Model Study and Fault Detection for the Railway System

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Abstract—The wheel-rail-sleepers system is simulated as a series of moving point loads on an Euler–Bernoulli beam resting on a visco-elastic half space. This paper concentrates on the rail-sleepers interaction system (railway system) and the fault detection. The main objective is to mathematically develop and implement a dynamic model of a railway system then the diagnosis of system defects using a Luenberger observer (LO). The simulation results are based on a physical description, mathematical equations and simulations with MATLAB simulation program.

Keywords—*Dynamic model; the wheel-rail–sleepers system; interaction system; Euler–Bernoulli; Luenberger observer (LO); fault detection*

I. INTRODUCTION

Because of the importance of the safety and the great rail-sleepers interaction effect on it, in the last few years there has been a growing interest in the diagnosis of the railway system and the rail-sleepers interaction induced by moving high-speed trains. The early detection and isolation are very important to avoid costly breakdowns and improve equipment reliability as reported in [1] and [2]. In the past decades, various analytical, semi-analytical and numerical methods with different accuracies have been attracted much attention from research teams to investigate the rail vibrations then the diagnosis of this system and isolation of the defect [23], [24], [25] and [26]. The finite elements method [29] is a very effective method for the railway model calculates. This paper is focused on this method for the model calculation.

There are essentially two opposing attitudes for diagnosis system [22] and [27]. First one is corrective maintenance or diagnosis after the failure, second one is the preventive maintenance [28] or diagnosis before the failure. While the second attitude seems more attractive, it's not systematically applied on industrial process. In practice, these two approaches still coexist. Previous studies indicate that the good maintenance is to implement the most appropriate technique for each device, sub-assembly or each element. The maintenance type choice is based on the knowledge available on the installation and objectives.

To diagnose and isolate the fault of the railway system, it must first calculate its dynamic model [30] and [31]. The model considers the rail and the sleepers as a whole system and couples the vertical interaction with the lateral interaction. To calculate the railway model it must consider the wheel-rail interaction. The vertical contact forces between wheels and rails are described by Hertzian nonlinear elastic contact theory and the tangential wheel-rail forces are decided by the creep theory. Generally, there are three main classes of dynamic interactions between rails and sleepers: vertical, lateral and torsional. Vertical rail-sleepers interactions are responsible for damage and service life of the system [3] demonstrated that. Lateral and torsional interactions usually influence the safety against derailment of train and wear of wheels and rails. In fact, the vertical and lateral rail-sleepers interactions can't be separated from each other.

So this work is a mathematical calculation of the railway model, then the isolation of default with Luenberger Observer (LO) [21].

Section II describes the real system in which this work is applied. Section III analyzes the system model where the dynamic model of the system is calculated. Section IV the different variation cases of the system where there is the global model study of the railway system. Section V concentrates for the fault detection where there is study of observer type used in this work. Choice of Luenberger observer for the diagnosis and fault isolation, calculation of parameters and experimental results are presented in Section VI; Section VII concludes the paper.

II. SYSTEM DESCRIPTION

The system is a train rail. The rail treated as a rigid model in which it is supported on two double-axles with the primary and the secondary suspension systems. The primary suspensions, connecting the wheels, are represented by ordinary Kelvin elements. The secondary suspensions, connecting the sleepers, are also modeled as ordinary Kelvin elements.

The wheels and sleepers are coupled through the suspension elements. The rail and sleepers each undergo lateral displacement and vertical, but only vertical displacement is considered in the four wheels. More knowledge about this system presented in [4]-[6].

The dynamic model of railway system can be presented in the form of equations of motions vertical, lateral and torsional [7].

III. MATHEMATIC MODEL

There are several experiments in order to verify the validity of the infinite element method [8]-[15].

The research in this work started by these equations that describe the motion of the rail is as follows:

- Vertical Motion:

$$EI_y \frac{d^4 Z_r(x,t)}{dx^4} + m_r \frac{d^2 Z_r(x,t)}{dt^2}$$
$$= -\sum_{i=1}^{N} F_{vi}(t)\delta(x - x_i) + \sum_{j=1}^{4} G_{vj}(t)\delta(x - x_g) \qquad (1)$$

- Lateral Motion:

$$EI_z \frac{d^4 Y_r(x,t)}{dx^4} + m_r \frac{d^2 Y_r(x,t)}{dt^2}$$
$$= -\sum_{i=1}^{N} F_{Li}(t)\delta(x - x_i) + \sum_{j=1}^{4} G_{Lj}(t)\delta(x - x_g) \qquad (2)$$

- Torsional Motion:

$$\rho I_0 \frac{d^2 \phi_r(x,t)}{dt^2} - GK \frac{d^2 \phi_r(x,t)}{dx^2}$$
$$= -\sum_{i=1}^{N} M_{Fi}(t)\delta(x - x_i) + \sum_{j=1}^{4} M_{Gj}(t)\delta(x - x_g) \qquad (3)$$

Where $Z_r(x,t)$, $Y_r(x,t)$ and $\phi_r(x,t)$ are respectively the vertical, lateral and torsional displacements of de rail, EIy and EIz are the rail bending stiffness to the y-axle and to the z-axels respectively, mr the rail mass per unit length, ρ the rail density, I_0 the torsional inertia of rail, GK the rail torsional stiffness and $\delta(x)$ the Dirac delta function.

Equation (4) represents the vertical and lateral dynamic forces of the i-th rail/sleeper point.

$$\begin{cases} F_{vi}(t) = K_{pv}\left(Z_r(x_i,t) - Z_{si}(t) - a\phi_{si}(t)\right) + C_{pv}\left(\dot{Z}_r(x_i,t) - \dot{Z}_{si}(t) - a\dot{\phi}_{si}(t)\right) \\ F_{Li}(t) = K_{ph}\left(Y_r(x_i,t) - Y_{si}(t) - a\phi_{si}(t)\right) + C_{ph}\left(\dot{Y}_{si}(t) - a\dot{Y}_{si}(t)\right) \end{cases} \tag{4}$$

Where, $Z_{si}(t)$, $Y_{si}(t)$ and $\phi_{si}(t)$ Vertical, lateral and torsional displacements of the i-th sleepers They can be assumed to be zero, K_{pv} and K_{ph} are the vertical and lateral stiffness between the rail and the sleepers and C_{pv} et C_{ph} are the vertical and lateral rail/sleeper damping.

