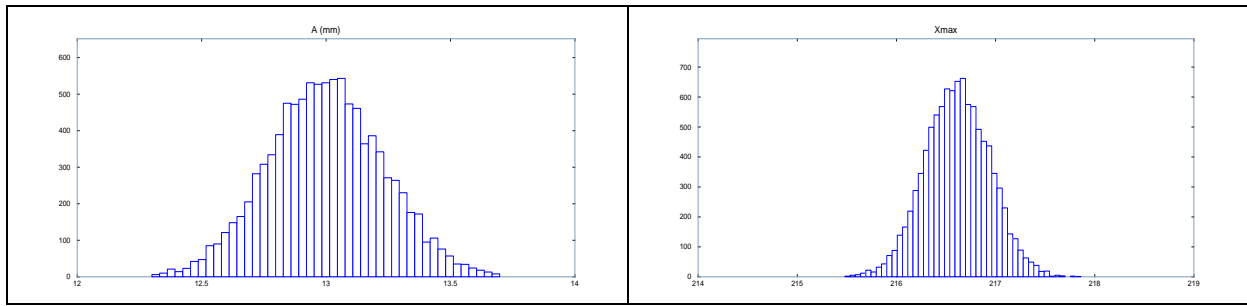
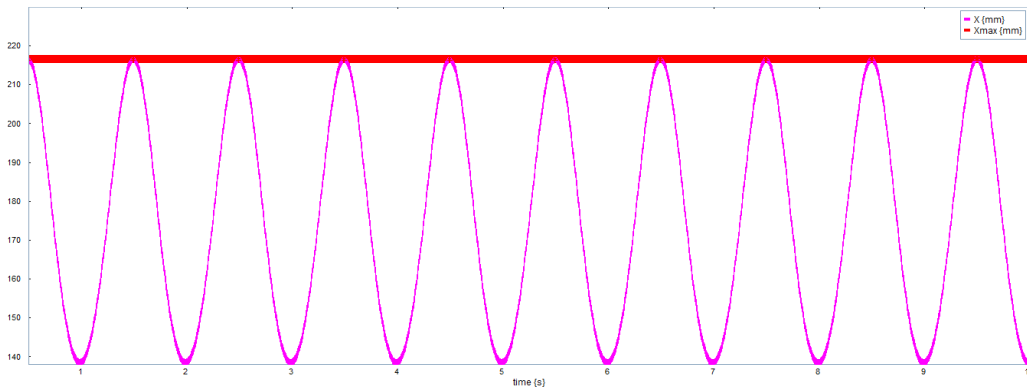Fig. 8.   Variation of the dimensions L, A,r and C_f in their tolerance bands

Fig. 9.   The effects of changes in parameters X and X_{max}

TABLE III.   RESULTS OF DIMENSIONAL TOLERANCING WITH MCS

Parameters	Nominal values	L_i	L_s	IT_{MCS}	σ
L	178	177,300	178,700	±0,70	0,233
R	39	38,300	39,700	±0,70	0,233
A	13	12,300	13,700	±0,70	0,233
X_{max}	216,610249	215,41491	217,75478	**±1,16993**	0,390

By comparing the results obtained by the worst case method to those found with MCS, (Table IV), we find that the statistical variation zone is within the area determined by the arithmetic method WC ($IT_{MCS} < IT_{WC}$).

TABLE IV.   COMPARISON OF IT_{WC} AND IT_{MCS}

C_f	L_i (Cf)	L_s (Cf)	IT
$IT_{\_WC}$	215,1643	218,0534	±1,4445
$IT_{\_MCS}$	215,4149	217,7548	±1,1699

D. Tolerances optimization

Since the statistical variation area is included in the arithmetic area, the optimization of parameters IT_{MCS} intervals is possible. For this, we proceed to the expansion of each parameter tolerances intervals, until the new value of the interval IT_{MCS} is acceptable ($IT_{MCS} < IT_{wc}$). The optimization cycle is stopped. For the studied mechanism, the optimization cycle yielded the values shown in the table V.

TABLE V.   IT'S OPTIMIZED WITH MCS

Parameters	Nominal values	L_i	L_s	IT_{opt}	σ
L	178	177,15	178,85	± 0,85	0,283
r	39	38,15	39,85	± 0,85	0,283
A	13	12,15	13,85	± 0,85	0,283
C_{f_opti}	216,610249	215,11767	217,978238	±1,43028	0,477

VI.   CONCLUSION

In this work, a new method based on Monte Carlo simulation and the Worst Case was proposed to analyze nonlinear condition tolerance interval. Tolerance intervals were expanded for different dimensions without affecting the whole system functionality. This will reduce manufacturing costs while maintaining the overall system functionality.

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