The dynamic of wheel/rail forces can be written as (5), where $G_{vj}(t)$ vertical forces and $G_{LJ}(t)$ lateral forces.

$$\begin{cases} G_{vj}(t) = P_j(t) - P_0 \\ G_{Lj}(t) = Q_j(t) \end{cases} \tag{5}$$

$P_j(t)$ and $Q_j(t)$ are the j-th wheel/rail forces and P_0 the static wheel load.

Equation (6) defines the moments of forces acting on the rail due to fastening system and due to wheel / rail forces.

$$\begin{cases} M_{fi}(t) = K_{\psi}\phi_r(x_i,t) \\ M_{gj}(t) = G_{Lj}(t)e_1 - G_{vj}(t)e_2 \end{cases} \tag{6}$$

Where, K_{ψ} the torsional stiffness of the fastening system, e_1 and e_2 are the arms of the lateral and vertical wheel/rail forces to the torsional canter of the rail.

To facilitate the solution of motion differential equations of the rail, the following railway mode shape functions are assumed:

- Vertical Vibration

$$Z_k(x) = \sqrt{\frac{2}{m_r l}} \sin\left(\frac{k\pi x}{l}\right) \tag{7}$$

- Lateral Vibration

$$Y_k(x) = \sqrt{\frac{2}{m_r l}} \sin\left(\frac{k\pi x}{l}\right) \tag{8}$$

- Torsional Vibration

$$\phi_k(x) = \sqrt{\frac{2}{\rho I_0 l}} \sin\left(\frac{k\pi x}{l}\right) \tag{9}$$

Where l is the length of the rail and k is the mode number.

Thus, the characteristic variables of (1), (2) and (3) can be expressed as:

$$Z_r(x,t) = \sum_{k=1}^{K} Z_k(x) q_{zk}(t) \tag{10}$$

$$Y_r(x,t) = \sum_{k=1}^{K} Y_k(x) q_{yk}(t) \tag{11}$$

$$\phi_r(x,t) = \sum_{k=1}^{K} \phi_k(x) q_{tk}(t) \tag{12}$$

$q_{zk}(t)$, $q_{yk}(t)$ and $q_{tk}(t)$ are the k-th mode time coordinate and K is the total number of modes considered.

Then the second order ordinary differential equations simplified in terms of the time coordinates (1), (2) and (3) can be computed by the following equation:

- Vertical Motion

$$\ddot{q}_{zk}(t) + \frac{EI_y}{m_r}\left(\frac{k\pi}{l}\right)^4 q_{zk}(t) = -\sum_{i=1}^{N} F_{vi}(t) Z_k(x_i) + \sum_{j=1}^{4} G_{vj}(t) Z_k(x_{gi})$$
$$K = 1, 2, \dots, k \tag{13}$$

- Lateral Motion

$$\ddot{q}_{yk}(t) + \frac{EI_z}{m_r}\left(\frac{k\pi}{l}\right)^4 q_{yk}(t) = -\sum_{i=1}^{N} F_{Li}(t)Y_k(x_i) + \sum_{j=1}^{4} G_{Lj}(t)Y_k(x_{gi})$$

$$K = 1, 2, \ldots, k \qquad (14)$$

- Torsional Motion

$$\ddot{q}_{tk}(t) + \frac{GK}{\rho I_0}\left(\frac{k\pi}{l}\right)^4 q_{tk}(t) = -\sum_{i=1}^{N} M_{fi}(t)\phi_k(x_i) + \sum_{j=1}^{4} M_{gj}(t)\phi_k(x_{gi})$$

$$K = 1, 2, \ldots, k \qquad (15)$$

If K fixed by the value 1 and with a small change of the variables the vertical, lateral and torsional vibration system of the rail could be mathematically written as:

- Vertical movement equation

$$M_z \ddot{q}_{z1}(t) + C_z \dot{q}_{z1}(t) + K_z q_{z1}(t) + C_{zy}\ddot{q}_{t1}(t) + K_{zy}q_{t1}(t) = P_z(t) \quad (16)$$

Where

$$M_z = 1, \; C_z = \sum_{i=1}^{N} C_{pv}\left((Z_1(x_i) - Z_1(x_{si}))Z_1(x_i)\right)$$

$$C_{zy} = -a\, C_{pv}\sum_{i=1}^{N}\left(\phi_1(x_{si})Z_1(x_i)\right)$$

$$K_z = \sum_{i=1}^{N}\left(K_{pv}\left(Z_1(x_i) - Z_1(x_{si})\right)Z_1(x_i)\right) + \frac{EI_y}{m_r}\left(\frac{k\pi}{l}\right)^4,$$

$$K_{zy} = -a\, K_{pv}\sum_{i=1}^{N}\left(\phi_1(x_{si})q_{t1}(t)Z_1(x_i)\right)$$

and $P_z(t) = \sum_{j=1}^{4}\left(P_j(t) - P_0\right)Z_1(x_{gi})$

- Lateral movement equation

$$M_y \ddot{q}_{y1}(t) + C_y \dot{q}_{y1}(t) + K_y q_{y1}(t) = P_y(t) \qquad (17)$$

Where $M_y = 1, \; C_y = C_{ph}\sum_{i=1}^{N}\left(Y_1(x_i) - Y_1(x_{si})\right)Y_1(x_i)$,

$$K_y = \frac{EI_z}{m_r}\left(\frac{k\pi}{l}\right)^4 + K_{ph}\sum_{i=1}^{N}\left((Y_1(x_i) - Y_1(x_{si}))\, Y_1(x_i)\right)$$ and

$$P_y(t) = \sum_{j=1}^{4} G_{Lj}(t)Y_1(x_{gi})$$
.

- Torsional movement equation

$$M_t \ddot{q}_{t1}(t) + C_t \dot{q}_{t1}(t) + K_t q_{t1}(t) = P_t(t) \qquad (18)$$

Where $M_t = 1, \; C_t = 0, \; K_t = \frac{GK}{\rho I_0}\left(\frac{\pi}{l}\right)^2 + \sum_{i=1}^{N} K_\psi \phi_1^2(x_i)$ and

$$P_t(t) = \sum_{j=1}^{4}\left(G_{Lj}(t)e_1 - G_{vj}(t)e_2\right)\phi_1(x_{gi})$$
.

In order to consistent with rail vertical, lateral and torsional vibration matrix differential equation, the rail vibration system is mathematically written as (19).

$$M\ddot{q}_1(t) + C\dot{q}_1(t) + Kq_1(t) = P(t) \qquad (19)$$

Where M, C and K represent mass matrix, damping matrix and stiffness matrix of the rail vertical, lateral and torsional vibration system respectively $q_1(t)$, $\dot{q}_1(t)$ and $\ddot{q}_1(t)$ are rail displacement vector, velocity vector and acceleration vector, respectively, and P is the load vector [10].

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} C_z & 0 & C_{zy} \\ 0 & C_y & 0 \\ 0 & 0 & C_t \end{bmatrix}, \; K = \begin{bmatrix} K_z & 0 & K_{zy} \\ 0 & K_y & 0 \\ 0 & 0 & K_t \end{bmatrix},$$

$$\ddot{q}_1(t) = \begin{bmatrix} \ddot{q}_{z1}(t) \\ \ddot{q}_{y1}(t) \\ \ddot{q}_{t1}(t) \end{bmatrix}, \dot{q}_1(t) = \begin{bmatrix} \dot{q}_{z1}(t) \\ \dot{q}_{y1}(t) \\ \dot{q}_{t1}(t) \end{bmatrix}, q_1(t) = \begin{bmatrix} q_{z1}(t) \\ q_{y1}(t) \\ q_{t1}(t) \end{bmatrix} \text{ and } P(t) = \begin{bmatrix} P_z(t) \\ P_y(t) \\ P_t(t) \end{bmatrix}$$

IV. The Different Variation Cases of the System

After the mathematical calculation of railway dynamic model, this part focuses on this dynamic model study, where there is the study of each equation of motion and the presentation of different variation cases.

A. Vertical Motion Equation

This system is a SIMO system (Single Input Multi Output) with two outputs $q_{z1}(t)$ and $q_{t1}(t)$.

$$M_z\ddot{q}_{z1}(t) + C_z\dot{q}_{z1}(t) + K_z q_{z1}(t) + C_{zy}\ddot{q}_{t1}(t) + K_{zy}q_{t1}(t) = P_z(t)$$

We propose: $C_z = \dfrac{C}{M}, \; K_z = \dfrac{K}{M}, \; C_{zy} = \dfrac{C_1}{M}, \; K_{zy} = \dfrac{K_1}{M}$ and $P_z = \dfrac{F}{M}$.

C, C1 are respectively the damping constant of the first and second subsystem, K, K1 are the displacement constant of the first and second subsystem, respectively and F is the result force.

With the superposition method, this system can be divided on two SISO subsystems with $q_{z1}(t)$ and $q_{t1}(t)$ output. This method is presented in the following calculation of this paper.

The output $q_{t1}(t)$ supposed null then the first subsystem like:

$$\ddot{q}_{z1}(t) + C_z\dot{q}_{z1}(t) + K_z q_{z1}(t) = P_z(t) \qquad (20)$$

Now the second output $q_{z1}(t)$ supposed null then the second subsystem like:

$$C_{zy}\ddot{q}_{t1}(t) + K_{zy}q_{t1}(t) = P_z(t) \qquad (21)$$

The final solution of this system is the sum of two outputs $q_{z1}(t)$ and $q_{t1}(t)$ since both subsystems are parallel.

First subsystem (20) is a damped oscillating system with output $q_{z1}(t)$.

We can take:

$$\begin{cases} 2\varepsilon_z \omega_z = C_z \\ \omega_z^2 = K_z \end{cases} \tag{22}$$

εz the damping coefficient and ωz the natural frequency of the system.

Where
$$H_a(p) = \frac{1}{2\varepsilon_z \omega_z p + \omega_z^2 p + p^2} \tag{23}$$

If $\varepsilon z = 0$ the system unamortized and $Cz = 0$.

So $$\sum_{i=1}^{N} C_{pv}\left(\sin\left(\frac{\pi x_i}{l}\right) - \sin\left(\frac{\pi x_{si}}{l}\right)\right)\sin\left(\frac{\pi x_i}{l}\right) = 0 \tag{24}$$

The solution of this system is
$$\left\{ C_{pv} = 0 \ or \ x_i = nl \ or \ x_i = x_{si}; \ n \in N \right\}$$

Else if $\varepsilon z \neq 0$ the system is a damped oscillating system.

So the study of this part is like the following.

$$2\varepsilon_z \omega_z p + \omega_z^2 p + p^2 = 0$$
$$\Delta_z = 4\omega_z^2 (\varepsilon_z^2 - 1)$$

There are three cases to solve this system.

First case if $\varepsilon_z > 1$ then $\Delta_z < 0$ (a periodic regime).

Second case if $\varepsilon_z \in]0 \quad 1[$ then $\Delta_z > 0$ (pseudo periodic regime).

Third case if $\varepsilon_z = 1$ then $\Delta_z = 0$ (Critic regime).

First, the periodic regime study, $\varepsilon z > 1$.

$$F_z = C_{pv}^2 A^2 - 4 K_{pv} D - 4\frac{EI_y}{mr}\left(\frac{\pi}{\rho}\right)^4 > 0$$

$$A = \sum_{i=1}^{N} \left(Z_1(x_i) - Z_1(x_{si})\right) Z_1(x_i)$$

Then the solution of this equation is
$$A_z \in]-\infty \quad A_1[\cup]A_2 \quad +\infty[.$$

Where A1 and A2 are the two roots of this equation.

The solution is written as
$$\sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right) - \sin\left(\frac{\pi x_{si}}{l}\right)\right)\sin\left(\frac{\pi x_i}{l}\right) \in \left]-\infty \quad \frac{A_1 m_r l}{2}\right[\cup \left]\frac{A_2 m_r l}{2} \quad +\infty\right[\tag{25}$$

Second, the critic regime study. $\varepsilon_z = 1$ and $F_z = 0$.

The solution in this case is written as

$$\sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right) - \sin\left(\frac{\pi x_{si}}{l}\right)\right)\sin\left(\frac{\pi x_i}{l}\right) = \left\{ \frac{A_1 m_r l}{2} \quad \frac{A_2 m_r l}{2} \right\} \tag{26}$$

Third, the pseudo periodic regime study. $\varepsilon_z \in]0 \quad 1[$.

In this case the solution is written as

$$\sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right) - \sin\left(\frac{\pi x_{si}}{l}\right)\right)\sin\left(\frac{\pi x_i}{l}\right) \in \left]0 \quad \frac{A_2 m_r l}{2}\right[\tag{27}$$

If there is no wagon on the rail: $X_{gi} = 0$ and $P_z = 0$. So qz1(t) = 0 then it's the case where t<0.

Now if there are no sleepers: $X_{si} = 0$ and $\sin\left(\frac{\pi x_{si}}{l}\right) = 0$

then $$\sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right) - \sin\left(\frac{\pi x_{si}}{l}\right)\right)\sin\left(\frac{\pi x_i}{l}\right) = \sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right)\right)^2$$

Finally the general solution of the system is:

$$\begin{cases} \text{a periodic regime:} & \sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right)\right)^2 \in \left]-\infty \quad \frac{A_1 m_r l}{2}\right[\cup \left]\frac{A_2 m_r l}{2} \quad +\infty\right[\\ \text{Critic regime:} & \sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right)\right)^2 = \left\{ \frac{A_1 m_r l}{2} \quad \frac{A_2 m_r l}{2} \right\} \\ \text{Pseudo periodic regime:} & \sum_{i=1}^{N} \left(\sin\left(\frac{\pi x_i}{l}\right)\right)^2 \in \left]0 \quad \frac{A_2 m_r l}{2}\right[\end{cases}$$

Second subsystem is a system with uniform motion (constant speed system) so it can be written as a first order system with an output $q_{t1}(t)$.

We can take:

$$\begin{cases} C_{zy} = \dfrac{\tau}{k} \\ K_{zy} = \dfrac{1}{k} \end{cases} \tag{28}$$

Where τ is the time constant and k is the static gain. The equation (21) can be written like (29) and the transfer function of this system is written like (30).

$$\frac{\tau}{k}\dot{q}_{t1}(t) + \frac{1}{k} q_{t1}(t) = P_{zr}(t) \tag{29}$$

$$H_b = \frac{q_{t1}(p)}{P_{zr}(p)} = \frac{k}{p\tau + 1} \tag{30}$$

The general transfer function vertical motion equation is the sum of (23) and (30):

$$H_z(p) = H_a(p) + H_b(p) \tag{31}$$

B. Lateral Motion Equation

Apply the same principle of the calculation in the previous part for the vertical motion equation but in this case a SISO (Simple Input Simple Output) system.

$$M_y \ddot{q}_{y1}(t) + C_y \dot{q}_{y1}(t) + K_y q_{y1}(t) = P_y(t)$$

The general solutions of the system are:

$$
\begin{cases}
\text{a periodic regime:} & \sum_{i=1}^{N}\left(\sin\left(\dfrac{\pi x_i}{l}\right)\right)^2 \in \left]-\infty\ \dfrac{B_1 m_r l}{2}\right[\cup \left]\dfrac{B_2 m_r l}{2}\ +\infty\right[\\[2ex]
\text{Critic regime:} & \sum_{i=1}^{N}\left(\sin\left(\dfrac{\pi x_i}{l}\right)\right)^2 = \left\{\dfrac{B_1 m_r l}{2}\ \dfrac{B_2 m_r l}{2}\right\} \\[2ex]
\text{Pseudo periodique regime:} & \sum_{i=1}^{N}\left(\sin\left(\dfrac{\pi x_i}{l}\right)\right)^2 \in \left]0\ \dfrac{B_2 m_r l}{2}\right[
\end{cases}
$$

C. Torsional Motion Equation

In this part also select the same principle as the previous ones. This subsystem is a SISO system.

$$M_t \ddot{q}_{t1}(t) + C_t \dot{q}_{t1}(t) + K_t q_{t1}(t) = P_t(t)$$

Or $Ct=0$ so $\varepsilon_t = 0$ always, in this case the general system is unamortized.

This part presented the detailed study of general system which is divided into three essential parts: the lateral, vertical and torsional motion equations. There is a resolution for each part of nonlinear system based on accurate mathematical calculation in different regimes.

V. FAULT DETECTION

After the system study this section focuses on the quantitative diagnosis which contains many mandatory steps to have good results. The error detection this step should make it possible to decide whether the system is in a state of normal running or not, defect location step where the fault elements defined and the decision making step where the incorrect operation of the system being found so the decision how to keep the desired performance of the system under surveillance. The diagnosis can be generated using different methods, a few methods have studied and shown in [16]-[20].

This work deals with one of the finest methods for generating residuals it's the Luenberger Observer (LO) [21]. To prove the efficiency and accuracy of this method, analysis was performed and a result was presented on the following of this paper.

Suppose the linear system affected by a sensor fault, an actuator fault and a measurement noise following:

$$
\begin{cases}
\dot{x}(t) = Ax(t) + Bu(t) + F_x f(t) + D_x d(t) \\
y(t) = Cx(t) + F_y f(t)
\end{cases}
\tag{32}
$$

A dynamic observer of this model given by:

$$
\begin{cases}
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - \hat{y}) \\
\hat{y}(t) = C\hat{x}(t)
\end{cases}
\tag{33}
$$

Where, L is the Observer gain.

The error is: $e_x(t) = x(t) - \hat{x}(t) \Leftrightarrow \dot{e}_x(t) = \dot{x}(t) - \dot{\hat{x}}(t)$

$$
\begin{aligned}
\dot{e}_x(t) &= Ax(t) + Bu(t) + F_x f(t) + D_x d(t) - A\hat{x}(t) - Bu(t) - L(y(t) - \hat{y}(t)) \\
&= Ax(t) + Bu(t) + F_x f(t) + D_x d(t) - A\hat{x}(t) - Bu(t) - L(y(t) - C\hat{x}(t)) \\
&= Ax(t) - A\hat{x}(t) - L(y(t) - C\hat{x}(t)) + F_x f(t) + D_x d(t) \\
&= Ax(t) - LCx(t) - LF_y f(t) + F_x f(t) + D_x(t)d(t) - A\hat{x}(t) + LC\hat{x}(t) \\
&= (A - LC)(x(t) - \hat{x}(t)) - LF_y f(t) + F_x f(t) + D_x(t)d(t) \\
&= (A - LC)e_x(t) + (F_x - LF_y)f(t) + D_x(t)d(t)
\end{aligned}
$$

$$\dot{e}_x(t) = (A - LC)e_x(t) + (F_x - LF_y)f(t) + D_x(t)d(t)$$

In the absence of default (f (t) =0) and neglecting the effect of the unknown inputs (d (t) =0), the estimation error becomes $\dot{e}_x(t) = (A - LC)e_x(t)$. The objective is $\lim_{t \to +\infty} e_x(t) = 0$, this is the case by calculating L.

The dynamic of the state estimation is the residue of the system:

$$r(t) = y(t) - \hat{y}(t) = C\tilde{x}(t)$$

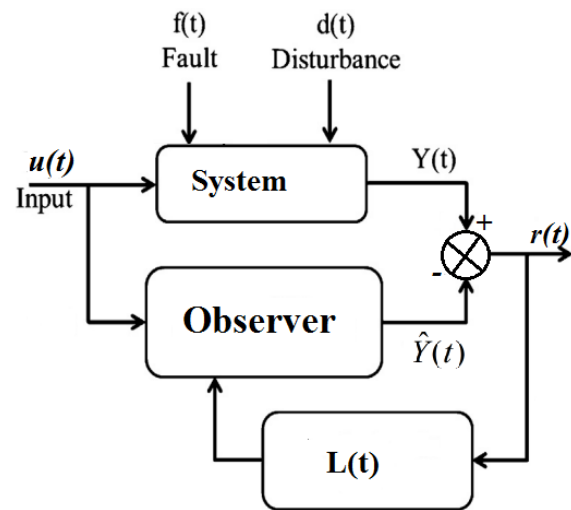As followed from Fig. 1 shown below, the principle of the generation of residues based on the observer.

Fig. 1. The principle of the generation of residues based on the observer.

The knowledge that the system contains a defect or not the residue value must be determined, the general conditions of residue presented in (34).

$$
\begin{cases}
r(t) \neq 0 & \text{if the default } f(t) \neq 0 \\
r(t) = 0 & \text{if the default } f(t) = 0
\end{cases}
\tag{34}
$$

VI. APPLICATIONS AND SIMULATIONS

A. The Global Model

This part presented the model and study simulations based on the MATLAB program.

Fig. 2 shows the three global outputs of the railway system model, vertical, lateral and torsional displacement.

The first output is the vertical movement; it is a damped system as Fig. 2(a) shows. It is amortized signal with maximum value in the begin movement equal 120 mm, with damping value C=0.0713 and stiffness value K=1.1891.

The second output is the lateral movement it's a damped system also as Fig. 2(b) shows, but it is more frequency than the first. It is amortized signal with maximum value in the begin movement equal 10 mm, with damping value C= 0.6454 and stiffness value K= 25.8164.

The third output is the torsional movement, it is unamortized system as Fig. 2(c) shows, but more frequency than the other outputs. It's unamortized signal with maximum value of movement equal 100 mm, with damping value C= 0 and stiffness value K= 173.7861.

a) The vertical displacement of the rail.

b) The lateral displacement of the rail.

c) The torsional displacement of the rail.
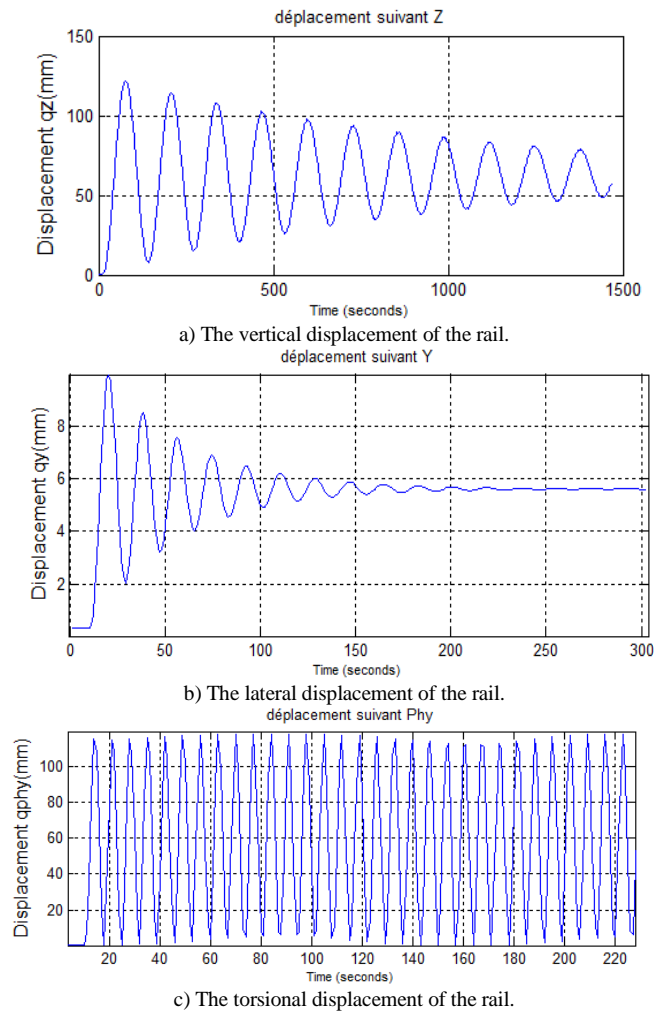
Fig. 2. The railway model system.

B. The Model Study

Some examples of numerical results in a few positions on the rail are studied such cases are depicted in the following figures:

- First case is the initial position of the system (X= 0).

Damping coefficients are $\varepsilon_z = 0$, $\varepsilon_y = 0$ and $\varepsilon_t = 0$. The natural frequency are $Wz = 0.0043$, $Wy = 0.0047$ and $Wt = 0.2602$.

The Transfer function Hz (vertical movement), Hy (Lateral movement) and Ht (Torsional movement) respectively like these:

$$Hz(p) = \frac{1}{p^2 + 1.83310^{-5}}, \; Hy(p) = \frac{1}{p^2 + 2.210^{-5}}$$

and $Ht(p) = \dfrac{1}{p^2 + 0.067}$

Fig. 3 illustrates the transfer functions of three motions directions. Like the simulations present they are unamortized then the system is in a Critic regime.
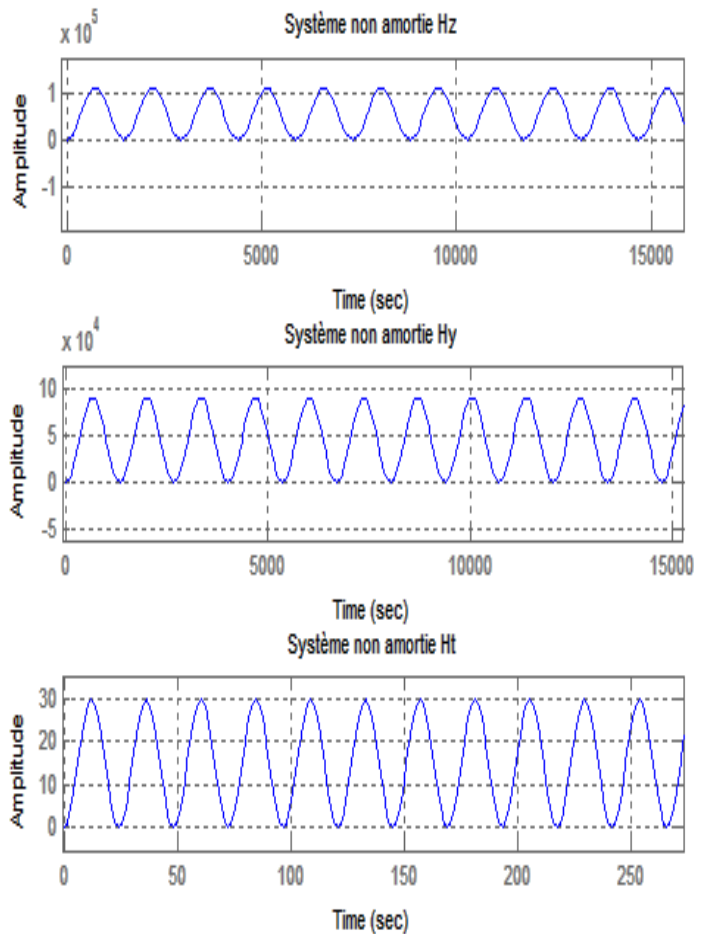

Fig. 3. Transfer function of the system in the first case.

- Second case is the seventh position for the system (X = 7).

Damping coefficients are $\varepsilon_z = 0.539$ and $\varepsilon_y = 0.305$. The natural frequency are $Wz = 1.295$, $Wy = 0.915$. The Transfer function Hz and Hy, respectively like these:

$$Hz(p) = Ha(p) + Hb(p) \quad \text{where} \begin{cases} Ha(p) = \dfrac{1}{p^2 + 1.398p + 1.678} \\ Hb(p) = \dfrac{1}{0.833p + 1.678} \end{cases}$$

$$Hy(p) = \frac{1}{p^2 + 0.559p + 0.838}$$

The simulation of these transfer functions illustrated in Fig. 4. Like these simulations present they are in a pseudo periodic regime then the system is amortized.
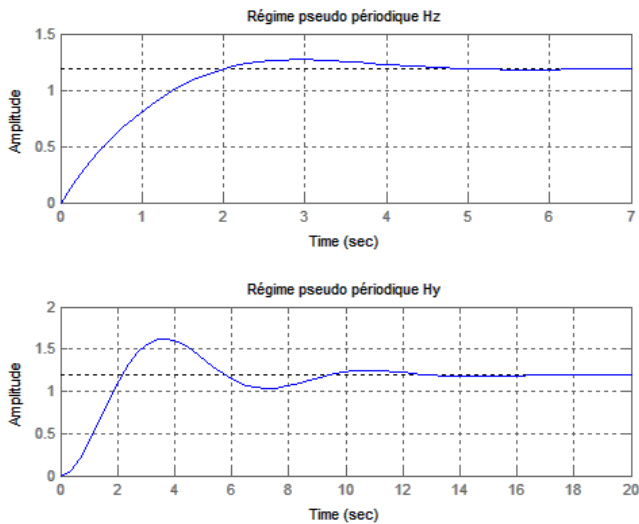

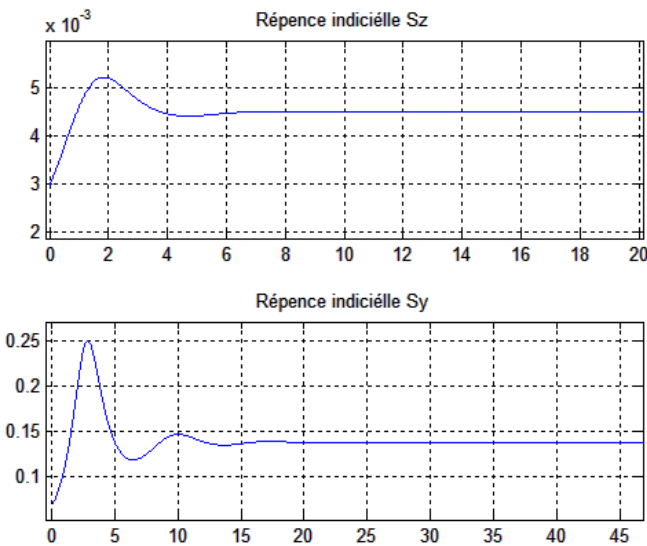Fig. 4. Transfer function of the system in the second case.

Fig. 5. The Step response of the system (Sz and Sy) in the second case.

The step response of this system with step unit function input is presented in Fig. 5.

- Third case is the final position for the system (X=30).

Damping coefficients in this case are $\varepsilon_y = 2.2045$ and $\varepsilon_z = 3.8971$. The natural frequency are $Wz = 9.3531$, $Wy = 6.6136$.

The Transfer function Hz and Hy:

$$Hz(p) = Ha(p) + Hb(p) \quad \text{where} \begin{cases} Ha(p) = \dfrac{1}{p^2 + 72.9p + 87.48} \\ Hb(p) = \dfrac{1}{0.83p + 87.48} \end{cases}$$

$$Hy(p) = \frac{1}{p^2 + 29.16p + 43.74}$$

The simulations of these transfer functions are given in Fig. 6. From this figure it can be see that the global system become a first order system and the simulation are in a periodic regime.
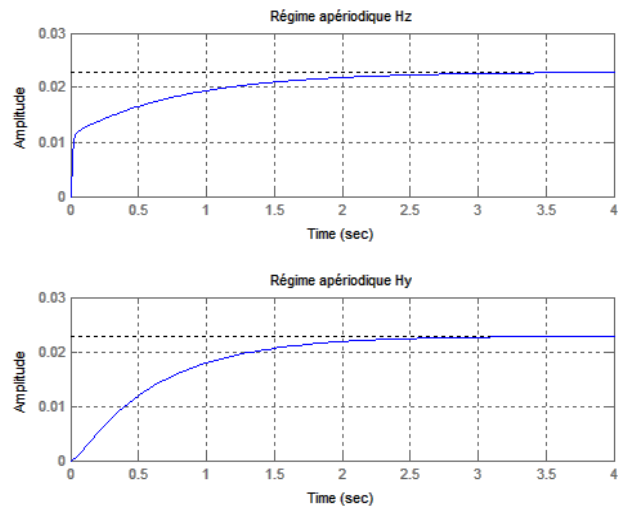

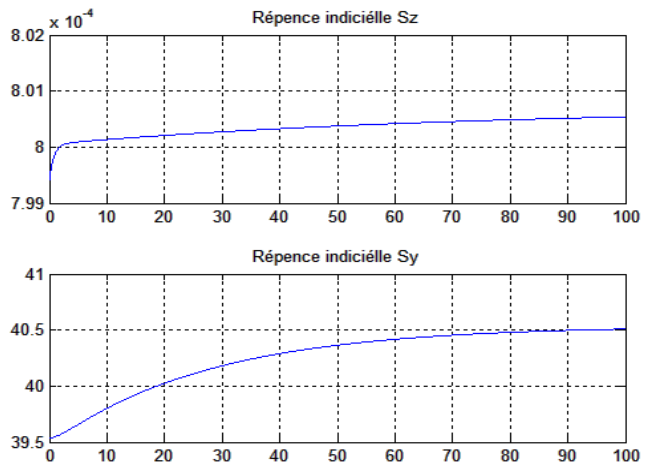Fig. 6. Transfer function of the system in the third case.

Fig. 7. The step response of the system (Sz,St) in the third case.

With the input of the system is step unit function, the step response being found in Fig. 7.

C. The Fault Detection

To calculate and isolate the system fault, first it must calculate the global system model which is done in the following. First there is a model calculation then an application for the LO, the numerical results are presented and the simulations are done.

The general model (19) demonstrate that the railway system have three defaults, one default on each side of vibration f(t). The simulation in this paper with no disturbance (d (t) =0).

To complete this work it must linearize the system to apply the luenberger observer. So after the linearization, the railway system described by the following state representation:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + Bu(t) \\ \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + F_y f(t) \end{cases}$$

Matrices of the state are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -K_z & -C_z & 0 & 0 & -K_{zy} & -C_{zy} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -K_y & -C_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -K_t & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, F_y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

$$\text{and } u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}$$

In the case N=20 the numerical values are founded as follow:

$K_z = 0.4951, C_z = 0.0297, K_{zy} = 0.6940, C_{zy} = 0.0416,$
$K_y = 25.8164, C_y = 0.6454, K_t = 173.7861.$

The state representation of system in the expanded form:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = A_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + B_1 U(t) \\ \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = C_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + D_1 U(t) \end{cases}$$

$$A_1 = A, B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, C_1 = C,$$

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

The actual structure of the LO is:

$$\begin{cases} \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{\hat{x}}_5 \\ \dot{\hat{x}}_6 \end{bmatrix} = A_{ob} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{x}_5 \\ \hat{x}_6 \end{bmatrix} + B_{ob} U(t) \\ \\ \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} = C_{ob} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{x}_5 \\ \hat{x}_6 \end{bmatrix} \end{cases}$$

$A_{ob} = A - LC, B_{ob} = \begin{bmatrix} B & L \end{bmatrix}$ and $C_{ob} = C$

The eigenvalues of A_{ob} are $\lambda_1 = -5, \lambda_2 = -9, \lambda_3 = -6, \lambda_4 = -5,$ $\lambda_5 = -2$ and $\lambda_6 = -10$. These eigenvalues gives the gain Matrix L.

$$L = \begin{bmatrix} 11.6844 & -1.4550 & -1.0819 \\ 19.7897 & -4.1470 & -9.1942 \\ -1.2283 & 6.9358 & -0.7012 \\ -2.4554 & -22.2438 & -2.7048 \\ -1.2962 & -0.8920 & 13.7047 \\ -9.2933 & -4.3545 & -130.8955 \end{bmatrix}$$

Therefore the numerical values of the state matrices for LO are:

$$\begin{cases} A_{ob} = \begin{bmatrix} -11.6844 & 1 & 1.4550 & 0 & 1.0819 & 0 \\ -20.2848 & -0.0297 & 4.1470 & 0 & 8.5002 & -0.0416 \\ 1.2283 & 0 & -6.9358 & 1 & 0.7012 & 0 \\ 2.4554 & 0 & -3.5725 & -0.6454 & 2.7048 & 0 \\ 1.2962 & 0 & 0.8920 & 0 & -13.7047 & 1 \\ 9.2933 & 0 & 4.3545 & 0 & -42.8907 & 0 \end{bmatrix} \\[2em] B_{ob} = \begin{bmatrix} 0 & 0 & 0 & 11.6844 & -1.4550 & -1.0819 \\ 1 & 0 & 0 & 19.7897 & -4.1470 & -9.1942 \\ 0 & 0 & 0 & -1.2283 & 6.9358 & -0.7012 \\ 0 & 1 & 0 & -2.4554 & -22.2438 & -2.7048 \\ 0 & 0 & 0 & -1.2962 & -0.8920 & 13.7047 \\ 0 & 0 & 1 & -9.2933 & -4.3545 & -130.8955 \end{bmatrix} \\[2em] C_{ob} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{cases}$$

The system provides three outputs $q_z(t)$, $q_y(t)$ and $q_t(t)$ are respectively the vertical, lateral and torsional displacement of rail because the vibration. The simulation of the real and estimate outputs of the railway system then the simulation of the residual of each default in the following of the paper finally the conclusion of fault detection part.

In the simulation, the default value, $f(t) = \begin{bmatrix} 0.5 \\ -0.6 \\ 0.1 \end{bmatrix}$.

Fig. 8 defines the real and estimate vertical displacements of rail they are respectively Y1r and Y1es.
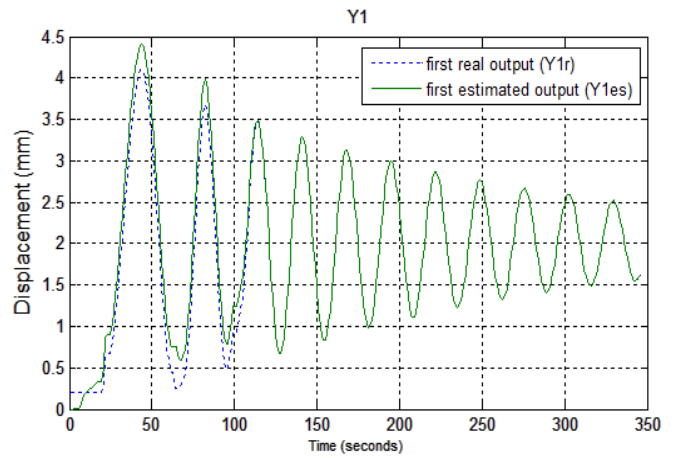

Fig. 8. Real and estimated outputs.

Fig. 9 indicates the real and estimate lateral displacements of rail they are respectively Y2r and Y2es.
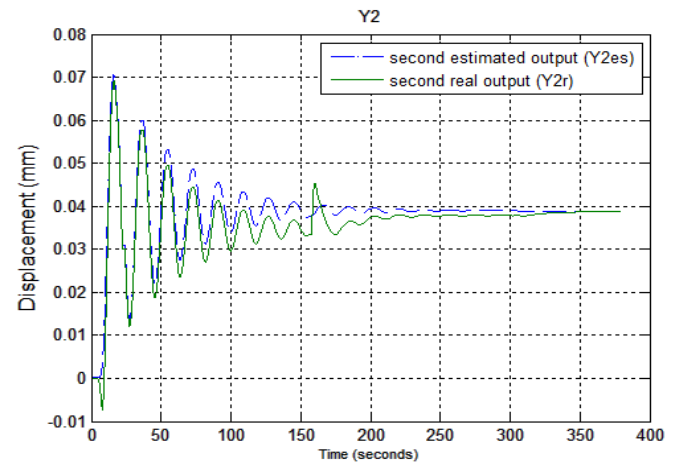

Fig. 9. Real and estimate second outputs.

Fig. 10 depicts the real and estimate torsional displacements of rail they are respectively Y3r and Y3es.
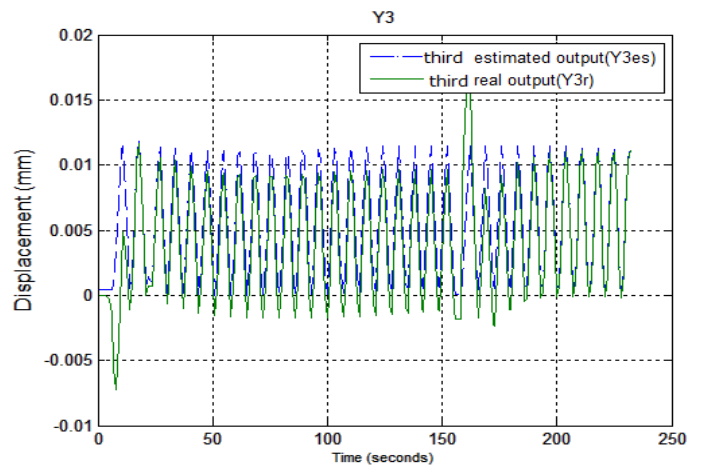

Fig. 10. Real and estimate third outputs.

This fault case of $f(t)$ gives a residue value to each output of the railway system, these values are described in the simulations of the following figures.

The fault on the first output in this case where:

$$f_1(t) = \begin{cases} 0.5 & \text{if } t=500s \\ 0 & \text{elsewhere} \end{cases}$$

This case is shown in Fig. 11 where the result of the residue r1 is sensitive to the fault.
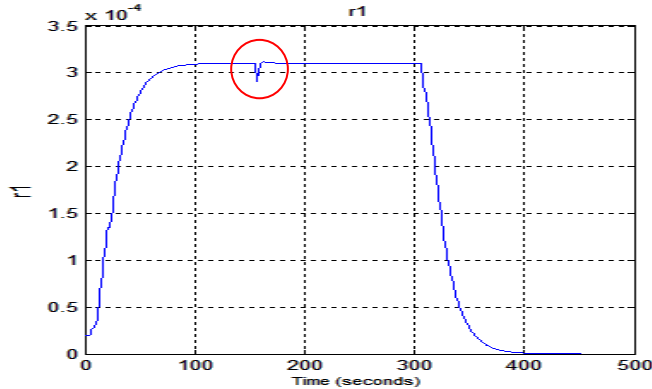

Fig. 11. The residue r1 with default $f_1(t)$.

Now the simulation for the effect of the second value $f_2(t)$ on the residue r2 of second output found is shown in Fig. 12.

$$f_2(t) = \begin{cases} -0.6 & t=\{10s \text{ and } 301s\} \\ 0 & \text{elsewhere} \end{cases}$$
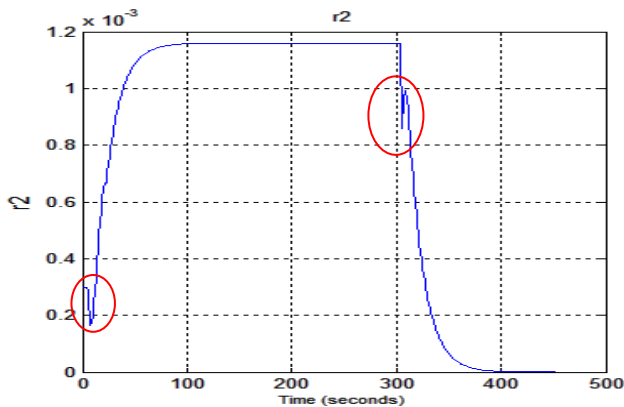

Fig. 12. Residual r2 with default $f_2(t)$.

Finally, the case of the effect of the third default on third residue r3 of the third output is:

$$f_3(t) = \begin{cases} 0.1 & t=57s \\ 0 & \text{elsewhere} \end{cases}$$

With this value of default, Fig. 13 shows the result of the residual r3 is sensitive to the fault $f_3(t)$.
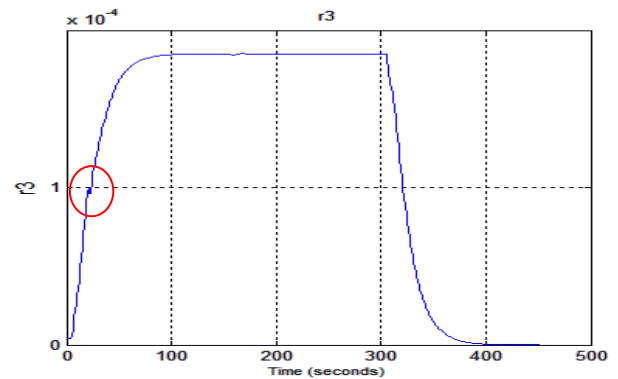

Fig. 13. Residual r3 with fault $f_3(t)$.

VII. CONCLUSION

Based on the results, it can be concluded that the research into the detection and isolation the default of a railway system has been very successful. A complete model for simulating dynamic interactions between rail and wheels has been presented. An explicit integration algorithm has been employed to numerically solve the equations of motion of the system. After linearization, is studied the linear system and numerical results presented in this work. With the model of railway system we made an embedded system for the robust prediction and diagnosis that can guarantee the security of railway system. A Luenberger Observer (LO) used to generate residuals for detection of faults of a MIMO system.

The future research will concentrate on detection of default based on nonlinear local observer for the railway system and prediction for this default. The future work based on the Local exponential Observer and will apply the results of Sundarapandian research on this type of observer [32], [33].

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